Image transformations

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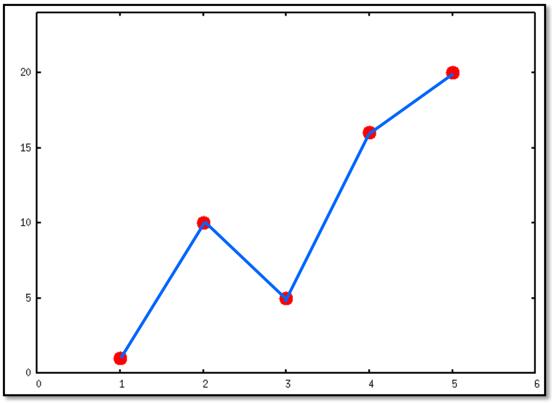
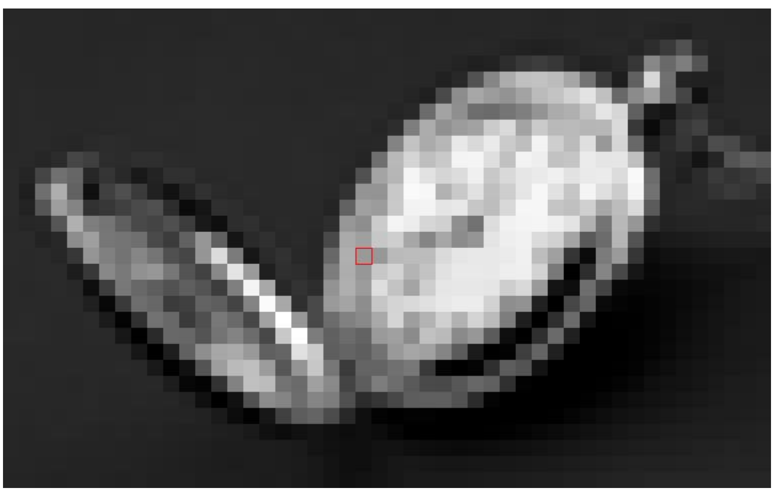
Administrivia

Last time: Interpolation



Nearest neighbor interpolation



Bilinear interpolation

Bicubic interpolation

Gray2Color

http://www.cs.huji.ac.il/~yweiss/Colorization/
(Matlab code available)

Example algorithm that can't exist

- Consider a compression algorithm, like zip
  - Take a file F, produce a smaller version F'
  - Given F', we can uncompress to recover F
  - This is lossless compression, because we can "invert" it
    - MP3, JPEG, MPEG, etc. are not lossless

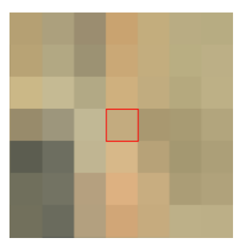
- Claim: there is no such algorithm that always produces a **smaller** file F' for every input file F

Proof of claim (by contradiction)

- Pick a file F, produce F′ by compression
  - F′ is smaller than F, by assumption
- Now run compression on F′
  - Get an even smaller file, F′′
- At the end, you've got a file with only a single byte (a number from 0 to 255)
  - Yet by repeatedly uncompressing this you can eventually get F
- However, there are more than 256 different files F that you could start with!

Conclusions

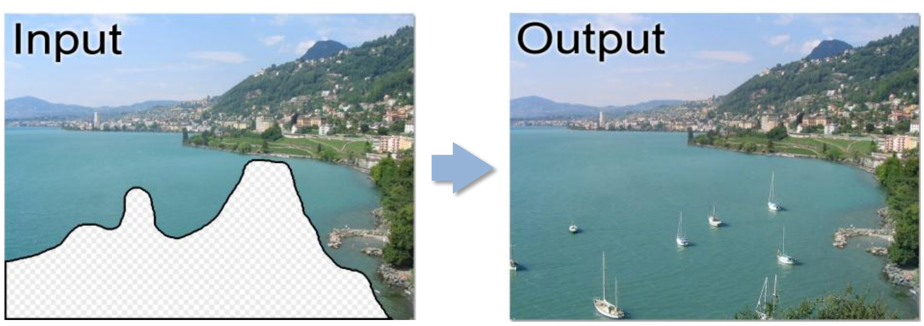
1. Some files will get larger if you compress them (usually files with random data)
2. We can't (always) correctly recover missing data using interpolation
3. A low-res image can represent multiple high-res images



Extrapolation

- Suppose you only know the values f(1), f(2), f(3), f(4) of a function
  - What is f(5)?
- This problem is called extrapolation
  - Much harder than interpolation: what is outside the image?
  - For the particular case of temporal data, extrapolation is called prediction (what will the value of MSFT stock be tomorrow?)
  - If you have a good model, this can work well

Image extrapolation



http://graphics.cs.cmu.edu/projects/scene-completion/
Computed using a database of millions of photos

Questions?

