NAME: _____ NETID: _____

CS2110 Fall 2009 Prelim 2
November 17, 2009

Write your name and Cornell netid . There are 6 questions on 8 numbered pages. Check now that you have all the pages. Write your answers in the boxes provided. Use the back of the pages for workspace. Ambiguous answers will be considered incorrect. The exam is closed book and closed notes. Do not begin until instructed. You have 90 minutes. Good luck!

	1	2	3	4	5	6	Total
Score	/15	/20	/30	/10	/5	/20	
Grader							

1. (15 points) You probably know the game of "Towers of Hanoi". Here's a picture of a typical initial configuration: there is a stack of N disks on the first of three poles (call them A, B and C) and your job is to move the disks from pole A to pole B without ever putting a larger disk on top of a smaller disk. The game can be solved recursively using the code shown below.

```
class Hanoi
{
  public static void main (String args[])
  {
    playHanoi(Integer.parse(args[0]), "A", "B", "C");
  }

// Moves n disks from pole "from" to pole "to" using "other" as a
// temporary place to hold n-1 disks
static void playHanoi(int n, String from, String to, String other)
  {
    if (n > 0) {
      playHanoi (n-1, from, other, to);
      System.out.printf("Move one disk from pole %s to pole %d\n", from, to);
      playHanoi (n-1, other, to, from);
    }
  }
}
```

(a) 5 points. If moving a single disk from one pole to another is the operation we're counting, give a formula for the cost of moving N disks in terms of the cost of moving N-1 disks.

C(N) = C(N-1) + 1 + C(N-1) = 2C(N-1)+1

(b) 5 points. Solve the formula in (a), obtaining a polynomial that gives the cost.

C(N) = 2(2….2(2*0+1)+1)….)+1 = $2^N+2^{N-1}+2^{N-2}+….+2^0$

(c) 5 points. Simplify formula (b). *Hint: Think about the binary representation of an integer.*

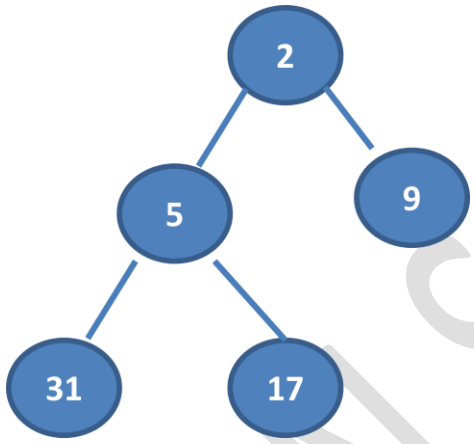
C(N) = $2^{N+1}-1$

2. (20 points) True or false?

a	T	F	Multicore machines have a few cores, usually 2 or 4 and perhaps as many as 16. Nonetheless, the complexity of comparison sorting on such a system is still at least O(n log n).
b	T	F	For a program to gain a speedup by exploiting concurrency on a multicore machine it must use threads
c	T	F	A heap data structure (as used in HeapSort) would be a fantastic choice for implementing a priority queue.
d	T	F	Items pop from a stack in "last in, first out" or LIFO, order
e	T	F	If you incorrectly implement the priority queue interface (your version of poll() has a bug and sometimes doesn't return the smallest element), Java's type checking will catch your mistake and the code won't compile.
f	T	F	If the same element is inserted multiple times into a Java set, the set will only contain a single instance of that element.
g	T	F	If the same element is inserted multiple times into a Java list, the list will only contain a single instance of that element.
h	T	F	When you refresh a GUI the user might not see the actual updated screen immediately.
i	T	F	Java won't garbage collect an object if there is any way to reference that object from some active thread in your program.
j	T	F	A cache typically contains some limited set of previously computed results and is used to avoid recomputing those results again and again.
k	T	F	If a method always takes exactly 10 minutes to compute something, we would say that it has complexity O(10mins).
l	T	F	The complexity of looking up an item in a balanced BST containing N items is O(log(N))
m	T	F	The complexity of looking up *k* items in a balanced BST containing N items is O(k log(N)), which is the same as $O(\log(N^k))$.
n	T	F	If you use a very badly chosen hash function, then looking up an item in a HashMap might have complexity much worse than O(1)
o	T	F	The worst case complexity of QuickSort is $O(N^2)$ but normally, it runs in time O(N log N)
p	T	F	If a method does something that requires N^2 operations and then does something else that requires N log N operations but does it N/2 times, the complexity of the method is $O(N^2)$
q	T	F	Nested for loops always result in complexity $O(N^2)$
r	T	F	If Dog and Cat are two classes that extend class Animal, then if cleopatra is an instance of Cat, (Dog)((Animal) cleopatra).bark() will upcast cleopatra to Animal, then downcast to Dog, and then run its bark() method. *Assume that only class Dog defines the bark method.*
s	T	F	In a recursive procedure the base case does all the work.
t	T	F	One difference between Java and other languages is that in Java, type checking is used to create documentation, also known as Java Doc.

