STANDARD ADTS

Lecture 17

CS2110 – Fall 2013

Textbook reference

- Stacks: Chapters 5,6
- Queues: Chapters 10,11

- Self-test problem: Suppose that you are given a list of Integers. Using a foreach loop you run down the list, pushing each element onto a stack. Now, you create a new List<Integer> and item by item, pop items from the stack and add them to the end of your new list. What will the list contain when you are done?

Abstract Data Types (ADTs)

- A method for achieving abstraction for data structures and algorithms
- ADT = model + operations
- Describes what each operation does, but not how it does it
- An ADT is independent of its implementation

- In Java, an interface corresponds well to an ADT
  - The interface describes the operations, but says nothing at all about how they are implemented
- Example: Stack interface/ADT

```
public interface Stack {
  public void push(Object x);
  public Object pop();
  public Object peek();
  public boolean isEmpty();
  public void clear();
}
```

Queues & Priority Queues

- ADT Queue
  - Operations:
    ```
    void add(Object x);
    Object poll();
    Object peek();
    boolean isEmpty();
    void clear();
    ```
- Where used:
  - Simple job scheduler (e.g., print queue)
  - Wide use within other algorithms

- ADT PriorityQueue
  - Operations:

```
void insert(Object x);
Object getMax();
Object peekAtMax();
boolean isEmpty();
void clear();
```

- Where used:
  - Job scheduler for OS
  - Event-driven simulation
  - Can be used for sorting
  - Wide use within other algorithms

A (basic) queue is "first in, first out". A priority queue ranks objects: getMax() returns the "largest" according to the comparator interface.

Sets

- ADT Set
  - Operations:

```
void insert(Object element);
boolean contains(Object element);
void remove(Object element);
boolean isEmpty();
void clear();
for(Object o: mySet) { ... }
```

- Where used:
  - Wide use within other algorithms

- Note: no duplicates allowed
  - A “set” with duplicates is sometimes called a *multiset* or *bag*

A set makes no promises about ordering, but you can still iterate over it.

Dictionaries

- ADT Dictionary (aka Map)
  - Operations:
    - `void insert(Object key, Object value);`
    - `void update(Object key, Object value);`
    - `Object find(Object key);`
    - `void remove(Object key);`
    - `boolean isEmpty();`
    - `void clear();`

- Think of: key = word; value = definition
- Where used:
  - Symbol tables
  - Wide use within other algorithms

A HashMap is a particular implementation of the Map interface

Data Structure Building Blocks

- These are *implementation* "building blocks" that are often used to build more-complicated data structures
  - Arrays
  - Linked Lists
    - Singly linked
    - Doubly linked
  - Binary Trees
  - Graphs
    - Adjacency matrix
    - Adjacency list

From interface to implementation

- Given that we want to support some interface, the designer still faces a choice
  - What will be the best way to implement this interface for my expected type of use?
  - Choice of implementation can reflect many considerations

- Major factors we think about
  - Speed for typical use case
  - Storage space required

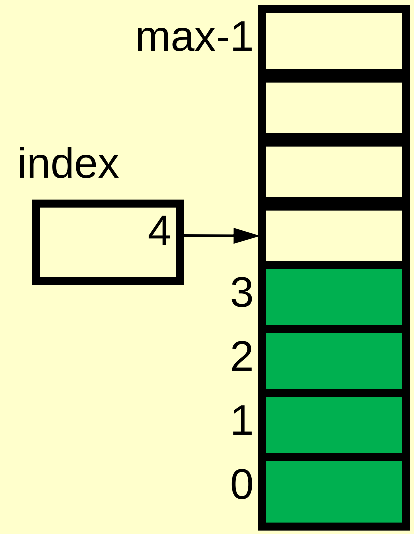
Array Implementation of Stack


```
class ArrayStack implements Stack {

   private Object[] array; //Array that holds the Stack
   private int index = 0; //First empty slot in Stack

   public ArrayStack(int maxSize)
      { array = new Object[maxSize]; }

   public void push(Object x) { array[index++] = x; }
   public Object pop() { return array[--index]; }
   public Object peek() { return array[index-1]; }
   public boolean isEmpty() { return index == 0; }
   public void clear() { index = 0; }
}
```



O(1) worst-case time for each operation

Question: What can go wrong?

