Sorting and Selection, Part 1

Prof. Noah Snavely
CS1114
http://www.cs.cornell.edu/courses/cs1114

Cornell University
Computer Science

Administrivia

- Assignment 1 due Friday by 5pm
  - Please sign up for a demo slot using CMS (or demo before Friday)
- Assignment 2 out on Friday

“Corner cases”

- Sometimes the input to a function isn’t what you expect
  - What is the maximum element of a vector of length 0?
- When writing a function, you should try and anticipate such corner cases

Recap from last time

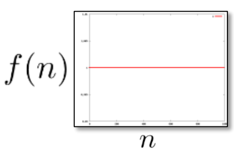
- We looked at the “trimmed mean” problem for locating the lightstick
  - Remove 5% of points on all sides, find centroid
- This is a version of a more general problem:
  - Finding the k^{th} largest element in an array
  - Also called the “selection” problem
- We considered an algorithm that repeatedly removes the largest element
  - How fast is this algorithm?

Recap from last time

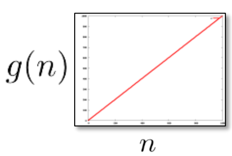
- Big-O notation allows us to reason about speed without worrying about
  - Getting lucky on the input
  - Depending on our hardware

- Big-O of repeatedly removing the biggest element?
  - Worst-case (k = n/2, i.e., median) is quadratic, O(n^2)

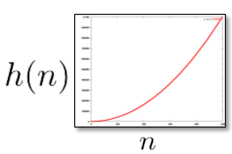
Classes of algorithm speed (complexity)



- Constant time algorithms, $O(1)$
  - Do not depend on the input size
  - Example: find the first element



- Linear time algorithms, $O(n)$
  - Constant amount of work for every input item
  - Example: find the largest element

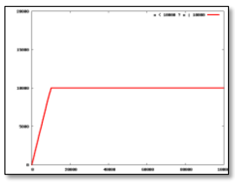


- Quadratic time algorithms, $O(n^2)$
  - Linear amount of work for every input item
  - Example: repeatedly removing max element

Asymptotic complexity

- Big-O only cares about the number of operations as *n* (the size of the input) grows large ($n \rightarrow \infty$)

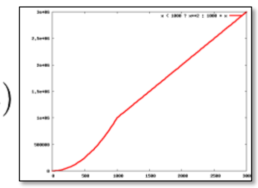
$f(n)$



n

→ O(1)

$g(n)$



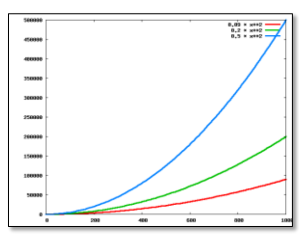
n

→ O(n)

Complexity classes

- Big-O doesn't care about constant coefficients
  - "Constant of proportionality" doesn't matter

$$0.001n = O(n)$$
$$1{,}000{,}000n = O(n)$$



→ $O(n^2)$

♦ What is the complexity of:

1. Finding the 2nd biggest element (> all but 1)?

2. Finding the element bigger than all but 2%?
   - Assume we do this by repeated "find biggest"

3. Multiplying two *n*-digit numbers (using long multiplication)?

How to do selection better?

- If our input were sorted, we can do better
  - Given 100 numbers in increasing order, we can easily figure out the k^{th} biggest or smallest (with what time complexity?)
- Very important principle! (encapsulation)
  - Divide your problem into pieces
    - One person (or group) can provide `sort`
    - The other person can use `sort`
  - As long as both agree on what `sort` does, they can work independently
  - Can even "upgrade" to a faster `sort`

How to sort?

- Sorting is an ancient problem, by the standards of CS
  - First important "computer" sort used for 1890 census, by Hollerith (the 1880 census took 8 years, 1890 took just one)
- There are many sorting algorithms

How to sort?

- Given an array of numbers:

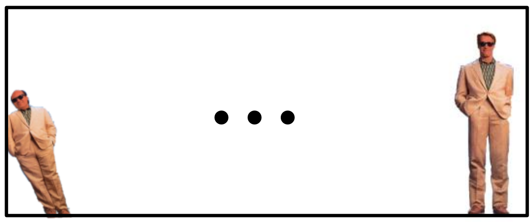
  [10 2 5 30 4 8 19 102 53 3]

- How can we produce a sorted array?

  [2 3 4 5 8 10 19 30 53 102]

How to sort?

- A concrete version of the problem
  - Suppose I want to sort all actors by height



  - How do I do this?

Sorting, 1st attempt

- Idea: Given *n* actors

1. Find the shortest actor, put him/her first
2. Find the shortest actor in the remaining group, put him/her second

… Repeat …

n. Find the shortest actor in the remaining group (one left), put him/her last

Sorting, 1st attempt

Algorithm 1

1. Find the shortest actor put him first
2. Find the shortest actor in the remaining group, put him/her second

   … Repeat …

n. Find the shortest actor in the remaining group put him/her last

- What does this remind you of?
- This is called *selection sort*
- After round *k*, the first *k* entries are sorted

Selection sort – pseudocode

```
function [ A ] = selection_sort(A)
% Returns a sorted version of array A
%   by applying selection sort
%   Uses in place sorting
n = length(A);
for i = 1:n
    % Find the smallest element in A(i:n)
    % Swap that element with something (what?)
end
```

Filling in the gaps

- `% Find the smallest element in A(i:n)`
- We pretty much know how to do this

```
m = A(i); m_index = i;
for j = i+1:n
    if A(j) < m
        m = A(j); m_index = j;
    end
end
```

```
[ 10 13 41 6 51 11 ]
% After round 1,
%  m = 6, m_index = 4
```


Filling in the gaps

- `% Swap the smallest element with something`
- `% Swap element A(m_index) with A(i)`

~~A(i) = A(m_index);~~
~~A(m_index) = A(i);~~

```
tmp = A(i);
A(i) = A(m_index);
A(m_index) = tmp;
```

```
[ 10 13 41 6 51 11 ]

[ 6 13 41 10 51 11 ]
```

Putting it all together

```
function [ A ] = selection_sort(A)
% Returns a sorted version of array A
n = length(A);
for i = 1:n
    % Find the smallest element in A(i:len)
    m = A(i); m_index = i;
    for j = i:n
        if A(j) < m
            m = A(j); m_index = j;
        end
    end
    % Swap element A(m_index) with A(i)
    tmp = A(i);
    A(i) = A(m_index);
    A(m_index) = tmp;
end
```


Example of selection sort

[10 13 41 6 51 11]

[6 13 41 10 51 11]

[6 10 41 13 51 11]

[6 10 11 13 51 41]

[6 10 11 13 51 41]

[6 10 11 13 41 51]

[6 10 11 13 41 51]

Speed of selection sort

- Let n be the size of the array
- How fast is selection sort?

O(1)   O(n)   O(n^2)   ?

- How many comparisons (<) does it do?
- First iteration: n comparisons
- Second iteration: $n - 1$ comparisons

  …
- n^{th} iteration: 1 comparison

Speed of selection sort

- Total number of comparisons:

  $n + (n - 1) + (n - 2) + \ldots + 1$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

- Work grows in proportion to n^2 → selection sort is O(n^2)

Other ideas for sorting?