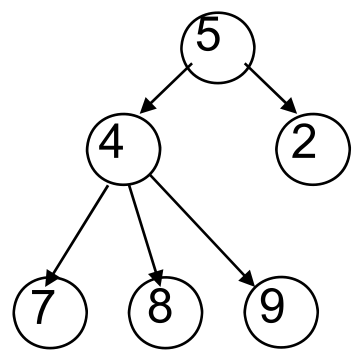
TREES

Lecture 9

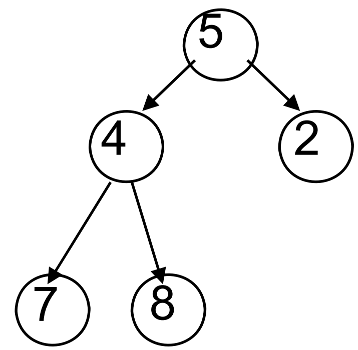
CS2110 – Fall 2009

Tree Overview

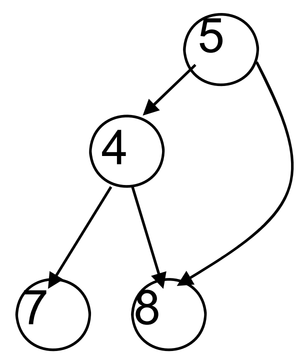
- *Tree*: recursive data structure (similar to list)
  - Each cell may have zero or more *successors* (children)
  - Each cell has exactly one *predecessor* (parent) except the *root*, which has none
  - All cells are reachable from *root*
- *Binary tree*: tree in which each cell can have at most two children: a left child and a right child



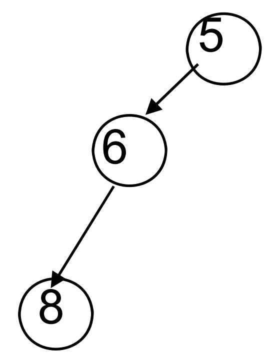
General tree



Binary tree



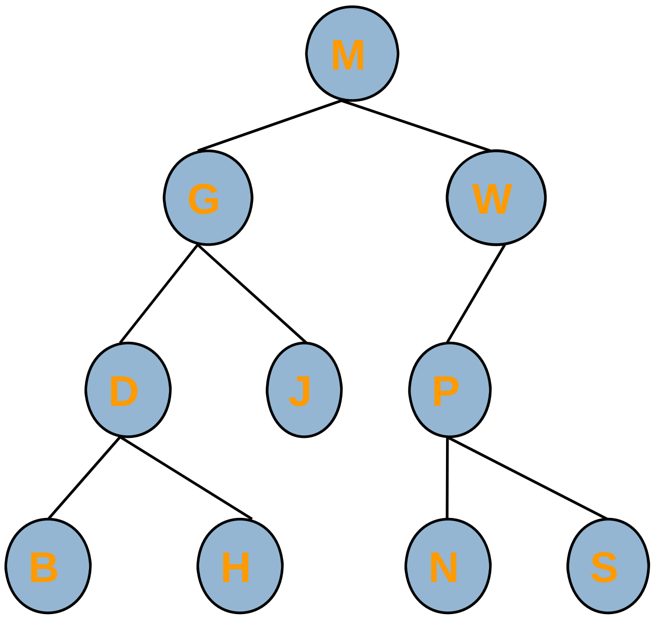
Not a tree



List-like tree

Tree Terminology

- M is the *root* of this tree
- G is the *root* of the *left subtree* of M
- B, H, J, N, and S are *leaves*
- N is the *left child* of P; S is the *right child*
- P is the *parent* of N
- M and G are *ancestors* of D
- P, N, and S are *descendants* of W
- Node J is at *depth* 2 (i.e., *depth* = length of path from root = number of edges)
- Node W is at *height* 2 (i.e., *height* = length of longest path to a leaf)
- A collection of several trees is called a ...?