3. (30 points) Suppose we create a heap (of maximum size eight) by inserting five elements 9, 17, 5, 31, 2 in the order shown (leaving room for three more). Draw the resulting balanced binary heap (exactly as seen in class) and then fill in the vector with the elements in "heap representation" (again, using the vector representation of a heap seen in class).

(a) 10 points. The heap:



(b) 10 points. The corresponding vector (put a slash through "empty" elements):

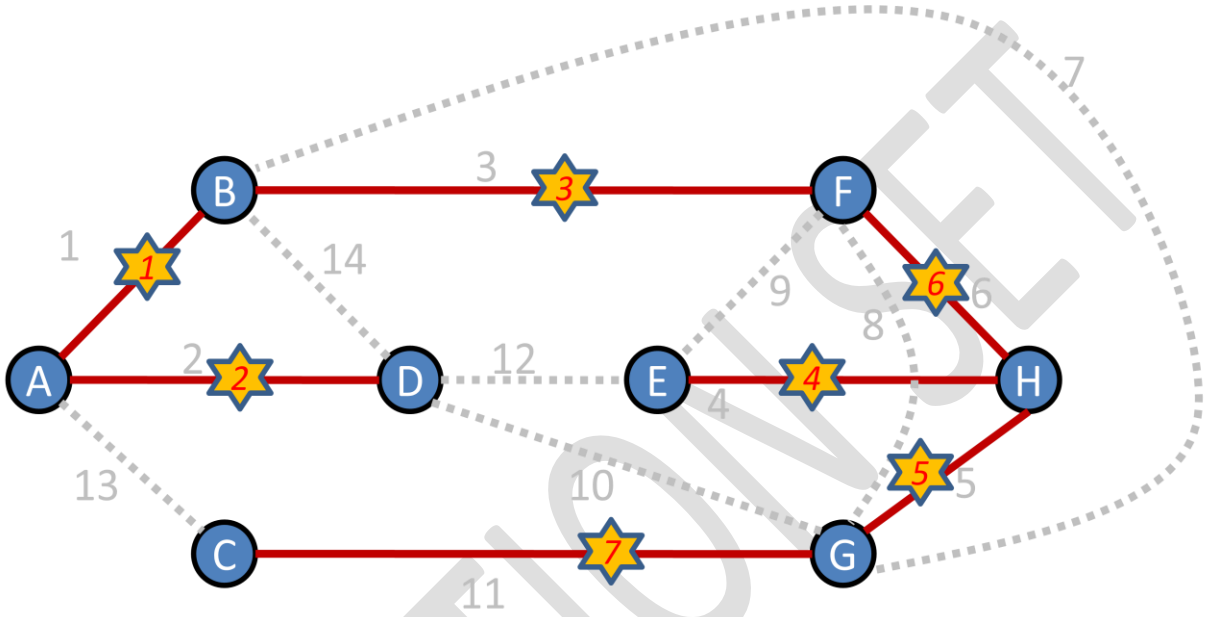
2	5	9	31	17	/	/	/

(c) 10 points. Now show the vector after we call poll() once, to extract the smallest element.

