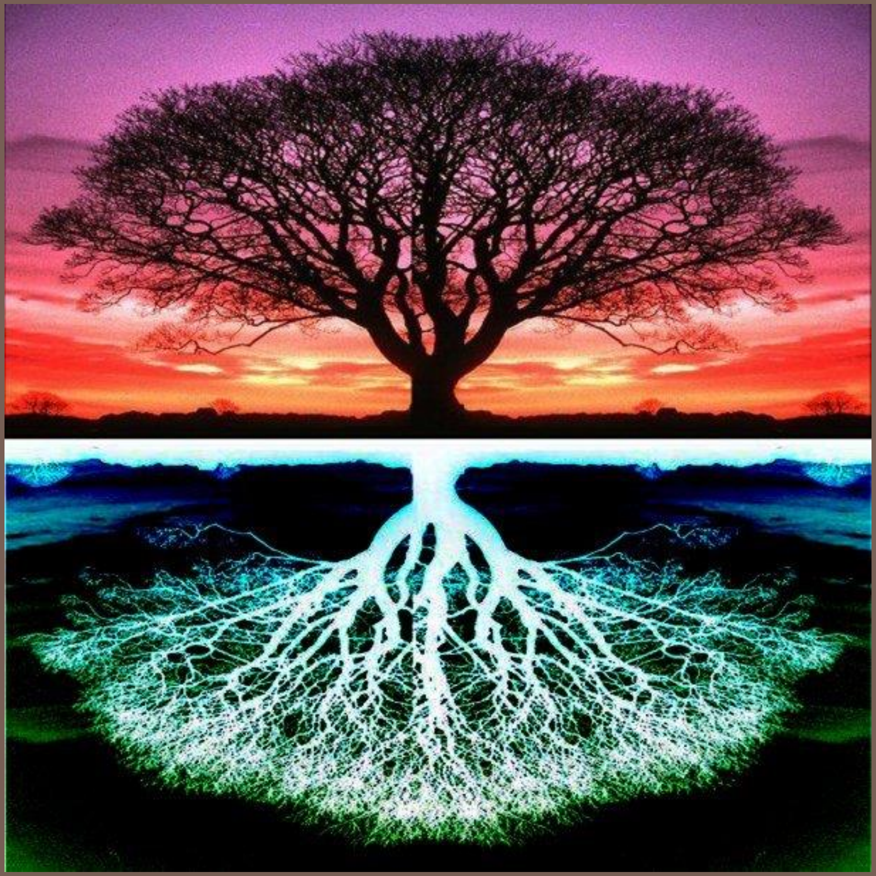


TREES

Lecture 10

CS2110 – Fall 2013

Readings and Homework

- Textbook, Chapter 23, 24

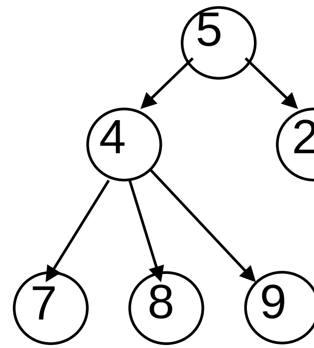
- Homework: A thought problem (draw pictures!)
  - Suppose you use trees to represent student schedules. For each student there would be a general tree with a root node containing student name and ID. The inner nodes in the tree represent courses, and the leaves represent the times/places where each course meets. Given two such trees, how could you determine whether and where the two students might run into one-another?

Which Oct 10 exam session?