Today: image transformations

2D Transformations

- 2D Transformation:
  - Function from 2D → 2D

    $$f(x, y) = (x', y')$$

  - We'll apply this function to every pixel to get a new pixel location

- Examples:

  $$f(x, y) = (0.5x, 1.5y)$$

  $$f(x, y) = (y, x)$$

2D Transformations

$f(x, y) = (0.5x, 2y)$

2D Transformations

$f(x, y) = (y, x)$

2D Transformations

- Can be non-linear:

2D Transformations

image credit: Matt Brown

Linear transformations

- We will focus on linear transformations
- 1D:

  f(x) = ax
- 2D:

  f(x, y) = (ax + by, cx + dy)

- Examples
  1. f(x, y) = (0.5x, 1.5y)
  2. f(x, y) = (y, x)

2D Linear Transformations

- Can be represented with a 2D matrix

$$T = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

- And applied to a point using matrix multiplication

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

2D Linear Transformations

- Can be represented with a 2D matrix

$$T = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad p = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$Tp = q$$

Examples

- f(x, y) = (0.5x, 1.5y)

$$T = \begin{bmatrix} 0.5 & 0 \\ 0 & 1.5 \end{bmatrix}$$

- f(x, y) = (y, x)

$$T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Common linear transformations

- Uniform scaling:

$$S = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \qquad \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} sx \\ sy \end{bmatrix}$$

Common linear transformations

- Rotation by angle θ

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Common linear transformations

- Shear

$$H = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$$

Composing linear transformations

- What if we want to scale *and* rotate?
- Answer: multiply the matrices together

$$S = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \qquad R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

scale *rotation*

$$RS = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} = \begin{bmatrix} s\cos\theta & -s\sin\theta \\ s\sin\theta & s\cos\theta \end{bmatrix}$$

scale and rotation

Questions?

Implementing transformations

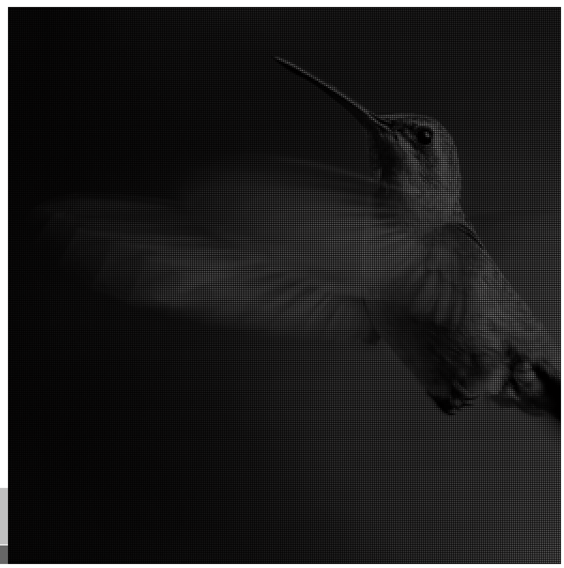
- First approach (grayscale image)

```
function img_out = transform_image(img_in, T)

[num_rows, num_cols] = size(img_in);
img_out = zeros(num_rows, num_cols);

for row = 1:num_rows
    for col = 1:num_cols
        p = [col; row];
        p_new = T * p;
        img_out(p_new(2), p_new(1)) = img_in(row, col);
    end
end
```


Forward mapping



How do we fix this?

- Answer: do the opposite

1. Create an output image
2. For each pixel in the output image, find the corresponding pixel in the input image
3. Give that output pixel the same color

- Requires that we invert the mapping

Inverse mapping

- How do we invert the mapping?

- With linear transformations T, we invert T

$$T = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad T^{-1}T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$T^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Inverse mapping

Resampling

- Suppose we scale image by 2
- T = [2 0 ; 0 2]
- inv(T) =
- Pixel (5,5) in img_out should be colored with pixel (2.5, 2.5) in img_in
- How do we find the intensity at (2.5, 2.5)?