Class for Binary Tree Cells


```
class TreeCell<T> {
   private T datum;
   private TreeCell<T> left, right;

   public TreeCell(T x) { datum = x; }
   public TreeCell(T x, TreeCell<T> lft,
                        TreeCell<T> rgt) {
      datum = x;
      left = lft;
      right = rgt;
   }
   more methods: getDatum, setDatum,
   getLeft, setLeft, getRight, setRight
}
```

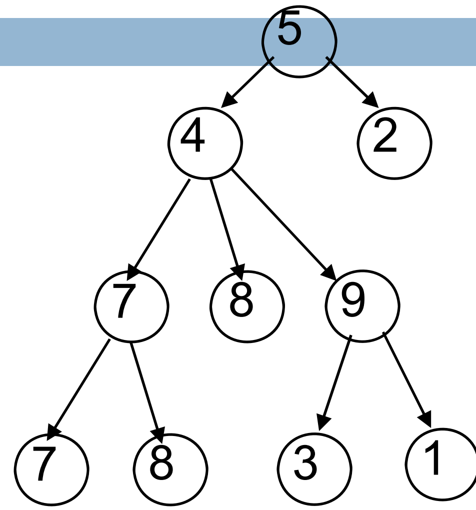
```
... new TreeCell<String>("hello") ...
```

Class for General Trees

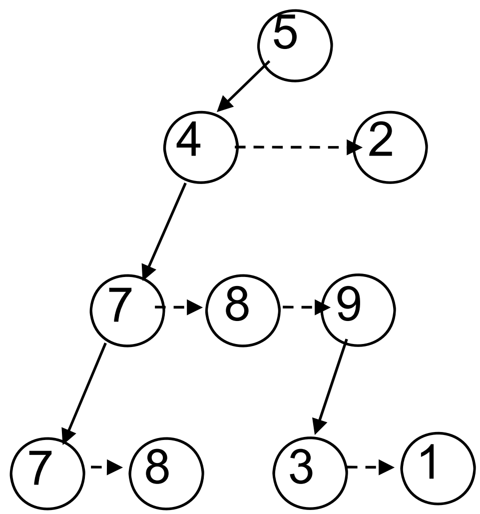

```
class GTreeCell {
    private Object datum;
    private GTreeCell left;
    private GTreeCell sibling;

    appropriate getter and
    setter methods
}
```

Parent node points directly only to its leftmost child

Leftmost child has pointer to next sibling, which points to next sibling, etc.



General tree



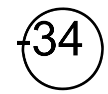
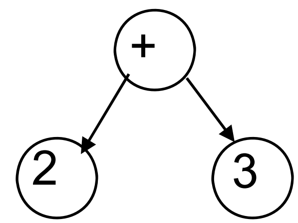
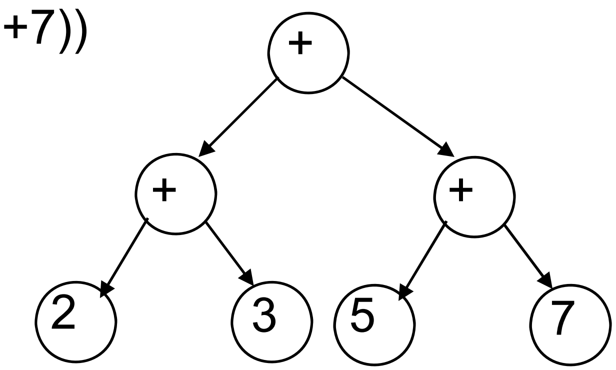
Tree represented using GTreeCell

Applications of Trees

- Most languages (natural and computer) have a recursive, hierarchical structure
- This structure is *implicit* in ordinary textual representation
- Recursive structure can be made *explicit* by representing sentences in the language as trees: Abstract Syntax Trees (ASTs)
- ASTs are easier to optimize, generate code from, etc. than textual representation
- A parser converts textual representations to AST

Example

- Expression grammar:
  - E ⟶ integer
  - E ⟶ (E + E)
- In textual representation
  - Parentheses show hierarchical structure
- In tree representation
  - Hierarchy is explicit in the structure of the tree

Text	AST Representation
-34	
(2 + 3)	
((2+3) + (5+7))	