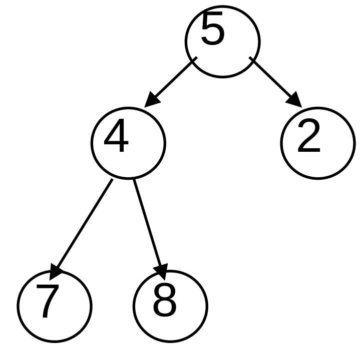
- Spoke to Amy about a conflict? Follow her instructions.
- Your student ID is an even number? 116 Kennedy Hall, "Call" Auditorium. 7:30-9:00pm.
- ***Your student ID is an odd number? 116 Kennedy Hall, "Call" Auditorium. 5:30-7:00pm***
  - ***Your ID # is odd, but you have a conflict with this earlier time? Contact Amy; with her permission you can come at 7:30-9:00pm.***

Tree Overview

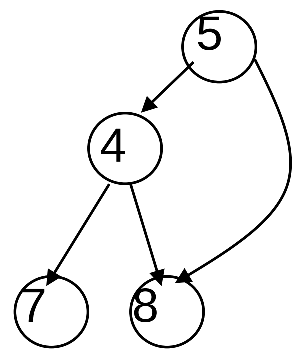
- *Tree*: recursive data structure (similar to list)
  - Each cell may have zero or more *successors* (children)
  - Each cell has exactly one *predecessor* (parent) except the *root*, which has none
  - All cells are reachable from *root*
- *Binary tree*: tree in which each cell can have at most two children: a left child and a right child



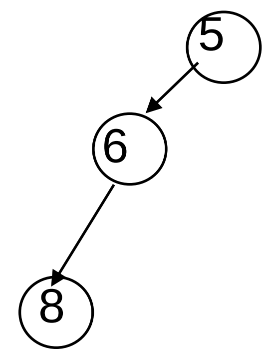
General tree



Binary tree



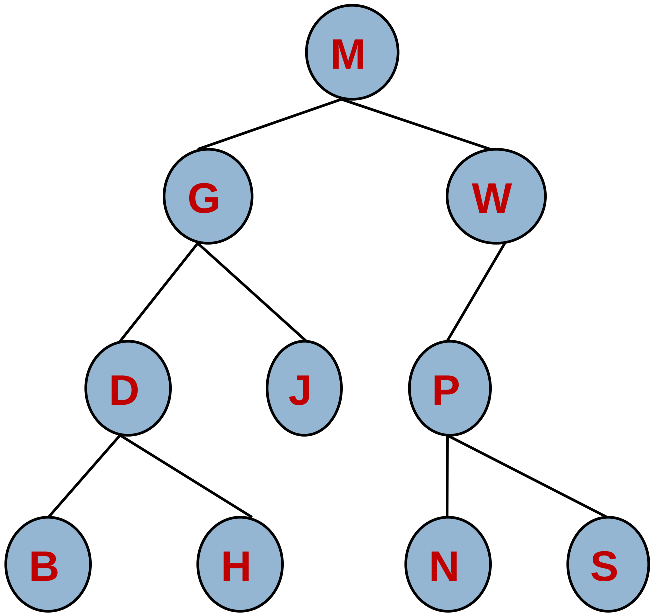
Not a tree



List-like tree

Tree Terminology

- M is the *root* of this tree
- G is the *root* of the *left subtree* of M
- B, H, J, N, and S are *leaves*
- N is the *left child* of P; S is the *right child*
- P is the *parent* of N
- M and G are *ancestors* of D
- P, N, and S are *descendants* of W
- Node J is at *depth* 2 (i.e., *depth* = length of path from root = number of edges)
- Node W is at *height* 2 (i.e., *height* = length of longest path to a leaf)
- A collection of several trees is called a ...?



Class for Binary Tree Cells


```
class TreeCell<T> {
   private T datum;
   private TreeCell<T> left, right;

   public TreeCell(T x) { datum = x; }
   public TreeCell(T x, TreeCell<T> lft, TreeCell<T> rgt) {
      datum = x;
      left = lft;
      right = rgt;
   }
   more methods: getDatum, setDatum, getLeft, setLeft,
                 getRight, setRight
}
```

