Standard ADTs

Lecture 16
CS2110 – Fall 2008

Abstract Data Types (ADTs)

- A method for achieving abstraction for data structures and algorithms
- ADT = model + operations
- Describes what each operation does, but not how it does it
- An ADT is independent of its implementation

- In Java, an interface corresponds well to an ADT
  - The interface describes the operations, but says nothing at all about how they are implemented
- Example: Stack interface/ADT

```
public interface Stack {
  public void push(Object x);
  public Object pop();
  public Object peek();
  public boolean isEmpty();
  public void clear();
}
```

Queues & Priority Queues

- ADT Queue
  - Operations:

```
void add(Object x);
Object poll();
Object peek();
boolean isEmpty();
void clear();
```

- Where used:
  - Simple job scheduler (e.g., print queue)
  - Wide use within other algorithms

- ADT PriorityQueue
  - Operations:

```
void insert(Object x);
Object getMax();
Object peekAtMax();
boolean isEmpty();
void clear();
```

- Where used:
  - Job scheduler for OS
  - Event-driven simulation
  - Can be used for sorting
  - Wide use within other algorithms

Sets

- ADT Set
  - Operations:

```
void insert(Object element);
boolean contains(Object element);
void remove(Object element);
boolean isEmpty();
void clear();
```

- Where used:
  - Wide use within other algorithms
- Note: no duplicates allowed
  - A “set” with duplicates is sometimes called a *multiset* or *bag*

Dictionaries

- ADT Dictionary (aka Map)
  - Operations:

```
void insert(Object key, Object value);
void update(Object key, Object value);
Object find(Object key);
void remove(Object key);
boolean isEmpty();
void clear();
```

- Think of: key = word; value = definition
- Where used:
  - Symbol tables
  - Wide use within other algorithms

Data Structure Building Blocks

- These are *implementation* “building blocks” that are often used to build more-complicated data structures
  - Arrays
  - Linked Lists
    - Singly linked
    - Doubly linked
  - Binary Trees
  - Graphs
    - Adjacency matrix
    - Adjacency list

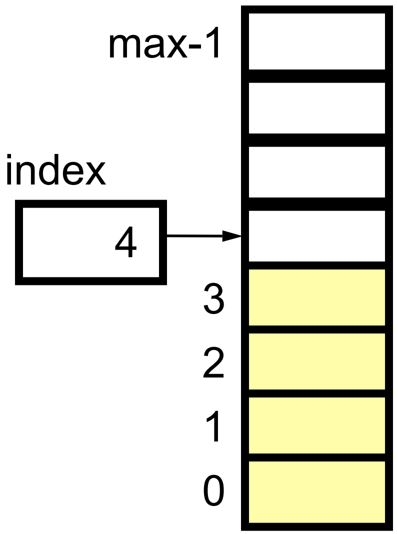
Array Implementation of Stack

```
class ArrayStack implements Stack {

   private Object[] array; //Array that holds the Stack
   private int index = 0; //First empty slot in Stack

   public ArrayStack(int maxSize)
      { array = new Object[maxSize]; }

   public void push(Object x) { array[index++] = x; }
   public Object pop() { return array[--index]; }
   public Object peek() { return array[index-1]; }
   public boolean isEmpty() { return index == 0; }
   public void clear() { index = 0; }
}
```



O(1) worst-case time for each operation

Question: What can go wrong?

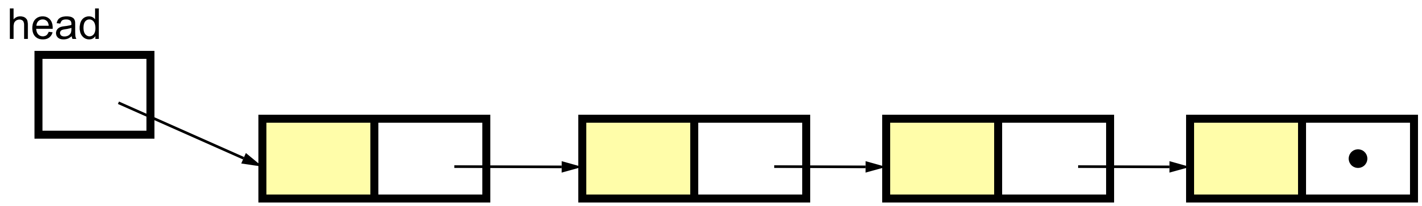
Linked List Implementation of Stack

```
class ListStack implements Stack {
   private Node head = null;  //Head of list that
                              //holds the Stack

   public void push(Object x) { head = new Node(x, head); }
   public Object pop() {
      Node temp = head;
      head = head.next;
      return temp.data;
   }
   public Object peek() { return head.data; }
   public boolean isEmpty() { return head == null; }
   public void clear() { head = null; }
}
```

