Quickselect

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http://www.cs.cornell.edu/courses/cs1114

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Administrivia

- Assignment 2 is out
  - First part due on Friday by 4:30pm
  - Second part due next Friday by 4:30pm
  - Demos in the lab
- Quiz 2 on Thursday
  - Coverage through today
    (topics include running time, sorting)
  - Closed book / closed note

Recap from last time

- We can solve the selection problem by sorting the numbers first
- We've learned two ways to do this so far:
    1. Selection sort
    2. *Quicksort*

Quicksort

1. Pick an element (**pivot**)
2. **Partition** the array into elements < pivot, = to pivot, and > pivot
3. Quicksort these smaller arrays separately

- What is the worst-case running time?
- What is the expected running time (on a random input)?

Back to the selection problem

- Can solve with quicksort
  - Faster (on average) than "repeated remove biggest"
- Is there a better way?

- Rev. Charles L. Dodgson's problem
  - Based on how to run a tennis tournament
  - Specifically, how to award 2nd prize fairly

- How many teams were in the tournament?
- How many games were played?
- Which is the second-best team?

Standard Tournament

- Example

  [8 3 1 2 4 6 7 5]

- Compare everyone to their neighbor, keep the larger one

  [8 2 6 7]

  [8 7]

  [8]

Finding the second best team

- Could use quicksort to sort the teams
- Step 1: Choose one team as a pivot (say, Arizona)
- Step 2: Arizona plays every team
- Step 3: Put all teams worse than Arizona in Group 1, all teams better than Arizona in Group 2 (no ties allowed)
- Step 4: Recurse on Groups 1 and 2
- … eventually will rank all the teams …

Quicksort Tournament

Quicksort Tournament
Step 1: Choose one team (say, Arizona)
Step 2: Arizona plays every team
Step 3: Put all teams worse than Arizona in Group 1, all teams better than Arizona in Group 2 (no ties allowed)
Step 4: Recurse on groups 1 and 2
… eventually will rank all the teams …

- (Note this is a bit silly – AZ plays 63 games)
- This gives us a ranking of all teams
  - What if we just care about finding the 2nd-best team?

Modifying quicksort to select

- Suppose Arizona beats 36 teams, and loses to 27 teams

$\underbrace{\text{36 teams}}_{\text{Group 1}} < \text{A} < \underbrace{\text{27 teams}}_{\text{Group 2}}$

- If we just want to know the 2nd-best team, how can we save time?

Modifying quicksort to select – Finding the 2nd best team

$\underbrace{\text{36 teams}}_{\text{Group 1}} < \quad < \underbrace{\text{27 teams}}_{\text{Group 2}}$

$\underbrace{\text{16 teams}}_{\text{Group 2.1}} < \text{Cornell} < \underbrace{\text{10 teams}}_{\text{Group 2.2}}$

7 teams < < 2 teams

Modifying quicksort to select – Finding the 32nd best team

$\underbrace{\text{36 teams}}_{\text{Group 1}} < \quad < \underbrace{\text{27 teams}}_{\text{Group 2}}$

$\underbrace{\text{20 teams}}_{\text{Group 1.1}} < \text{GEORGE MASON UNIVERSITY} < \underbrace{\text{15 teams}}_{\text{Group 1.2}}$

- Q: Which group do we visit next?
- The 32nd best team overall is the 4th best team in Group 1

Find kth largest element in A (< than k-1 others)

A = [6.0 5.4 5.5 6.2 5.3 5.0 5.9]

MODIFIED QUICKSORT(A, k):

- Pick an element in A as the pivot, call it x
- Divide A into A1 (<x), A2 (=x), A3 (>x)
- If k < length(A3)
  - MODIFIED QUICKSORT (A3, k)
- If k > length(A2) + length(A3)
  - Let j = k – [length(A2) + length(A3)]
  - MODIFIED QUICKSORT (A1, j)
- Otherwise, return x

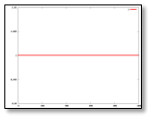
Modified quicksort

MODIFIED QUICKSORT(A, k):