Recursion on Trees

- Recursive methods can be written to operate on trees in an obvious way

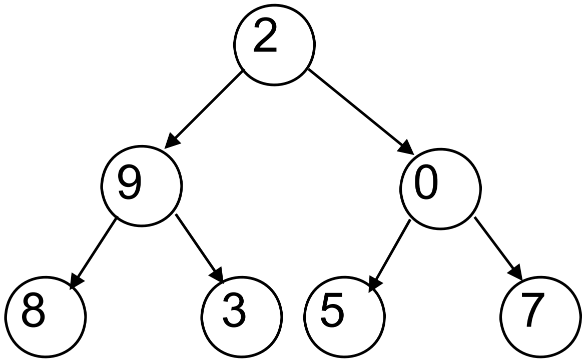
- Base case
  - empty tree
  - leaf node

- Recursive case
  - solve problem on left and right subtrees
  - put solutions together to get solution for full tree

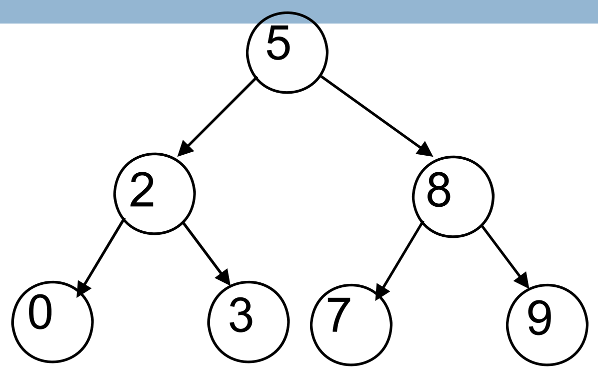
Searching in a Binary Tree


```
public static boolean treeSearch(Object x,
                                 TreeCell node) {
   if (node == null) return false;
   if (node.datum.equals(x)) return true;
   return treeSearch(x, node.left) ||
          treeSearch(x, node.right);
}
```

- Analog of linear search in lists: given tree and an object, find out if object is stored in tree
- Easy to write recursively, harder to write iteratively



Binary Search Tree (BST)

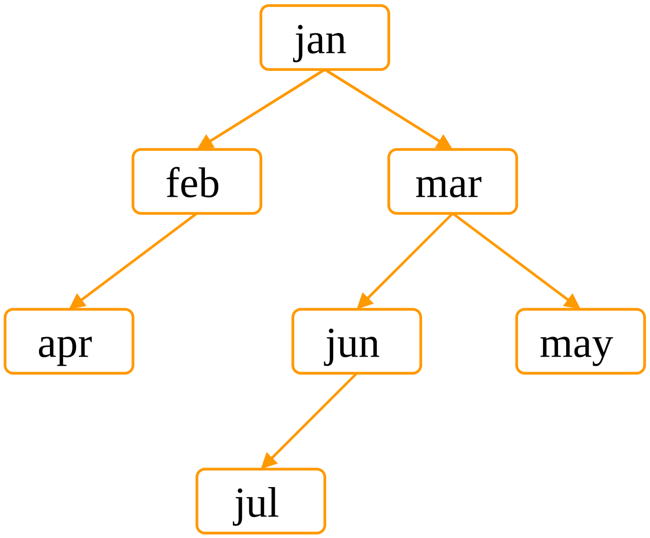


- If the tree data are *ordered* – in any subtree,
  - All *left* descendents of node come *before* node
  - All *right* descendents of node come *after* node
- This makes it *much* faster to search