Points to left subtree

Points to right subtree

Constructor: datum x, no children

Constructor: datum x and children

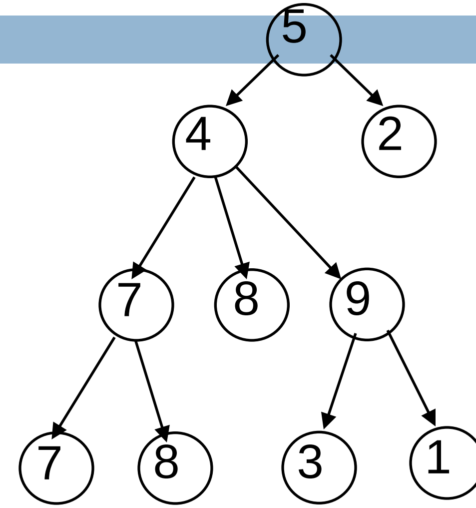
```
... new TreeCell<String>("hello") ...
```

Binary versus general tree

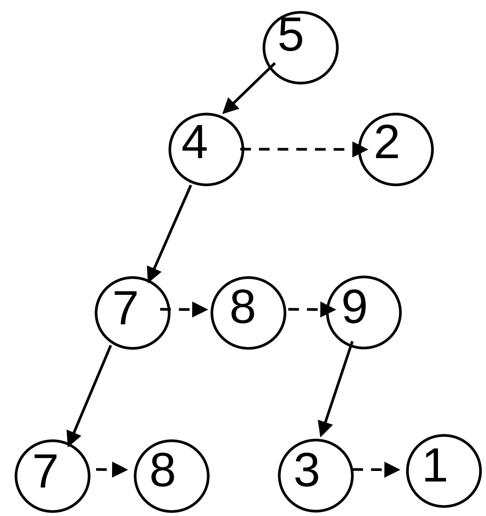
- In a binary tree each node has exactly two pointers: to the left subtree and to the right one
  - Of course one or both could be *null*
- In a general tree a node can have any number of child nodes
  - Very useful in some situations …
  - … one of which will be our assignments!

Class for General Tree nodes


```
class GTreeCell {
   private Object datum;
   private GTreeCell left;
   private GTreeCell sibling;

   appropriate getter and
   setter methods
}
```



General tree



Tree represented using GTreeCell

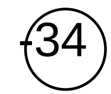
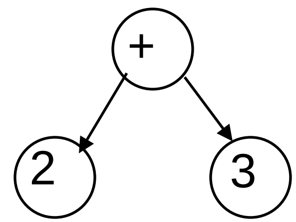
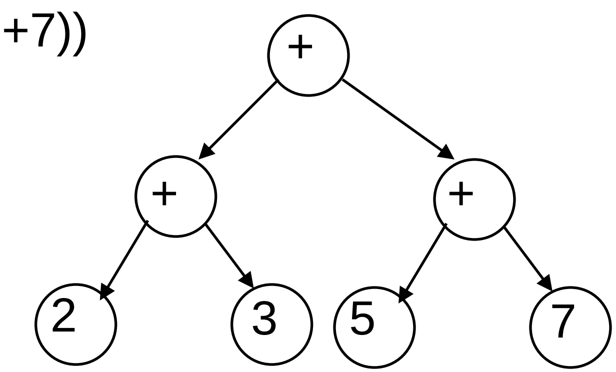
- Parent node points directly only to its leftmost child
- Leftmost child has pointer to next sibling, which points to next sibling, etc.

Applications of Trees

- Most languages (natural and computer) have a recursive, hierarchical structure
- This structure is *implicit* in ordinary textual representation
- Recursive structure can be made *explicit* by representing sentences in the language as trees: Abstract Syntax Trees (ASTs)
- ASTs are easier to optimize, generate code from, etc. than textual representation
- A parser converts textual representations to AST

Example

- Expression grammar:
  - E ⟶ integer
  - E ⟶ (E + E)
- In textual representation
  - Parentheses show hierarchical structure
- In tree representation
  - Hierarchy is explicit in the structure of the tree

Text	AST Representation
-34	
(2 + 3)	
((2+3) + (5+7))	