…. What if maxSize is too small?

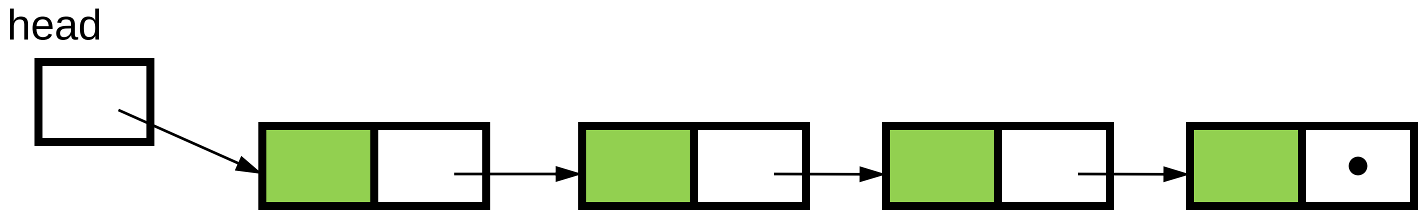
Linked List Implementation of Stack


```
class ListStack implements Stack {
   private Node head = null;  //Head of list that
                              //holds the Stack

   public void push(Object x) { head = new Node(x, head); }
   public Object pop() {
      Node temp = head;
      head = head.next;
      return temp.data;
   }
   public Object peek() { return head.data; }
   public boolean isEmpty() { return head == null; }
   public void clear() { head = null; }
}
```

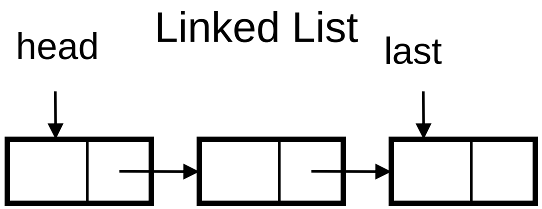
O(1) worst-case time for each operation (but constant is larger)

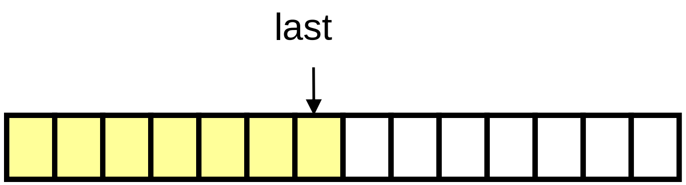
Note that array implementation can overflow, but the linked list version cannot



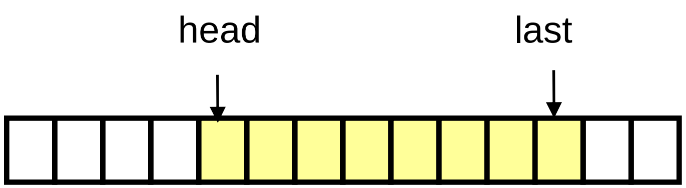
Queue Implementations

- Possible implementations





Array with head always at A[0]
(poll( ) becomes expensive)
(can overflow)



Array with wraparound
(can overflow)

- Recall: operations are **`add`**, **`poll`**, **`peek`**,…

- For linked-list
  - All operations are O(1)

- For array with head at A[0]
  - poll takes time O(n)
  - Other ops are O(1)
  - Can overflow

- For array with wraparound
  - All operations are O(1)
  - Can overflow

A Queue From 2 Stacks

- Add pushes onto stack A
- Poll pops from stack B
- If B is empty, move all elements from stack A to stack B
- Some individual operations are costly, but still O(1) time per operations over the long run

Dealing with Overflow

- For array implementations of stacks and queues, use *table doubling*
- Check for overflow with each insert op
- If table will overflow,
  - Allocate a new table twice the size
  - Copy everything over
- The operations that cause overflow are expensive, but still constant time per operation over the long run (proof later)

Goal: Design a *Dictionary* (aka *Map*)

- Operations
  - `void insert(key, value)`
  - `void update(key, value)`
  - `Object find(key)`
  - `void remove(key)`
  - `boolean isEmpty()`
  - `void clear()`

