Robustness and speed

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Administrivia

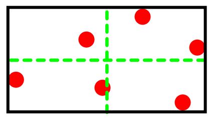
- Assignment 1 is due next Friday by 5pm
  - Lots of TA time available (check the web)

- For grading, please get checked off by a TA, or sign up for a demo slot
  - These will be posted to CMS soon

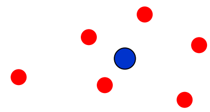
Finding the lightstick center

- Last time: two approaches

Bounding box



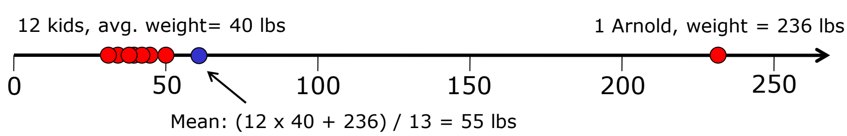
Centroid



- Both have problems…

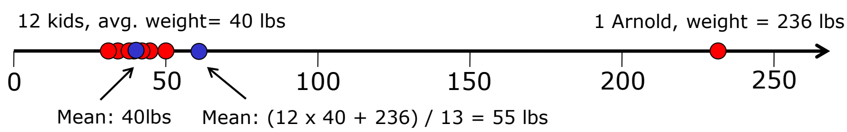
How can we do better?

- What is the average weight of the people in this kindergarten class photo?



How can we do better?

- Idea: remove maximum value, compute average of the rest



5% Trimmed mean

```
% A is a vector of length 100
for i = 1:5
  % 1. Find the maximum element of A
  % 2. Remove it
end
```

- Is it correct?
- Is it fast?
- Is it *the fastest* way?

How do we define fast?

- We want to think about this issue in a way that doesn't depend on either:
  A. Getting really lucky input
  B. Happening to have really fast hardware

How fast is our algorithm?

- An elegant answer exists
- You will learn it in later CS courses
  - But I'm going to steal their thunder and explain the basic idea to you here
  - It's called "big-O notation"

- Two basic principles:
  - Think about the average / worst case
    - Don't depend on luck
  - Think in a hardware-independent way
    - Don't depend on Intel!

Simple example: finding the max

```
function m = find_max(A)
% Find the maximum element of an array A
m = A(1);
n = length(A);
for i = 2:n
    if (A(i) > m)
        m = A(i);
    end
end
```

how many times will this comparison be done?

- How much work is this?

Big-O Notation

```
function m = find_max(A)
% Find the maximum element of an array A
m = -1;
for i = 1:length(A)
    if (A(i) > m)
        m = A(i);
    end
end
```

- Let's call the length of the array n
- The amount of work grows in proportion to n
- We say that this algorithm runs in time $O(n)$
- (Or that it is a *linear-time* algorithm)

Another version of the trimmed mean

- Given an array of n numbers, find the k^{th} largest number in the array
- Strategy:
  1. Find the biggest number in the array
  2. Remove it
  - Repeat k times
  - The answer is the last number you found

Performance of our algorithm

- Given an array of n numbers, find the k^{th} largest number in the array
- Strategy:
  1. Find the biggest number in the array
  2. Remove it
  - Repeat k times
  - The answer is the last number you found

- How many operations do we need to do, in terms of k and n?

Performance of our algorithm

- How much work will we do?

1. Examine *n* elements to find the biggest
2. Examine *n*-1 elements to find the biggest

… keep going …

k. Examine *n*-(*k*-*1*) elements to find the biggest

Performance of our algorithm

- What value of *k* is the worst case?
  - ~~*k* = *n*~~ we can easily fix this
  - *k* = *n*/2

- How much work will we do in the worst case?
  1. Examine *n* elements to find the biggest
  2. Examine *n*-1 elements to find the biggest

     … keep going …

  n/2. Examine *n*/2 elements to find the biggest

How much work is this?

- How many elements will we examine in total?

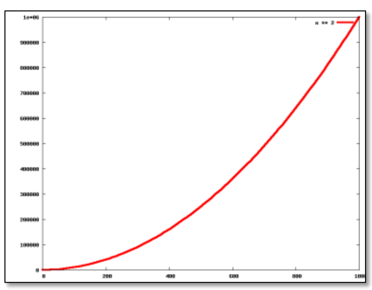
$$\underbrace{n + (n - 1) + (n - 2) + \ldots + n/2}_{n\ /\ 2\ \text{terms}}$$

$$= ?$$

- We don't really care about the exact answer
  - It's bigger than $(n / 2)^2$ and smaller than n^2

How much work in the worst case?

- The amount of work grows in proportion to n^2

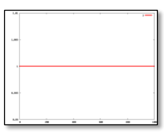


- We say that this algorithm is $O(n^2)$

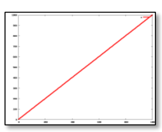
How much work is this?

- The amount of work grows *in proportion* to n^2
- We say that this algorithm is $O(n^2)$
- If it takes 10 seconds when n = 1,000, how long will it take when n = 2,000?
  - A: 20 seconds
  - B: 40 seconds

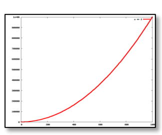
Classes of algorithm speed



- Constant time algorithms, $O(1)$
  - Do not depend on the input size
  - Example: find the first element

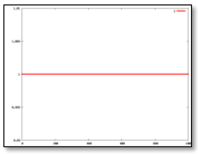


- Linear time algorithms, $O(n)$
  - Constant amount of work for every input item
  - Example: find the largest element

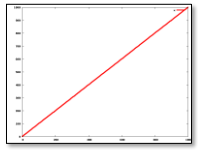


- Quadratic time algorithms, $O(n^2)$
  - Linear amount of work for every input item
  - Example: slow median method

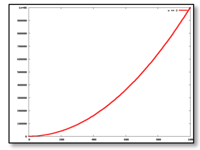
Asymptotic analysis picture



$O(1)$



$O(n)$



$O(n^2)$

- Different hardware only affects the parameters (i.e., line slope)
- As *n* gets big, the "dumber" algorithm by this measure always loses eventually

Where we are so far

- Finding the lightstick
  - Attempt 1: Bounding box (not so great)
  - Attempt 2: Centroid isn't much better
  - Attempt 3: Trimmed mean
    - Seems promising
    - But how do we compute it quickly?
    - The obvious way doesn't seem so great…

Questions?