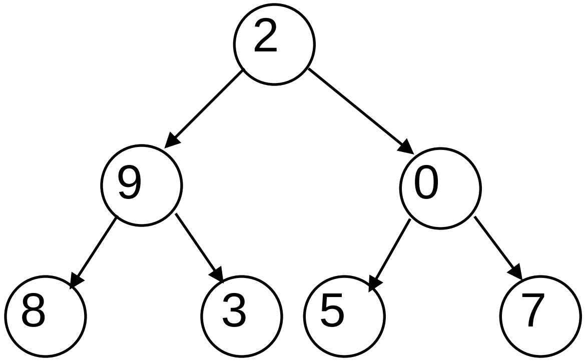
Recursion on Trees

- Recursive methods can be written to operate on trees in an obvious way
- Base case
  - empty tree
  - leaf node
- Recursive case
  - solve problem on left and right subtrees
  - put solutions together to get solution for full tree

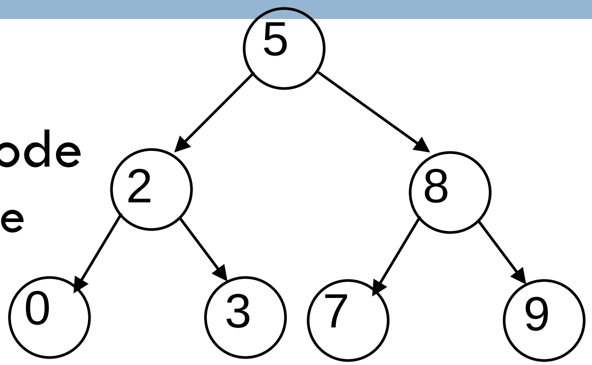
Searching in a Binary Tree


```
/** Return true iff x if the datum in a cell of tree node */
public static boolean treeSearch(Object x, TreeCell node) {
   if (node == null) return false;
   if (node.datum.equals(x)) return true;
   return treeSearch(x, node.left) ||
          treeSearch(x, node.right);
}
```

- Analog of linear search in lists: given tree and an object, find out if object is stored in tree
- Easy to write recursively, harder to write iteratively



Binary Search Tree (BST)

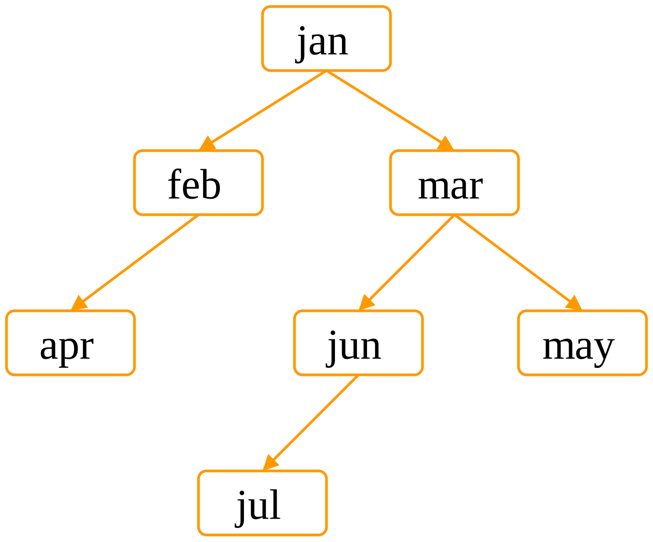


- If the tree data are *ordered* – in any subtree,
  - All *left* descendents of node come *before* node
  - All *right* descendents of node come *after* node
- Search is MUCH faster