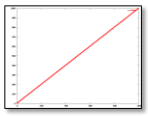
- Pick an element in A as the pivot, call it x
- Divide A into A1 (<x), A2 (=x), A3 (>x)
- If k < length(A3)
  - Find the element < k others in A3
- If k > length(A2) + length(A3)
  - Let j = k – [length(A2) + length(A3)]
  - Find the element < j others in A1
- Otherwise, return x

- We'll call this *quickselect*
- Let's consider the running time…

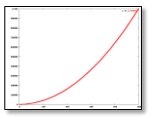
What is the running time of:



- Finding the 1st element?
  - O(1) (effort doesn't depend on input)



- Finding the biggest element?
  - O(n) (constant work per input element)



- Finding the median by repeatedly finding and removing the biggest element?
  - $O(n^2)$ (linear work per input element)

- Finding the median using quickselect?
  - Worst case? O(_________)
  - Best case? O(_________)

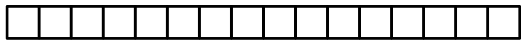
Quickselect – "medium" case

- Suppose we split the array in half each time (i.e., happen to choose the median as the pivot)

- How many comparisons will there be?

How many comparisons? ("medium" case)

- Suppose `length(A) == n`



- Round 1: Compare *n* elements to the pivot

  … now break the array in half, quickselect one half …

- Round 2: For remaining half, compare *n* / 2 elements to the pivot (total # comparisons = n / 2)

  … now break the half in half …

- Round 3: For remaining quarter, compare *n* / 4 elements to the pivot (total # comparisons = n / 4)

How many comparisons? ("medium" case)

Number of comparisons =

$$n + n / 2 + n / 4 + n / 8 + \ldots + 1$$

$$= ?$$

→ The "medium" case is O(n)!

Quickselect

- For random input this method actually runs in linear time (beyond the scope of this class)
- The worst case is still bad
- Quickselect gives us a way to find the k^{th} element without actually sorting the array!

Quickselect

- It's possible to select in *guaranteed* linear time (1973)
  - Rev. Dodgson's problem
  - But the code is a little messy
    - And the analysis is messier
    http://en.wikipedia.org/wiki/Selection_algorithm
- Beyond the scope of this course

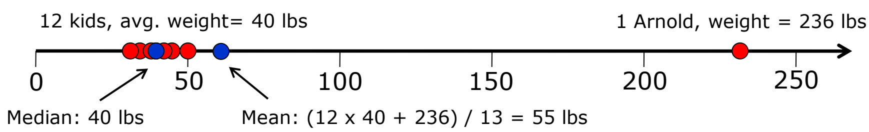
Questions?

Back to the lightstick

- By using quickselect we can find the 5% largest (or smallest) element
  - This allows us to efficiently compute the trimmed mean

What about the median?

- Another way to avoid our bad data points:
  - Use the median instead of the mean