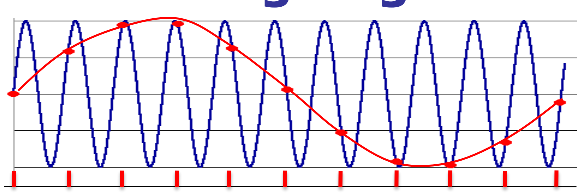
Inverse mapping

Downsampling

- Suppose we scale image by 0.25

Downsampling

What's going on?



- **Aliasing** can arise when you sample a continuous signal or image
- Occurs when the sampling rate is not high enough to capture the detail in the image
- Can give you the wrong signal/image—an alias

Questions?

Examples of aliasing

- Wagon wheel effect

- Moiré patterns

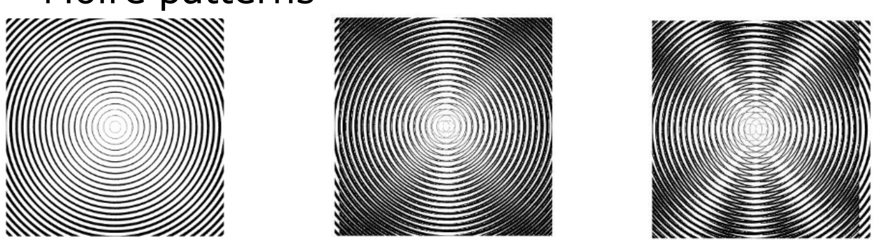


Image credit: Steve Seitz

- This image is too big to fit on the screen. How can we create a half-sized version?

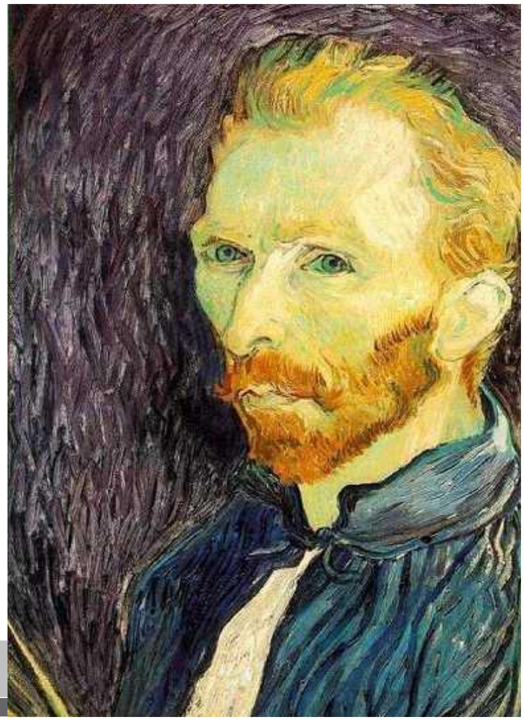
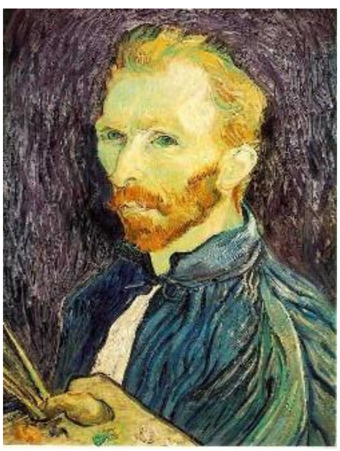