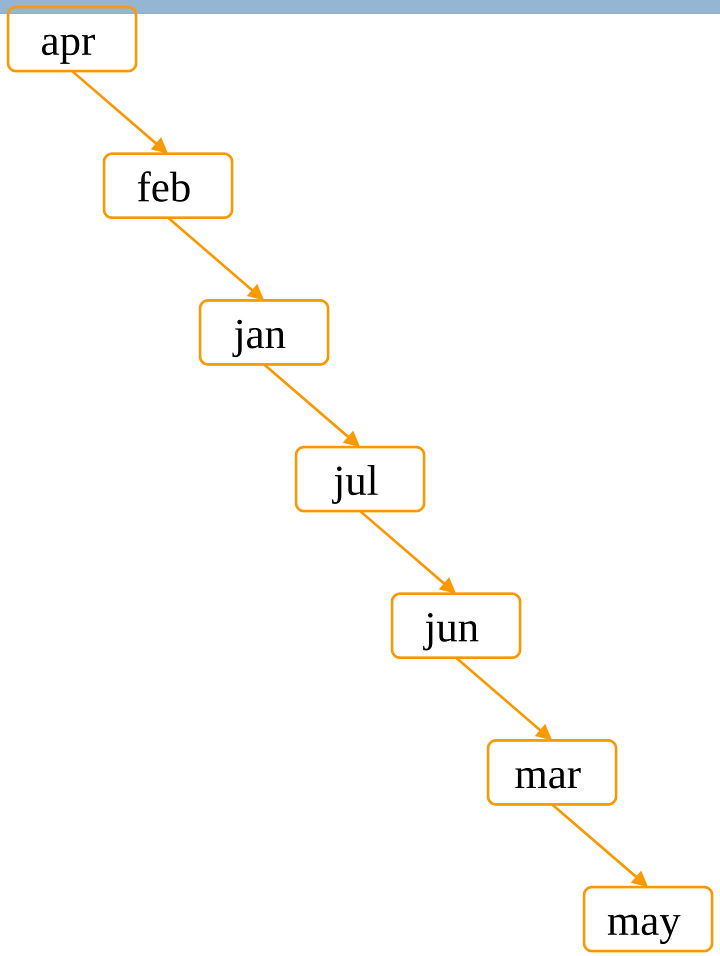
```
public static boolean treeSearch (Object x, TreeCell node) {
   if (node == null) return false;
   if (node.datum.equals(x)) return true;
   if (node.datum.compareTo(x) > 0)
        return treeSearch(x, node.left);
   else return treeSearch(x, node.right);
}
```

Building a BST

- To insert a new item
  - Pretend to look for the item
  - Put the new node in the place where you fall off the tree
- This can be done using either recursion or iteration
- Example
  - Tree uses *alphabetical order*
  - Months appear for insertion in *calendar order*



What Can Go Wrong?



- A BST makes searches very fast, *unless…*
  - Nodes are inserted in alphabetical order
  - In this case, we're basically building a linked list (with some extra wasted space for the **`left`** fields that aren't being used)

- BST works great if data arrives in random order

Printing Contents of BST

- Because of the ordering rules for a BST, it's easy to print the items in alphabetical order
  - Recursively print everything in the left subtree
  - Print the node
  - Recursively print everything in the right subtree

```
/**
* Show the contents of the BST in
* alphabetical order.
*/
public void show () {
   show(root);
   System.out.println();
}

private static void show(TreeNode node) {
   if (node == null) return;
   show(node.lchild);
   System.out.print(node.datum + " ");
   show(node.rchild);
}
```

Tree Traversals

- "Walking" over the whole tree is a tree traversal
  - This is done often enough that there are standard names
  - The previous example is an inorder traversal
    - Process left subtree
    - Process node
    - Process right subtree
- Note: we're using this for printing, but any kind of processing can be done

There are other standard kinds of traversals

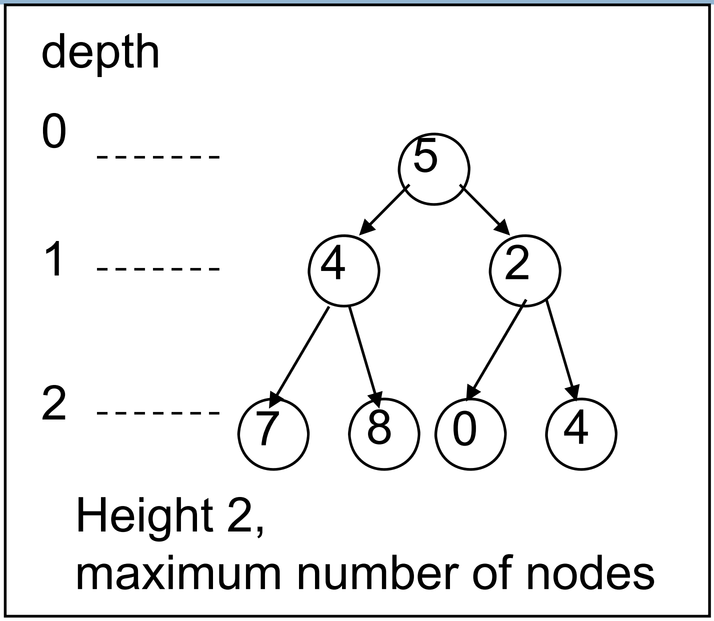
- Preorder traversal
  - Process node
  - Process left subtree
  - Process right subtree
- Postorder traversal
  - Process left subtree
  - Process right subtree
  - Process node
- Level-order traversal
  - Not recursive
  - Uses a queue