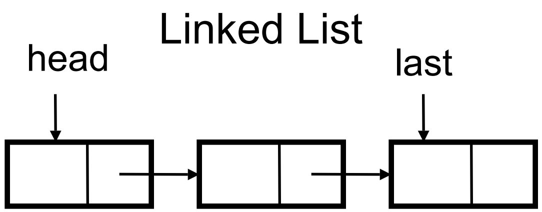
O(1) worst-case time for each operation (but constant is larger)

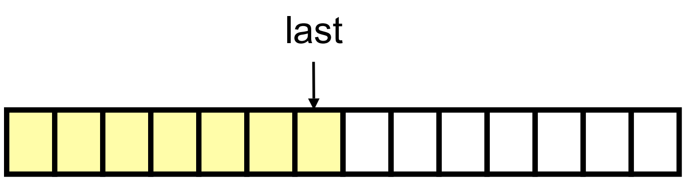
Note that array implementation can overflow, but the linked list version cannot



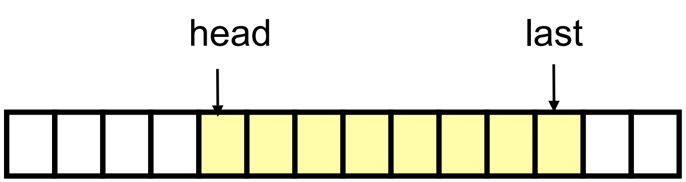
Queue Implementations

- Possible implementations





Array with head always at A[0]
(poll( ) becomes expensive)
(can overflow)



Array with wraparound
(can overflow)

- Recall: operations are **`add`**, **`poll`**, **`peek`**,…
  - For linked-list
    - All operations are O(1)
  - For array with head at A[0]
    - poll takes time O(n)
    - Other ops are O(1)
    - Can overflow
  - For array with wraparound
    - All operations are O(1)
    - Can overflow

A Queue From 2 Stacks

- Add pushes onto stack A
- Poll pops from stack B
- If B is empty, move all elements from stack A to stack B
- Some individual operations are costly, but still O(1) time per operations over the long run

Dealing with Overflow

- For array implementations of stacks and queues, use *table doubling*
- Check for overflow with each insert op
- If table will overflow,
  - Allocate a new table twice the size
  - Copy everything over
- The operations that cause overflow are expensive, but still constant time per operation over the long run (proof later)

Goal: Design a *Dictionary* (aka *Map*)

- Operations

```
void insert(key, value)
void update(key, value)
Object find(key)
void remove(key)
boolean isEmpty()
void clear()
```

Array implementation: Using an array of (key,value) pairs

	Unsorted	Sorted
insert	O(1)	O(n)
update	O(n)	O(log n)
find	O(n)	O(log n)
remove	O(n)	O(n)

n is the number of items currently held in the dictionary

Hashing

- Idea: compute an array index via a *hash function* h
- U is the universe of keys
- h: U → [0,…,m-1]
  where m = hash table size
- Usually |U| is much bigger than m, so *collisions* are possible (two elements with the same hash code)
- h should
  - be easy to compute
  - avoid collisions
  - have roughly equal probability for each table position

Typical situation:

U = all legal identifiers

Typical hash function:

h converts each letter to a number, then compute a function of these numbers

A Hashing Example

- Suppose each word below has the following hashCode

jan	7
feb	0
mar	5
apr	2
may	4
jun	7
jul	3
aug	7
sep	2
oct	5

- How do we resolve collisions?
  - use chaining: each table position is the head of a list
  - for any particular problem, this *might* work terribly
- In practice, using a good hash function, we can assume each position is equally likely

Analysis for Hashing with Chaining

- Analyzed in terms of *load factor* λ = n/m = (items in table)/(table size)
- We count the expected number of *probes* (key comparisons)
- Goal: Determine expected number of probes for an *unsuccessful* search

- Expected number of probes for an unsuccessful search = average number of items per table position = n/m = λ
- Expected number of probes for a *successful* search = 1 + $\lambda/2 = O(\lambda)$
- Worst case is O(n)

Table Doubling

- We know each operation takes time $O(\lambda)$ where $\lambda=n/m$
- So it gets worse as n gets large relative to m
- Table Doubling:
  - Set a bound for λ (call it λ_0)
  - Whenever λ reaches this bound:
    - Create a new table twice as big
    - Then rehash all the data
  - As before, operations *usually* take time O(1)
    - But sometimes we copy the whole table

Analysis of Table Doubling

- Suppose we reach a state with n items in a table of size m and that we have just completed a table doubling

	Copying Work
Everything has just been copied	n inserts
Half were copied previously	n/2 inserts
Half of those were copied previously	n/4 inserts
…	…
Total work	n + n/2 + n/4 + … = 2n

Analysis of Table Doubling, Cont'd

- Total number of insert operations needed to reach current table = copying work + initial insertions of items = 2n + n = 3n inserts
- Each insert takes expected time $O(\lambda_0)$ or O(1), so total expected time to build entire table is O(n)
- Thus, expected time per operation is O(1)

- Disadvantages of table doubling:
  - Worst-case insertion time of O(n) is definitely achieved (but rarely)
  - Thus, not appropriate for time critical operations