Array implementation: Using an array of (key,value) pairs

	Unsorted	Sorted
insert	O(1)	O(n)
update	O(n)	O(log n)
find	O(n)	O(log n)
remove	O(n)	O(n)

n is the number of items currently held in the dictionary

Hashing

- Idea: compute an array index via a *hash function* h
- U is the universe of keys
- h: U → [0,…,m-1] where m = hash table size
- Usually |U| is much bigger than m, so *collisions* are possible (two elements with the same hash code)
- h should
  - be easy to compute
  - avoid collisions
  - have roughly equal probability for each table position

Typical situation:

U = all legal identifiers

Typical hash function:

h converts each letter to a number, then compute a function of these numbers

Best hash functions are highly random

This is connected to cryptography

We'll return to this in a few minutes

A Hashing Example

- Suppose each word below has the following hashCode
  - jan 7
  - feb 0
  - mar 5
  - apr 2
  - may 4
  - jun 7
  - jul 3
  - aug 7
  - sep 2
  - oct 5

- How do we resolve collisions?
  - use chaining: each table position is the head of a list
  - for any particular problem, this *might* work terribly

- In practice, using a good hash function, we can assume each position is equally likely

Analysis for Hashing with Chaining

- Analyzed in terms of *load factor* λ = n/m = (items in table)/(table size)
- We count the expected number of *probes* (key comparisons)
- Goal: Determine expected number of probes for an *unsuccessful* search

- Expected number of probes for an unsuccessful search = average number of items per table position = n/m = λ
- Expected number of probes for a *successful* search = 1 + λ = O(λ)
- Worst case is O(n)

Table Doubling

- We know each operation takes time O(λ) where λ λ =n/m
- So it gets worse as n gets large relative to m
- Table Doubling:
- Set a bound for λ (call it λ_0)
- Whenever λ reaches this bound:
  - Create a new table twice as big
  - Then rehash all the data
- As before, operations *usually* take time O(1)
  - But sometimes we copy the whole table

Analysis of Table Doubling

- Suppose we reach a state with n items in a table of size m and that we have just completed a table doubling

	Copying Work
Everything has just been copied	n inserts
Half were copied previously	n/2 inserts
Half of those were copied previously	n/4 inserts
…	…
Total work	n + n/2 + n/4 + … = 2n

Analysis of Table Doubling, Cont'd

- Total number of insert operations needed to reach current table = copying work + initial insertions of items = 2n + n = 3n inserts

- Each insert takes expected time $O(\lambda_0)$ or O(1), so total expected time to build entire table is O(n)

- Thus, expected time per operation is O(1)

- Disadvantages of table doubling:
  - Worst-case insertion time of O(n) is definitely achieved (but rarely)
  - Thus, not appropriate for time critical operations

Concept: "hash" codes

- Definition: a *hash code* is the output of a function that takes some input and maps it to a pseudo-random number (a *hash*)
  - Input could be a big object like a string or an Animal or some other complex thing
  - Same input always gives same out
  - Idea is that hashCode for distinct objects will have a very low likelihood of collisions
- Used to create *index* data structures for finding an object given its hash code

Java Hash Functions

- Most Java classes implement the **`hashCode()`** method
- **`hashCode()`** returns an int
- Java's **`HashMap`** class uses h(X) = X.hashCode() mod m
- h(X) in detail:
  - `int hash = X.hashCode();`
  - `int index = (hash & 0x7FFFFFFF) % m;`

- What **`hashCode()`** returns:
  - Integer:
    - uses the int value
  - Float:
    - converts to a bit representation and treats it as an int
  - Short Strings:
    - 37*previous + value of next character
  - Long Strings:
    - sample of 8 characters; 39*previous + next value

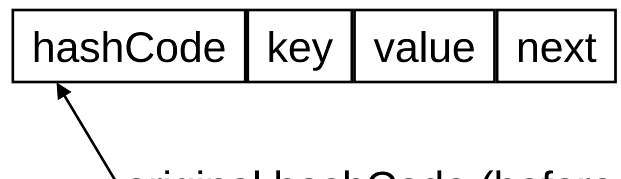
`hashCode()` Requirements

- Contract for **`hashCode()`** method:
  - Whenever it is invoked in the same object, it must return the same result
  - Two objects that are equal (in the sense of **`.equals(...)`**) must have the same hash code
  - Two objects that are not equal should return different hash codes, but are not required to do so (i.e., collisions are allowed)

