L12. More on Functions

Header, Specification, Body
Input Parameter List
Output Parameter List
Built-Ins: **randn, imag, real, max, min, ginput**

Eg. 1: "Gap N"

Keep tossing a fair coin until

| Heads - Tails | == N

Score = total # tosses

Write a function Gap(N) that returns the score and estimate the average value.

The Packaging…

```
function nTosses = Gap(N)
```

```
Heads = 0; Tails = 0; nTosses = 0;
while abs(Heads-Tails) < N
   nTosses = nTosses + 1;
   if rand <.5
       Heads = Heads + 1;
   else
       Tails = Tails + 1;
   end
end
```

The Header…

```
function nTosses = Gap(N)
```

output parameter list

input parameter list

The Body

```
Heads = 0; Tails = 0; nTosses = 0;
while abs(Heads-Tails) < N
   nTosses = nTosses + 1;
   if rand <.5
       Heads = Heads + 1;
   else
       Tails = Tails + 1;
   end
end
```

The necessary output value is computed.

Local Variables

```
Heads = 0; Tails = 0; nTosses = 0;
while abs(Heads-Tails) < N
   nTosses = nTosses + 1;
   if rand <.5
       Heads = Heads + 1;
   else
       Tails = Tails + 1;
   end
end
```

A Helpful Style

```
Heads = 0; Tails = 0; n = 0;
while abs(Heads-Tails) < N
   n = n + 1;
   if rand <.5
       Heads = Heads + 1;
   else
       Tails = Tails + 1;
   end
end
nTosses = n;
```

Explicitly assign output value at the end.

The Specification…

```
function nTosses = Gap(N)
```

```
% Simulates a game where you
% keep tossing a fair coin
% until |Heads - Tails| == N.
% N is a positive integer and
% nTosses is the number of
% tosses needed.
```

Estimate Expected Value of Gap(N)

Strategy:

Play "Gap N" a large number of times.

Compute the average "score."

That estimates the expected value.

Solution…

```
N = input('Enter N:');
nGames = 10000;
s = 0;
for k=1:nGames
   s = s + Gap(N);
end
ave = s/nGames;
```

A very common methodology for the estimation of expected value.

Sample Outputs

```
N = 10   Expected Value =  98.67
```

```
N = 20   Expected Value = 395.64
```

```
N = 30   Expected Value = 889.11
```

Solution…

```
N = input('Enter N:');
nGames = 10000;
s = 0;
for k=1:nGames
   s = s + Gap(N);
end
ave = s/nGames;
```

Program development is made easier by having a function that handles a single game.

What if the Game Was Not " Packaged"?

```
s = 0;
for k=1:nGames
   score = Gap(N)
   s = s + score;
end
ave = s/nGames;
```

```
s = 0;
for k=1:nGames
   Heads = 0; Tails = 0; nTosses = 0;
   while abs(Heads-Tails) < N
      nTosses = nTosses + 1;
      if rand <.5
         Heads = Heads + 1;
      else
         Tails = Tails + 1;
      end
   end
   score = nTosses;
   s = s + score;
end
ave = s/nGames;
```

A more cumbersome implementation

Is there a Pattern?

```
N = 10   Expected Value =  98.67
N = 20   Expected Value = 395.64
N = 30   Expected Value = 889.11
```

New Problem

Estimate expected value of Gap(N) for a range of N-values, say, N = 1:30

Pseudocode

for N=1:30

Estimate expected value of Gap(N)

Display the estimate.

end

Pseudocode

for N=1:30

Estimate expected value of Gap(N)

Display the estimate.

end

Refine this!

Done that..

```
nGames = 10000;
s = 0;
for k=1:nGames
   s = s + Gap(N);
end
ave = s/nGames;
```

Sol'n Involves a Nested Loop

```
for N = 1:30
% Estimate the expected value of Gap(N)
   s = 0;
   for k=1:nGames
      s = s + Gap(N);
   end
   ave = s/nGames;
   fprintf('%3d   %16.3f\n',N,ave))
end
```

Sol'n Involves a Nested Loop

```
for N = 1:30
% Estimate the expected value of Gap(N)
   s = 0;
   for k=1:nGames
      s = s + Gap(N);
   end
   ave = s/nGames;
   fprintf('%3d   %16.3f\n',N,ave))
end
```

But during derivation, we never had to reason about more than one loop.

Output

```
 N       Expected Value of Gap(N)
--------------------------------
 1              1.000
 2              4.009
 3              8.985
 4             16.094

28            775.710
29            838.537
30            885.672
```

Looks like N^2.

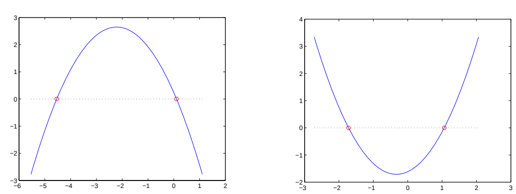
Maybe increase nTrials to solidify conjecture.

Eg. 2: Random Quadratics

Generate random quadratic
$q(x) = ax^2 + bx + c$

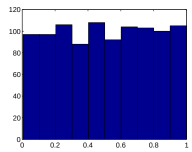
If it has real roots, then plot q(x) and highlight the roots.

Sample Output

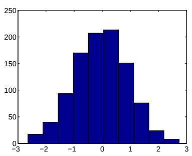


Built-In Function: randn

```
% Uniform
for k=1:1000
    x = rand;
end
```




```
% Normal
for k=1:1000
    x = randn;
end
```



Built-In Functions: imag and real

```
x = 3 + 4*sqrt(-1);

y = real(x)

z = imag(x)
```

Assigns 3 to y.