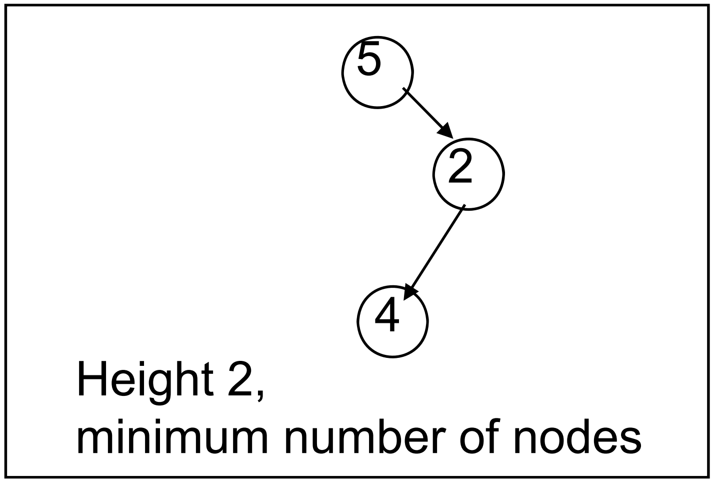
Some Useful Methods


```
//determine if a node is a leaf
public static boolean isLeaf(TreeCell node) {
   return (node != null) && (node.left == null)
                         && (node.right == null);
}

//compute height of tree using postorder traversal
public static int height(TreeCell node) {
   if (node == null) return -1; //empty tree
   if (isLeaf(node)) return 0;
   return 1 + Math.max(height(node.left),
                       height(node.right));
}

//compute number of nodes using postorder traversal
public static int nNodes(TreeCell node) {
   if (node == null) return 0;
   return 1 + nNodes(node.left) + nNodes(node.right);
}
```

Useful Facts about Binary Trees

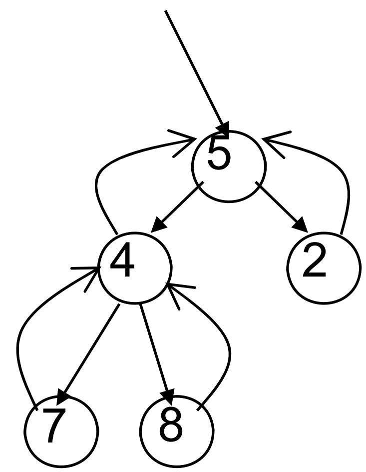
- 2^d = maximum number of nodes at depth d
- If height of tree is h
  - Minimum number of nodes in tree = $h + 1$
  - Maximum number of nodes in tree = $2^0 + 2^1 + \ldots + 2^h = 2^{h+1} - 1$
- Complete binary tree
  - All levels of tree down to a certain depth are completely filled