Hashtables in Java

- `java.util.HashMap`
- `java.util.HashSet`
- `java.util.Hashtable`

- A node in each chain looks like this:



original hashCode (before mod m)
Allows faster rehashing and (possibly) faster key comparison

- Use chaining
- Initial (default) size = 101
- Load factor = $\lfloor_{0}$ = 0.75
- Uses table doubling (2*previous+1)

Linear & Quadratic Probing

- These are techniques in which all data is stored directly within the hash table array

- Linear Probing
  - Probe at h(X), then at
    - h(X) + 1
    - h(X) + 2
    - …
    - h(X) + i
  - Leads to *primary clustering*
    - Long sequences of filled cells

- Quadratic Probing
  - Similar to Linear Probing in that data is stored within the table
  - Probe at h(X), then at
    - h(X)+1
    - h(X)+4
    - h(X)+9
    - …
    - h(X)+ i^2
  - Works well when
    - $\lambda < 0.5$
    - Table size is prime

Universal Hashing

- In in doubt, choose a hash function at random from a large parameterized family of hash functions (e.g., $h(x) = ax + b$, where a and b are chosen at random)
  - With high probability, it will be just as good as any custom-designed hash function you dream up

Dictionary Implementations

- Ordered Array
  - Better than unordered array because Binary Search can be used
- Unordered Linked List
  - Ordering doesn’t help
- Hashtables
  - O(1) expected time for Dictionary operations

Aside: Comparators

- When implementing a comparator interface you normally must
  - Override compareTo() method
  - Override hashCode()
  - Override equals()

- Easy to forget and if you make that mistake your code will be very buggy

`hashCode()` and `equals()`

- We mentioned that the hash codes of two equal objects must be equal — this is necessary for hashtable-based data structures such as **`HashMap`** and **`HashSet`** to work correctly

- In Java, this means if you override **`Object.equals()`**, you had better also override **`Object.hashCode()`**

- But how???

hashCode() and equals()


```
class Identifier {
   String name;
   String type;

   public boolean equals(Object obj) {
      if (obj == null) return false;
      Identifier id;
      try {
         id = (Identifier)obj;
      } catch (ClassCastException cce) {
         return false;
      }
      return name.equals(id.name) && type.equals(id.type);
   }





}
```

hashCode() and equals()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   String name;
   String type;

   public boolean equals(Object obj) {
      if (obj == null) return false;
      Identifier id;
      try {
         id = (Identifier)obj;
      } catch (ClassCastException cce) {
         return false;
      }
      return name.equals(id.name) && type.equals(id.type);
   }

   public int hashCode() {
      return 37 * name.hashCode() + 113 * type.hashCode() + 42;
   }
}
```

hashCode() and equals()


```
class TreeNode {
   TreeNode left, right;
   String datum;

   public boolean equals(Object obj) {
      if (obj == null || !(obj instanceof TreeNode)) return false;
      TreeNode t = (TreeNode)obj;
      boolean lEq = (left != null)?
         left.equals(t.left) : t.left == null;
      boolean rEq = (right != null)?
         right.equals(t.right) : t.right == null;
      return datum.equals(t.datum) && lEq && rEq;
   }
}
```

hashCode() and equals()


```
class TreeNode {
   TreeNode left, right;
   String datum;

   public boolean equals(Object obj) {
      if (obj == null || !(obj instanceof TreeNode)) return false;
      TreeNode t = (TreeNode)obj;
      boolean lEq = (left != null)?
         left.equals(t.left) : t.left == null;
      boolean rEq = (right != null)?
         right.equals(t.right) : t.right == null;
      return datum.equals(t.datum) && lEq && rEq;
   }

   public int hashCode() {
      int lHC = (left != null)? left.hashCode() : 298;
      int rHC = (right != null)? right.hashCode() : 377;
      return 37 * datum.hashCode() + 611 * lHC - 43 * rHC;
   }
}
```

Professional quality hash codes?

- For large objects we often compute an MD5 hash
  - MD5 is the fifth of a series of standard "message digest" functions
  - They are fast to compute (like an XOR over the bytes of the object)
  - But they also use a cryptographic key: without the key you can't guess what the MD5 hashcode will be
    - For example key could be a random number you pick when your program is launched
    - Or it could be a password
- With a password key, an MD5 hash is a "proof of authenticity"
  - If object is tampered with, the hashcode will reveal it!