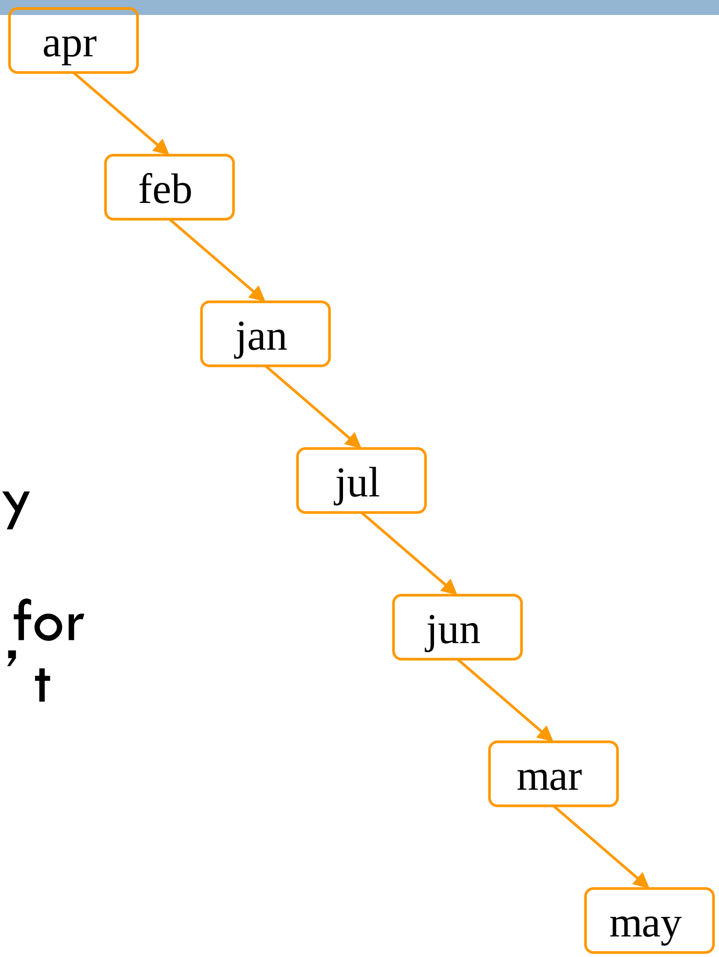
```
/** Return true iff x if the datum in a cell of tree node.
    Precondition: node is a BST */
public static boolean treeSearch (Object x, TreeCell node) {
   if (node == null) return false;
   if (node.datum.equals(x)) return true;
   if (node.datum.compareTo(x) > 0)
         return treeSearch(x, node.left);
   else return treeSearch(x, node.right);
}
```

Building a BST

- To insert a new item
  - Pretend to look for the item
  - Put the new node in the place where you fall off the tree
- This can be done using either recursion or iteration
- Example
  - Tree uses *alphabetical order*
  - Months appear for insertion in *calendar order*



What Can Go Wrong?



- A BST makes searches very fast, *unless*…
  - Nodes are inserted in alphabetical order
  - In this case, we're basically building a linked list (with some extra wasted space for the `left` fields that aren't being used)
- BST works great if data arrives in random order

Printing Contents of BST

Because of the ordering rules for a BST, it's easy to print the items in alphabetical order

- Recursively print left subtree
- Print the node
- Recursively print right subtree

```
/** Print the BST in alpha. order.  */
public void show () {
   show(root);
   System.out.println();
}
private static void show(
                     TreeNode node) {
   if (node == null) return;
   show(node.lchild);
   System.out.print(node.datum + " ");
   show(node.rchild);
}
```

Tree Traversals

- "Walking" over whole tree is a tree traversal
  - Done often enough that there are standard names
  - Previous example: inorder traversal
    - Process left subtree
    - Process node
    - Process right subtree
- Note: Can do other processing besides printing

Other standard kinds of traversals

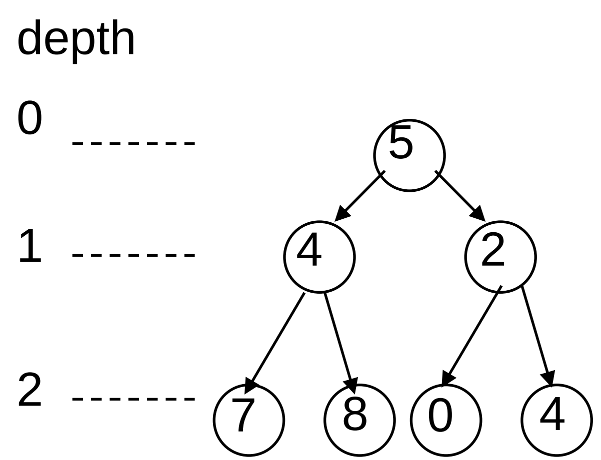
- Preorder traversal
  - Process node
  - Process left subtree
  - Process right subtree
- Postorder traversal
  - Process left subtree
  - Process right subtree
  - Process node
- Level-order traversal
  - Not recursive uses a queue