Image sub-sampling

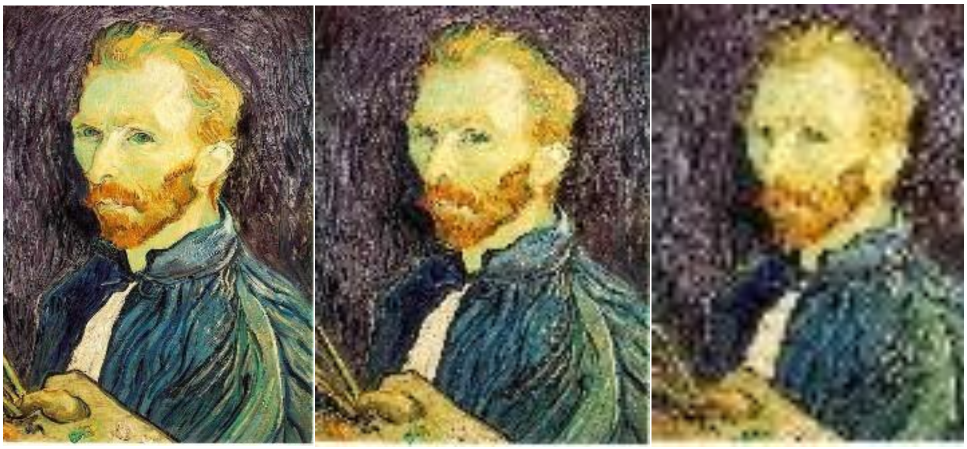


1/4

1/8

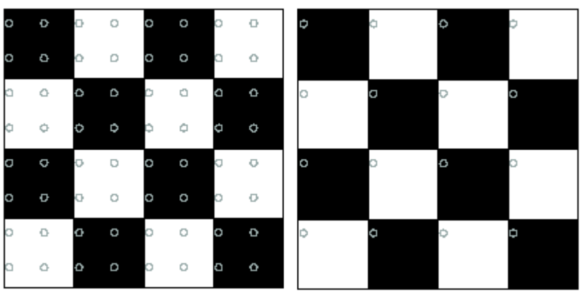
Current approach: throw away every other row and column (subsample)

Image sub-sampling

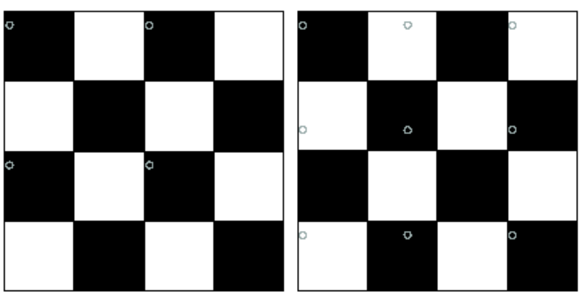


•1/2 •1/4 (2x zoom) •1/8 (4x zoom)

2D example



Good sampling



Bad sampling

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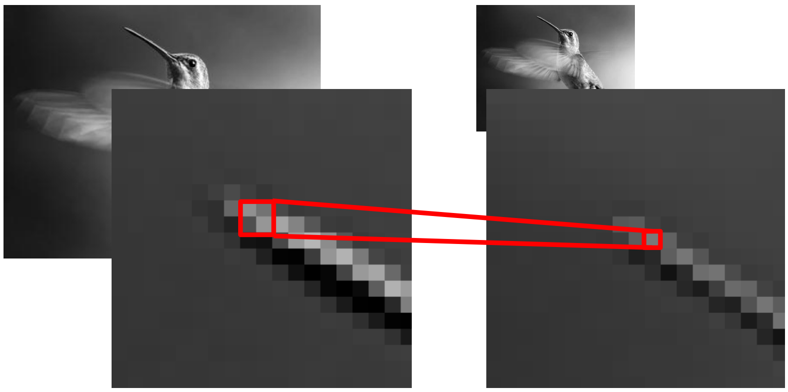
Image sub-sampling

- What's really going on?

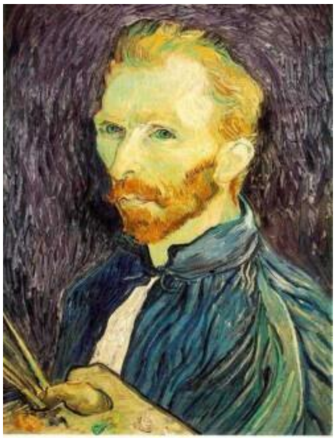


Subsampling with pre-filtering

Average 2x2 *Average 4x4* *Average 8x8*

- Solution: blur the image, then subsample
  - Filter size should double for each ½ size reduction.

Subsampling with pre-filtering

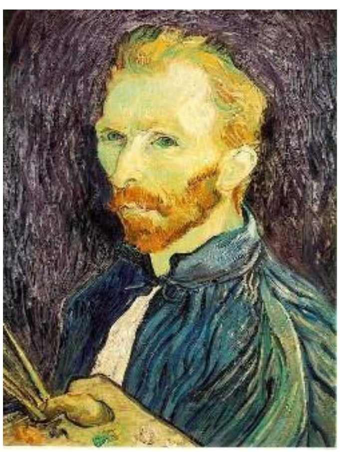

Average 2x2


Average 4x4


Average 8x8

- Solution: blur the image, then subsample
  - Filter size should double for each ½ size reduction.

Compare with




1/4


1/8