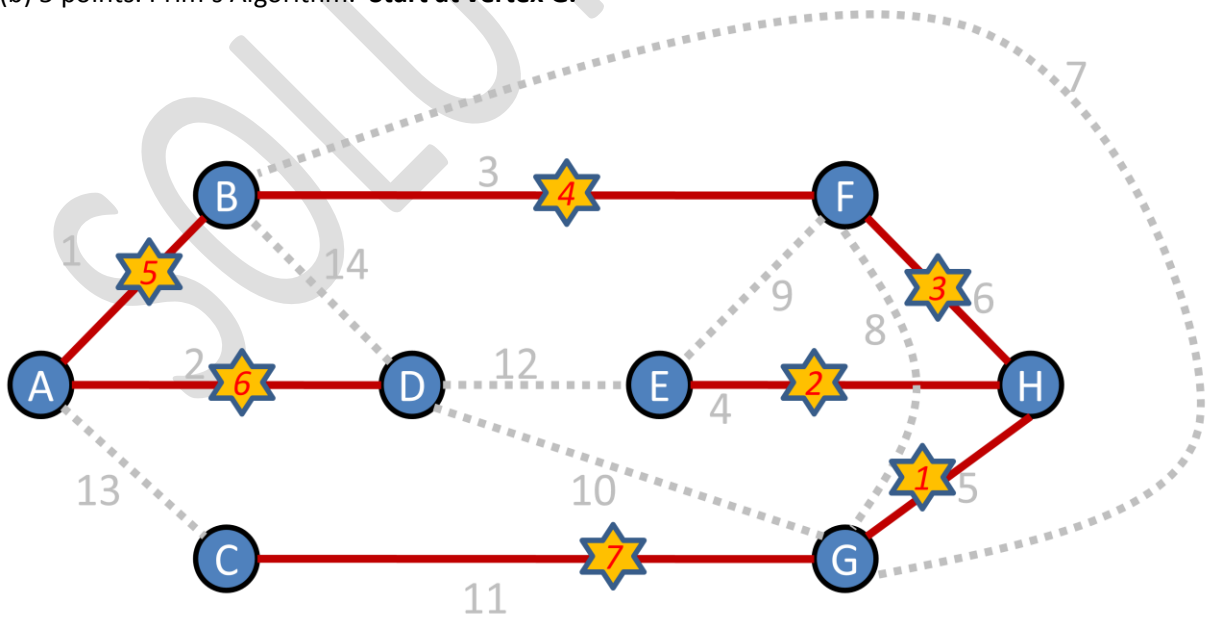
5	17	9	31	/	/	/	/

4. (10 points) Run Kruskal's and Prim's minimum spanning tree algorithms, as presented in class, by hand on the graphs shown below and draw the resulting graphs for us. These algorithms select edges to include as they run. Darken the selected edges and label them 1, 2, 3 so that we can see the order in which they were selected. Circle your edge numbers so that we can't confuse them with the gray-colored edge weights we provided.