Some Useful Methods

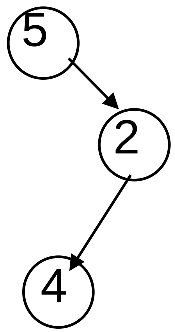

```
// Return true iff a node is a leaf
public static boolean isLeaf(TreeCell node) {
   return (node != null) && (node.left == null)
                         && (node.right == null);
}

//Return the height of a node using postorder traversal
public static int height(TreeCell node) {
   if (node == null) return -1; //empty tree
   if (isLeaf(node)) return 0;
   return 1 + Math.max(height(node.left),
                       height(node.right));
}

// Return number of nodes using postorder traversal
public static int nNodes(TreeCell node) {
   if (node == null) return 0;
   return 1 + nNodes(node.left) + nNodes(node.right);
}
```

Useful Facts about Binary Trees

- Max number of nodes at depth d: 2^d
- If height of tree is h
  - min number of nodes in tree: $h + 1$
  - Max number of nodes in tree:
  - $2^0 + \ldots + 2^h = 2^{h+1} - 1$
- Complete binary tree
  - All levels of tree down to a certain depth are completely filled



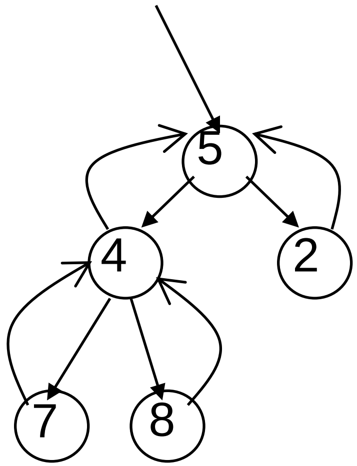
Height 2,
maximum number of nodes



Height 2,
minimum number of nodes

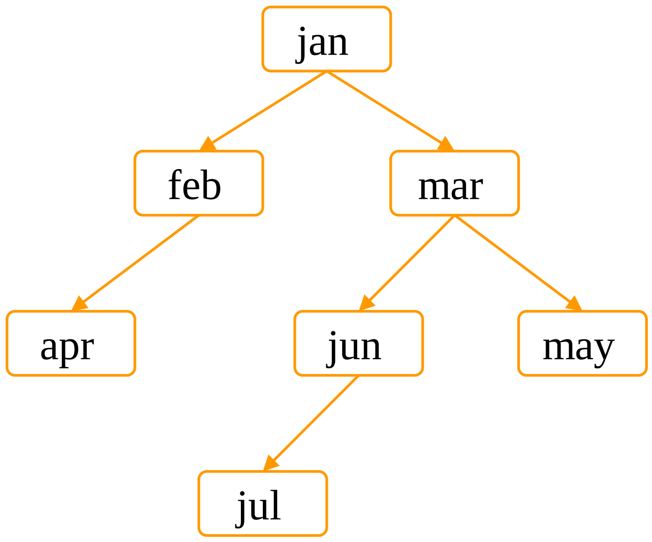
Tree with Parent Pointers

- In some applications, it is useful to have trees in which nodes can reference their parents
- Analog of doubly-linked lists



Things to Think About

- What if we want to *delete* data from a BST?
- A BST works great as long as it's *balanced*
  - How can we keep it balanced? *This turns out to be hard enough to motivate us to create other kinds of trees*



