Computation, Information, and Intelligence (COMS/ENGRI/INFO/COGST 172), Fall 2005
9/16/05: **Lecture 10 aid – Perceptron learning in the on-line setting**

Agenda: perceptrons as linear separators; the on-line learning setting and the need, at least in the perceptron-learning case, for restrictions on the oracle. We may also get to Rosenblatt's perceptron learning algorithm.

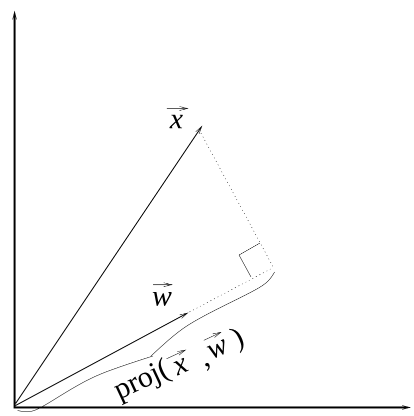
I. Recall: perceptron functions Given a weight vector $\overrightarrow{w}$ in $\Re^n$, $n \geq 1$, and a threshold value T, the function $f_{\overrightarrow{w},T} : \Re^n \rightarrow \{+1, -1\}$ is given by

$$f_{\overrightarrow{w},T}(\overrightarrow{x}) = \begin{cases} +1, & \overrightarrow{w} \cdot \overrightarrow{x} \geq T \\ -1 & \text{otherwise} \end{cases} .$$

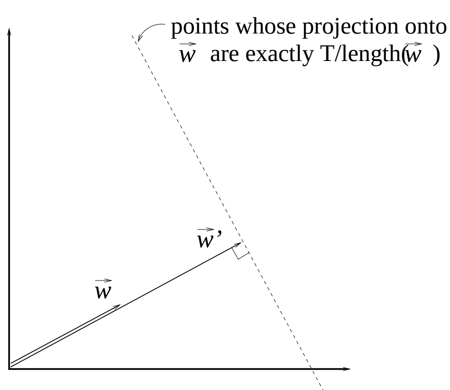
If length $(\overrightarrow{w}) > 0$, then this can be rewritten as

$$f_{\overrightarrow{w},T}(\overrightarrow{x}) = \begin{cases} +1, & \text{proj}(\overrightarrow{x}, \overrightarrow{w}) \geq \frac{T}{\text{length}(\overrightarrow{w})} \\ -1 & \text{otherwise} \end{cases} ,$$

where we are defining the projection of $\overrightarrow{x}$ onto $\overrightarrow{w}$ to be the (signed) length from the origin to the point on $\overrightarrow{w}$ that results from dropping a perpendicular from $\overrightarrow{x}$ to $\overrightarrow{w}$. (This is a signed distance because it can be negative if $\overrightarrow{x}$ points "behind" $\overrightarrow{w}$.)



II. Linear separation Continuing with the above conditions, if we let $\overrightarrow{w'}$ be the vector that points in the direction of $\overrightarrow{w}$ but has length $T/\text{length}\,(\overrightarrow{w})$, we get this picture:



Hence, a perceptron function is a *linear separator* corresponding to a *half-plane* concept.

III. Conventions for the oracle's sequence of labeled examples We denote the instances by $\overrightarrow{x}^{(1)}, \overrightarrow{x}^{(2)}, \ldots, \overrightarrow{x}^{(i)}, \ldots$. Those that are given label +1 are known as *positive* examples; those that are given label -1 are known as *negative* examples.

IV. Identification in the limit (a success criterion for on-line learning) After seeing a finite number of examples,

- the learner always outputs the same hypothesis ($\overrightarrow{w}$ and T) from then on, and
- this hypothesis correctly predicts the label of every subsequent example.