Java Hash Functions

- Most Java classes implement the **`hashCode()`** method
- **`hashCode()`** returns an int
- Java's **`HashMap`** class uses h(X) = X.hashCode() mod m
- h(X) in detail:

```
int hash = X.hashCode();
int index = (hash & 0x7FFFFFFF) % m;
```

- What **`hashCode()`** returns:
  - Integer:
    - uses the int value
  - Float:
    - converts to a bit representation and treats it as an int
  - Short Strings:
    - 37*previous + value of next character
  - Long Strings:
    - sample of 8 characters; 39*previous + next value

`hashCode()` Requirements

- Contract for `hashCode()` method:
  - Whenever it is invoked in the same object, it must return the same result
  - Two objects that are equal (in the sense of `.equals(...)`) must have the same hash code
  - Two objects that are not equal should return different hash codes, but are not required to do so (i.e., collisions are allowed)

Hashtables in Java

- **`java.util.HashMap`**
- **`java.util.HashSet`**
- **`java.util.Hashtable`**

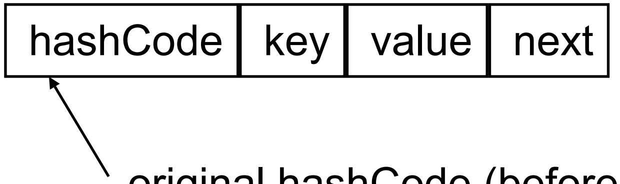
- Use chaining

- Initial (default) size = 101

- Load factor = $\lambda_0 = 0.75$

- Uses table doubling (2*previous+1)

- A node in each chain looks like this:



original hashCode (before mod m)
Allows faster rehashing and (possibly) faster key comparison

Linear & Quadratic Probing

- These are techniques in which all data is stored directly within the hash table array
- Linear Probing
  - Probe at h(X), then at
    - h(X) + 1
    - h(X) + 2
    - …
    - h(X) + i
  - Leads to *primary clustering*
    - Long sequences of filled cells
- Quadratic Probing
  - Similar to Linear Probing in that data is stored within the table
  - Probe at h(X), then at
    - h(X)+1
    - h(X)+4
    - h(X)+9
    - …
    - h(X)+ i^2
  - Works well when
    - $\lambda < 0.5$
    - Table size is prime

Universal Hashing

- Choose a hash function at random from a large parameterized family of hash functions (e.g., $h(x) = ax + b$, where a and b are chosen at random)
- With high probability, it will be just as good as any custom-designed hash function you can come up with

`hashCode()` and `equals()`

- We mentioned that the hash codes of two equal objects must be equal — this is necessary for hashtable-based data structures such as **`HashMap`** and **`HashSet`** to work correctly
- In Java, this means if you override **`Object.equals()`**, you had better also override **`Object.hashCode()`**
- But how???

hashCode() and equals()

```
class Identifier {
   String name;
   String type;

   public boolean equals(Object obj) {
      if (obj == null) return false;
      Identifier id;
      try {
         id = (Identifier)obj;
      } catch (ClassCastException cce) {
         return false;
      }
      return name.equals(id.name) && type.equals(id.type);
   }




}
```

hashCode() and equals()

```
class Identifier {
   String name;
   String type;

   public boolean equals(Object obj) {
      if (obj == null) return false;
      Identifier id;
      try {
         id = (Identifier)obj;
      } catch (ClassCastException cce) {
         return false;
      }
      return name.equals(id.name) && type.equals(id.type);
   }

   public int hashCode() {
      return 37 * name.hashCode() + 113 * type.hashCode() + 42;
   }
}
```

hashCode() and equals()

```
class TreeNode {
   TreeNode left, right;
   String datum;

   public boolean equals(Object obj) {
      if (obj == null || !(obj instanceof TreeNode)) return false;
      TreeNode t = (TreeNode)obj;
      boolean lEq = (left != null)?
         left.equals(t.left) : t.left == null;
      boolean rEq = (right != null)?
         right.equals(t.right) : t.right == null;
      return datum.equals(t.datum) && lEq && rEq;
   }

}
```

hashCode() and equals()

```
class TreeNode {
   TreeNode left, right;
   String datum;

   public boolean equals(Object obj) {
      if (obj == null || !(obj instanceof TreeNode)) return false;
      TreeNode t = (TreeNode)obj;
      boolean lEq = (left != null)?
         left.equals(t.left) : t.left == null;
      boolean rEq = (right != null)?
         right.equals(t.right) : t.right == null;
      return datum.equals(t.datum) && lEq && rEq;
   }

   public int hashCode() {
      int lHC = (left != null)? left.hashCode() : 298;
      int rHC = (right != null)? right.hashCode() : 377;
      return 37 * datum.hashCode() + 611 * lHC - 43 * rHC;
   }
}
```

Dictionary Implementations

- Ordered Array
  - Better than unordered array because Binary Search can be used
- Unordered Linked List
  - Ordering doesn’t help
- Hashtables
  - O(1) expected time for Dictionary operations