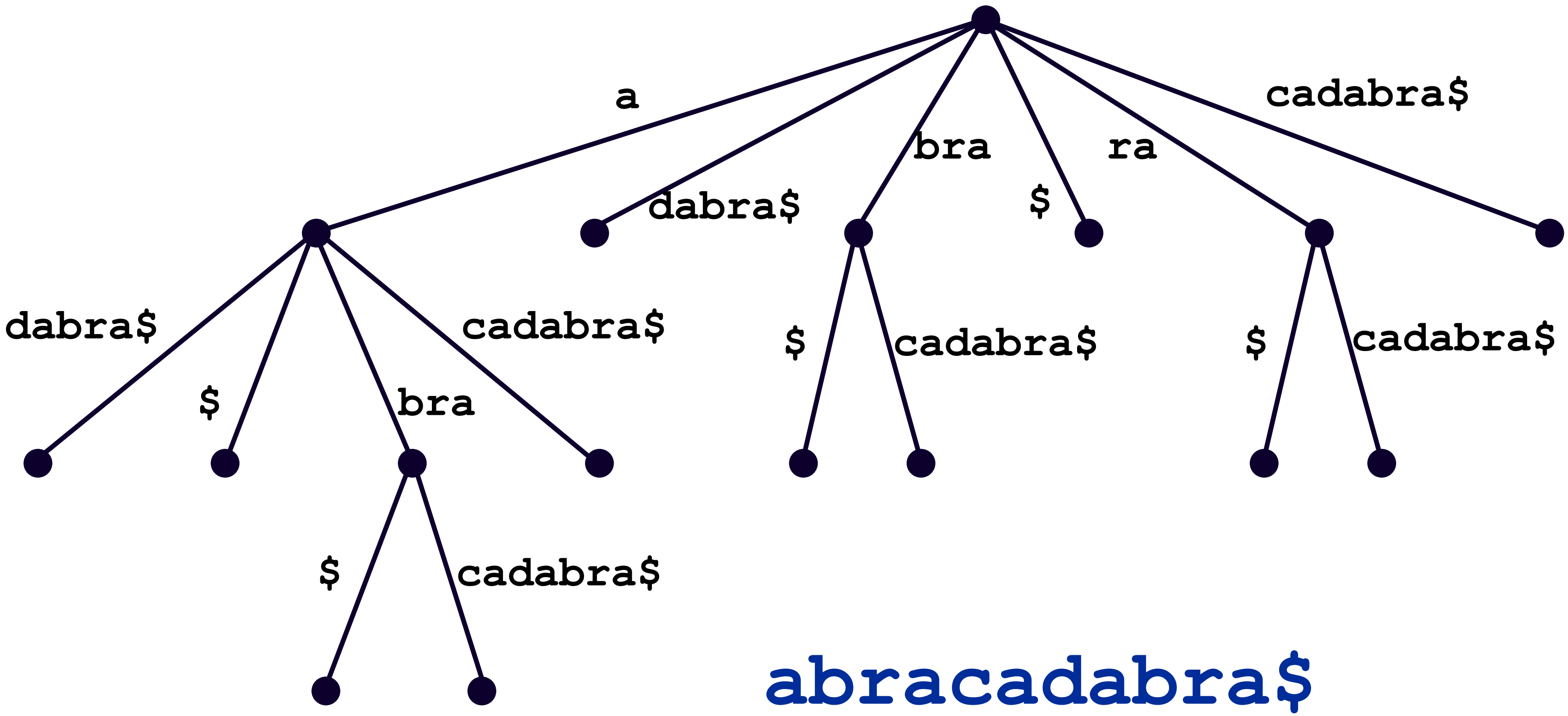
Suffix Trees

- Given a string s, a suffix tree for s is a tree such that

  - each edge has a unique label, which is a nonnull substring of s
  - any two edges out of the same node have labels beginning with different characters
  - the labels along any path from the root to a leaf concatenate together to give a suffix of s
  - all suffixes are represented by some path
  - the leaf of the path is labeled with the index of the first character of the suffix in s