Assigns 4 to z.

Built-In Functions: min and max

```
a = 3, b = 4;

y = min(a,b)

z = max(a,b)
```

Assigns 3 to y.

Assigns 4 to z.

Packaging the Coefficient Computation

```
 function [a,b,c] = randomQuadratic
% a, b, and c are random numbers,
% normally distributed.

a = randn;
b = randn;
c = randn;
```

Input & Output Parameters

```
function [a,b,c] = randomQuadratic
```

A function can have more than one output parameter.

Syntax: [v1,v2,...]

A function can have no input parameters.

Syntax: Nothing

Computing the Roots

```
  function [r1,r2] = rootsQuadratic(a,b,c)

% a, b, and c are real.
% r1 and r2 are roots of
% ax^2 + bx +c = 0.

  r1 = (-b - sqrt(b^2 - 4*a*c))/(2*a);
  r2 = (-b + sqrt(b^2 - 4*a*c))/(2*a);
```

Question Time

```
function [r1,r2] = rootsQuadratic(a,b,c)
r1 = (-b - sqrt(b^2 - 4*a*c))/(2*a);
r2 = (-b + sqrt(b^2 - 4*a*c))/(2*a);
```

```
a = 4; b = 0; c = -1;
[r2,r1] = rootsQuadratic(c,b,a);
r1 = r1
```

Output?

A. 2 B. -2 C. .5 D. -.5

Answer is B.

We are asking **rootsQuadratic** to solve

$-x^2 + 4 = 0$ roots = +2 and -2

Since the function ca ll is equivalent to [r2,r1] = rootsQuadratic(-1,0,4);

Script variable **r1** is assigned the value that **rootsQuadratic** returns through output parameter r2. That value is -2

Script Pseudocode

```
for k = 1:10
   Generate a random quadratic
   Compute its roots
   If the roots are real
       then plot the quadratic and
       show roots
end
```

Script Pseudocode

```
for k = 1:10
   Generate a random quadratic
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       then plot the quadratic and
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end
```

```
[a,b,c] = randomQuadratic;
```

Script Pseudocode

```
for k = 1:10
   [a,b,c] = randomQuadratic;
   Compute its roots
   If the roots are real
       then plot the quadratic and
       show roots
end
```

```
[r1,r2] = rootsQuadratic(a,b,c);
```

Script Pseudocode

```
for k = 1:10
   [a,b,c] = randomQuadratic;
   [r1,r2] = rootsQuadratic(a,b,c);
   If the roots are real
       then plot the quadratic and
       show roots
end
```

```
if imag(r1)==0 && imag(r2)===0
```

Script Pseudocode

```
for k = 1:10
  [a,b,c] = randomQuadratic;
  [r1,r2] = rootsQuadratic(a,b,c);
  if imag(r1)==0 && imag(r2)==0
      then plot the quadratic and
      show roots
  end
end
```

Plot the Quadratic and Show the Roots

```
m = min(r1,r2);
M = max(r1,r2);
x = linspace(m-1,M+1,100);
y = a*x.^2 + b*x + c;
plot(x,y,x,0*y,':k',r1,0,'or',r2,0,'or')
```

Plot the Quadratic and Show the Roots

```
m = min(r1,r2);
M = max(r1,r2);
x = linspace(m-1,M+1,100);
y = a*x.^2 + b*x + c;
plot(x,y,x,0*y,':k',r1,0,'or',r2,0,'or')
```

This determines a nice range of x-values.

Plot the Quadratic and Show the Roots

```
m = min(r1,r2);
M = max(r1,r2);
x = linspace(m-1,M+1,100);
y = a*x.^2 + b*x + c;
plot(x,y,x,0*y,':k',r1,0,'or',r2,0,'or')
```

Array ops get the y-values.

Plot the Quadratic and Show the Roots

```
m = min(r1,r2);
M = max(r1,r2);
x = linspace(m-1,M+1,100);
y = a*x.^2 + b*x + c;
plot(x,y,x,0*y,':k',r1,0,'or',r2,0,'or')
```

Graphs the quadratic.

Plot the Quadratic and Show the Roots

```
m = min(r1,r2);
M = max(r1,r2);
x = linspace(m-1,M+1,100);
y = a*x.^2 + b*x + c;
plot(x,y,x,0*y,':k',r1,0,'or',r2,0,'or')
```

A black, dashed line x-axis.

Plot the Quadratic and Show the Roots

```
m = min(r1,r2);
M = max(r1,r2);
x = linspace(m-1,M+1,100);
y = a*x.^2 + b*x + c;
plot(x,y,x,0*y,':k',r1,0,'or',r2,0,'or')
```

Highlight the root r1 with red circle.

Plot the Quadratic and Show the Roots

```
m = min(r1,r2);
M = max(r1,r2);
x = linspace(m-1,M+1,100);
y = a*x.^2 + b*x + c;
plot(x,y,x,0*y,':k',r1,0,'or',r2,0,'or')
```

Highlight the root r2 with red circle.

Complete Solution

```
for k=1:10
   [a,b,c] = randomQuadratic;
   [r1,r2] = rootsQuadratic(a,b,c);
   if imag(r1)==0
     m = min(r1,r2); M = max(r1,r2);
     x = linspace(m-1,M+1,100);
     y = a*x.^2 + b*x + c;
     plot(x,y,x,0*y,':k',r1,0,'or',r2,0,'or')
     shg
     pause(1)
   end
end
```