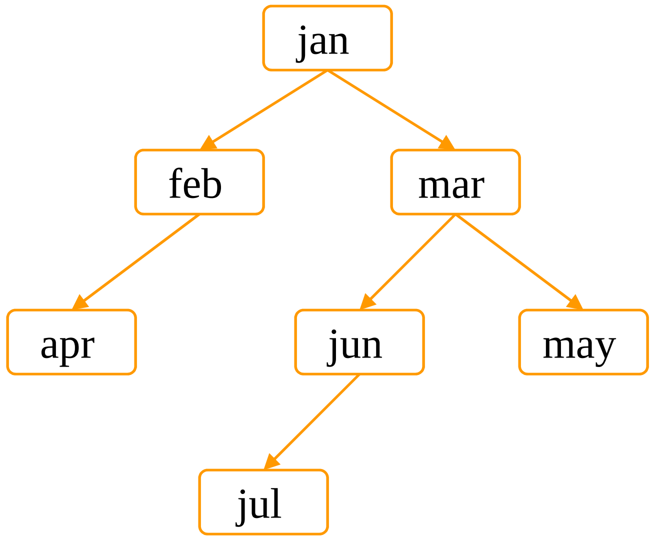
Tree with Parent Pointers

- In some applications, it is useful to have trees in which nodes can reference their parents
- Analog of doubly-linked lists



Things to Think About

- What if we want to *delete* data from a BST?
- A BST works great as long as it's *balanced*
  - How can we keep it balanced?