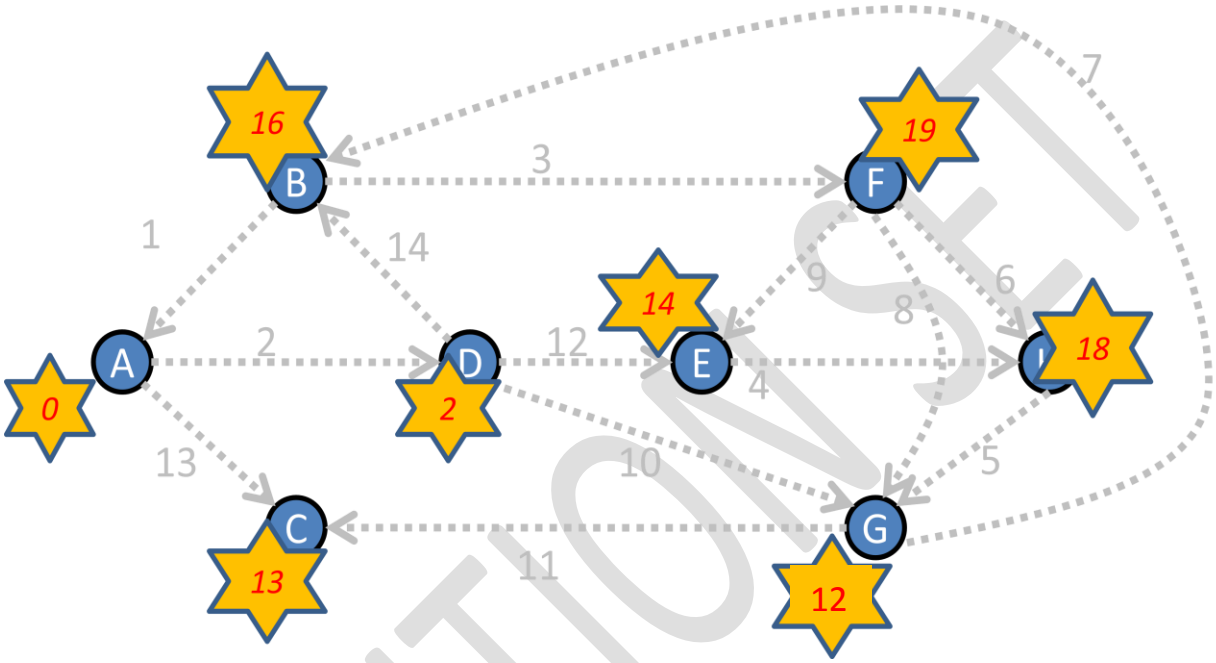
(a) 5 points. Kruskal's Algorithm



(b) 5 points. Prim's Algorithm. **Start at vertex G.**



5) 5 points. Run Dijkstra's algorithm on the graph below. Start Dijkstra's algorithm at node A and mark each node with the distance computed by Dijkstra's method. Be careful when writing the node distances so that we can read what you write and won't get confused by the gray edge weights we provided.



6. (20 points)

Suppose that we represent a dungeon as an undirected graph. Each node is a room which may contain treasures. Adjacent rooms are stored as an adjacency list. Note that this graph could have cycles.

Our goal is to print directions from a start room to the treasure. If we started in the ATRIUM and we were looking for a RING in the SECRET PASSAGE BEHIND THE WALL, our directions would look like this:

```
From ATRIUM go to DUNGEON.
From DUNGEON go to BROOM CLOSET.
From BROOM CLOSET go to SECRET PASSAGE BEHIND THE WALL
In SECRET PASSAGE BEHIND THE WALL is the RING you seek.
```

Or, if the dungeon does not contain the RING:

```
Sorry, master, I can't find you any RING.
```

a-i) 3 points. Read part (b). Explain how you will solve it here. Include details about which data structures you will use and how you will use them. Hint: Your master (e.g.: your grader) will know graph algorithms well. Focus your efforts not on explaining these algorithms, but explaining how you will implement them. If an algorithm has a name, use that name in your answer. *Limit: 3 sentences.*

I would use a recursive depth-first search to explore the maze, putting the Rooms already searched
into a Java LinkedList<Room> so that I could rapidly check to see if I've already visited a
given room. If the treasure is found my routine would remember the sequence of rooms visited.

a-ii) 2 points. Would your answer to (a-i) change if we had asked you to find the *shortest path* to the treasure? If not, explain why; if so, tell us what you would do differently.

Same solution but now I would use a breadth-first search (slightly more complex to code).

b) 10 points.

```
class Room {
        // Don't add more class variables…  You may assume that these three are never null
        String                  name;           // Name of this room
        ArrayList<String >      treasures;      // Treasures contained in this room.  Empty list if none
        ArrayList<Room >        adjacent;       // Rooms reachable from this room.  Empty list if none

        public LinkedList<Room>  findTreasurePath(String goal) {
                LinkedList<Room > visited = new LinkList<Room>();
                LinkedList<Room > path = new LinkList<Room>();
                if(explore(goal, visited, path) == true)
                        return path;
                return null;
        }

        private boolean  explore(String goal, LinkedList<Room > visited, LinkedList<Room > path) {

                // Been here before?
                if(visited.contains(this))
                        return false;

                // Remember that I was here once
                visited.addLast(this);

                // Optimistically, add this room to the end of the path
                path.addLast(this);

                // Did I find my goal?
                if(this.treasures.contains(goal))
                        return true;

                // Is there a path to my goal through some adjacent room?
                for(Room r: adjacent)
                        if(r.explore(goal, visited, path) == true)
                                return true;

                // Can't get to the goal from this room, remove it from the path
                path.remove(this);
                return false;
        }
}
```

c) 5 points.

```
public void printTreasurePath(String goal, LinkedList<Room> path) {
        // If the object wasn't found the argument will be null
        if(path == null)
        {
                System.out.printf("Sorry master.  I cannot find you any %s\n", goal);
                return;
        }

        // Last room I was in, initially null
        Room lastRoom = null;

        //  Room by room, list the path to follow.  Takes advantage of the fact that a
        // LinkedList can be traversed using "for" in the order the nodes were added
        for(Room r: path)  {

                // Print the next step on the path (except on the first room)
                if(from != null)
                        System.out.printf("From %s go to %s\n", lastRoom.name, r.name)

                // Remember which room I was in last
                lastRoom = r;
        }

        // After printing the path, print the "final line" of the output
        System.out.printf("In %s is the %s you seek\n", lastRoom, goal);
}
```