- Suffix trees can be constructed in linear time

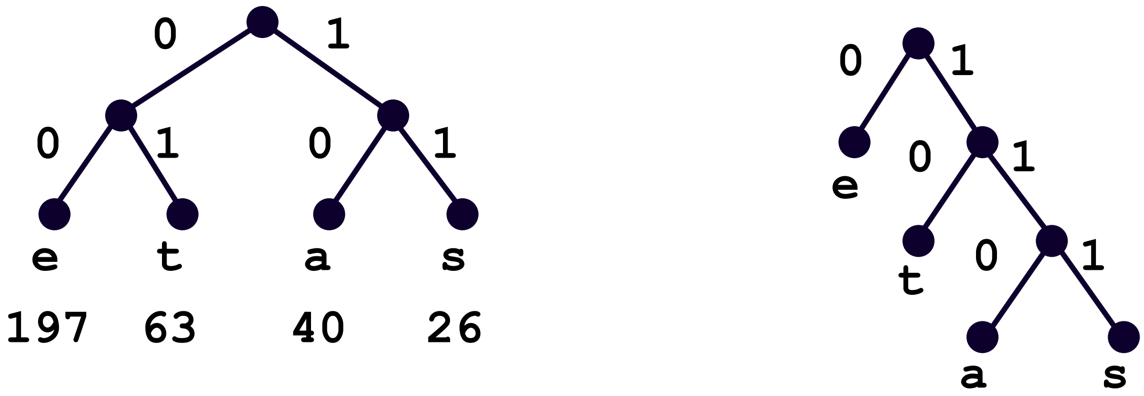
Suffix Trees



Suffix Trees

- Useful in string matching algorithms (e.g., longest common substring of 2 strings)
- Most algorithms linear time
- Used in genomics (human genome is ~4GB)

Huffman Trees



Fixed length encoding
197*2 + 63*2 + 40*2 + 26*2 = 652

Huffman encoding
197*1 + 63*2 + 40*3 + 26*3 = 521

Huffman Compression of “Ulysses”

- ' ' 242125 00100000 3 110
- 'e' 139496 01100101 3 000
- 't' 95660 01110100 4 1010
- 'a' 89651 01100001 4 1000
- 'o' 88884 01101111 4 0111
- 'n' 78465 01101110 4 0101
- 'i' 76505 01101001 4 0100
- 's' 73186 01110011 4 0011
- 'h' 68625 01101000 5 11111
- 'r' 68320 01110010 5 11110
- 'l' 52657 01101100 5 10111
- 'u' 32942 01110101 6 111011
- 'g' 26201 01100111 6 101101
- 'f' 25248 01100110 6 101100
- '.' 21361 00101110 6 011010
- 'p' 20661 01110000 6 011001

Huffman Compression of “Ulysses”

...

- '7' 68 00110111 15 111010101001111
- '/' 58 00101111 15 111010101001110
- 'X' 19 01011000 16 0110000000100011
- '&' 3 00100110 18 011000000010001010
- '%' 3 00100101 19 0110000000100010111
- '+' 2 00101011 19 0110000000100010110
- original size 11904320
- compressed size 6822151
- 42.7% compression

BSP Trees

- BSP = Binary Space Partition (not related to BST!)
- Used to render 3D images composed of polygons
- Each node n has one polygon p as data
- Left subtree of n contains all polygons on one side of p
- Right subtree of n contains all polygons on the other side of p
- Order of traversal determines occlusion (hiding)!

Tree Summary

- A *tree* is a recursive data structure
  - Each cell has 0 or more successors (*children*)
  - Each cell except the *root* has at exactly one predecessor (*parent*)
  - All cells are reachable from the *root*
  - A cell with no children is called a *leaf*
- Special case: *binary tree*
  - Binary tree cells have a left and a right child
  - Either or both children can be null
- Trees are useful for exposing the recursive structure of natural language and computer programs