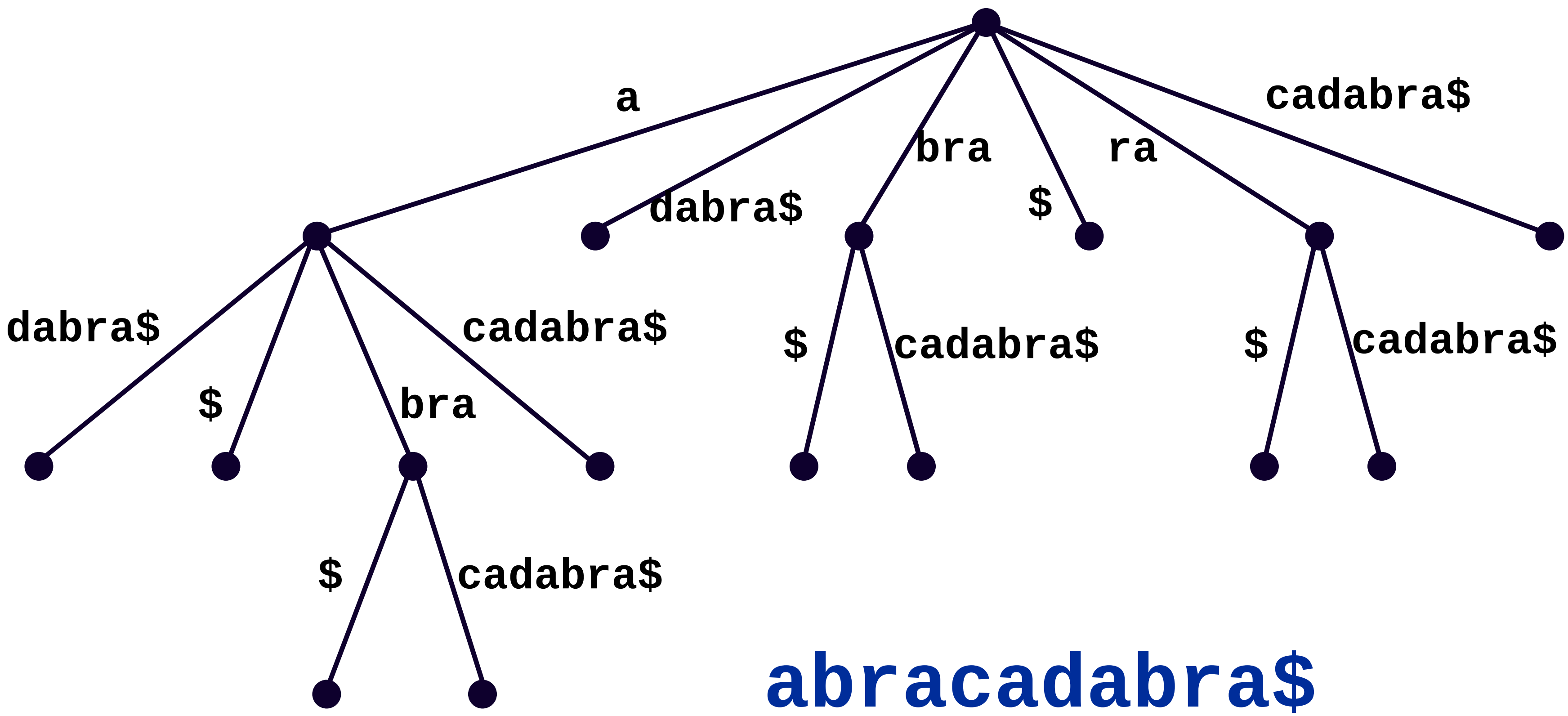
Suffix Trees

- Given a string s, a suffix tree for s is a tree such that

  - each edge has a unique label, which is a nonnull substring of s
  - any two edges out of the same node have labels beginning with different characters
  - the labels along any path from the root to a leaf concatenate together to give a suffix of s
  - all suffixes are represented by some path
  - the leaf of the path is labeled with the index of the first character of the suffix in s

- Suffix trees can be constructed in linear time

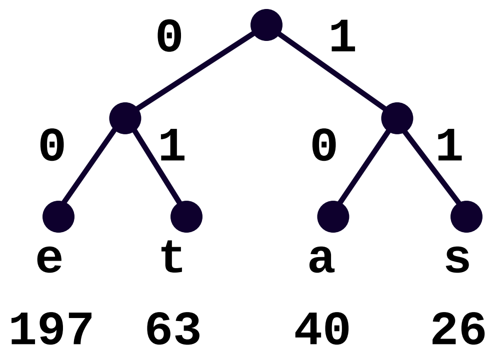
Suffix Trees

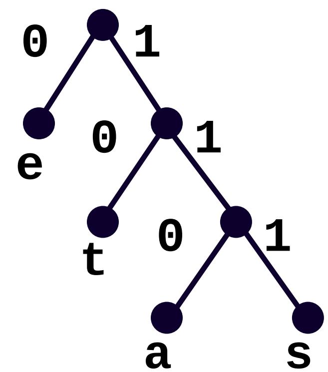


Suffix Trees

- Useful in string matching algorithms (e.g., longest common substring of 2 strings)
- Most algorithms linear time
- Used in genomics (human genome is ~4GB)

Huffman Trees





Fixed length encoding
197*2 + 63*2 + 40*2 + 26*2 = 652

Huffman encoding
197*1 + 63*2 + 40*3 + 26*3 = 521

Huffman Compression of “Ulysses”

- ' ' 242125 00100000 3 110
- 'e' 139496 01100101 3 000
- 't' 95660 01110100 4 1010
- 'a' 89651 01100001 4 1000
- 'o' 88884 01101111 4 0111
- 'n' 78465 01101110 4 0101
- 'i' 76505 01101001 4 0100
- 's' 73186 01110011 4 0011
- 'h' 68625 01101000 5 11111
- 'r' 68320 01110010 5 11110
- 'l' 52657 01101100 5 10111
- 'u' 32942 01110101 6 111011
- 'g' 26201 01100111 6 101101
- 'f' 25248 01100110 6 101100
- '.' 21361 00101110 6 011010
- 'p' 20661 01110000 6 011001

- ...

- '7' 68 00110111 15 111010101001111
- '/' 58 00101111 15 111010101001110
- 'X' 19 01011000 16 0110000000100011
- '&' 3 00100110 18 011000000010001010
- '%' 3 00100101 19 0110000000100010111
- '+' 2 00101011 19 0110000000100010110
- original size 11904320
- compressed size 6822151
- 42.7% compression

BSP Trees

- BSP = Binary Space Partition
- Used to render 3D images composed of polygons
- Each node n has one polygon p as data
- Left subtree of n contains all polygons on one side of p
- Right subtree of n contains all polygons on the other side of p
- Order of traversal determines occlusion!

Tree Summary

- A *tree* is a recursive data structure
  - Each cell has 0 or more successors (*children*)
  - Each cell except the *root* has at exactly one predecessor (*parent*)
  - All cells are reachable from the *root*
  - A cell with no children is called a *leaf*
- Special case: *binary tree*
  - Binary tree cells have a left and a right child
  - Either or both children can be null
- Trees are useful for exposing the recursive structure of natural language and computer programs