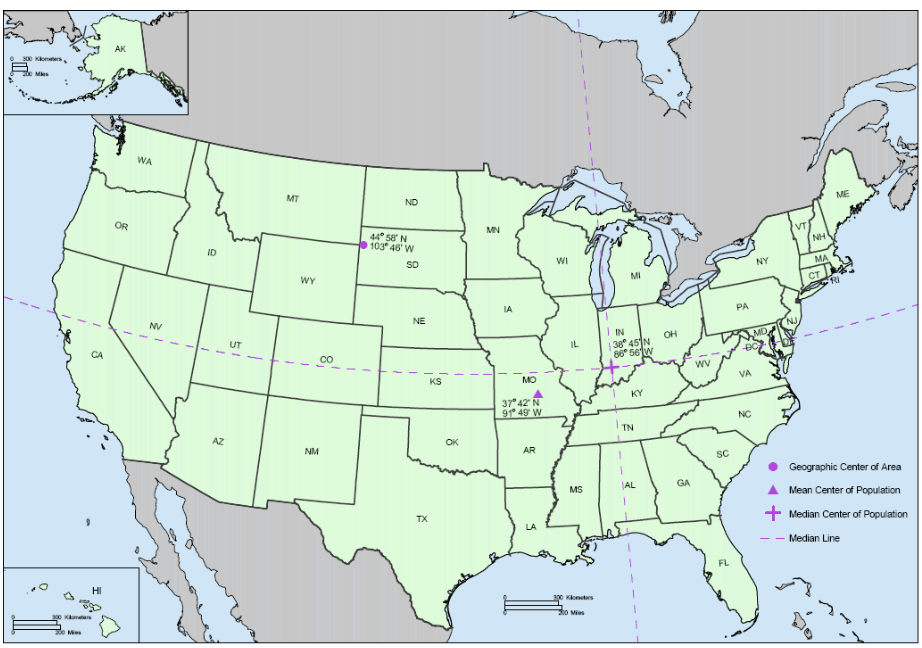
Median vector

- Mean, like median, was defined in 1D
  - For a 2D mean we used the centroid
  - Mean of x coordinates and y coordinates separately
    - Call this the “mean vector”
  - Does this work for the median also?

What is the median vector?

- In 1900, statisticians wanted to find the "geographical center of the population" to quantify westward shift
- Why not the centroid?
  - Someone being born in San Francisco changes the centroid much more than someone being born in Indiana
- What about the "median vector"?
  - Take the median of the x coordinates and the median of the y coordinates separately

Position of the Geographic Center of Area, Mean and Median Centers of Population: 2000

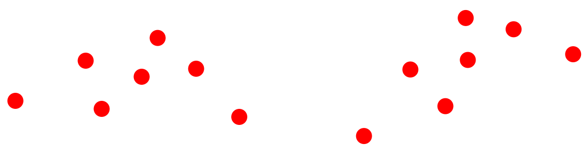


U.S. Department of Commerce Economics and Statistics Administration U.S. Census Bureau

Prepared by the Geography Division

Median vector

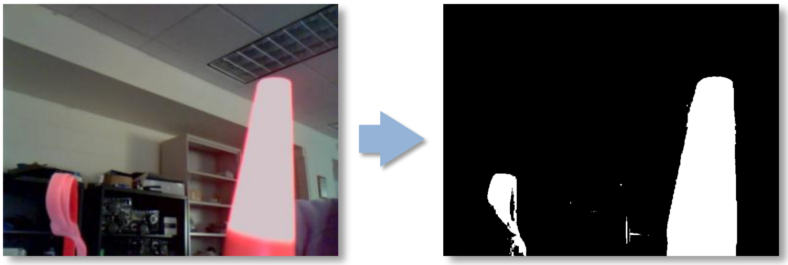
- A little thought will show you that this doesn't really make a lot of sense
  - Nonetheless, it's a common solution, and we will implement it for CS1114
  - In situations like ours it works pretty well
- It's almost never an actual datapoint
- It depends upon rotations!



Can we do even better?

- None of what we described works that well if we have widely scattered red pixels
  - And we can't figure out lightstick orientation
- Is it possible to do even better?
  - Yes!
- We will focus on:
  - Finding "blobs" (connected red pixels)
  - Summarizing the shape of a blob
  - Computing orientation from this
- We'll need brand new tricks!

Back to the lightstick



- The lightstick forms a large “blob” in the thresholded image (among other blobs)

What is a blob?

1	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	0	0	0	0	0	0

Finding blobs

1. Pick a 1 to start with, where you don't know which blob it is in
   – When there aren't any, you're done
2. Give it a new blob color
3. Assign the same blob color to each pixel that is part of the same blob

Finding blobs

1	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	1	0	0	0	0	0	0	0

Finding blobs

1	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	1	0	0	0	0	0	0	0

Finding blobs

1	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	1	0	0	0	0	0	0	0

Finding blobs

1	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	1	0	0	0	0	0	0	0

Finding blobs

1	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	1	0	0	0	0	0	0	0

Finding blobs

1	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	0	1	1	1	0	0	0	0
0	0	1	0	0	0	0	0	0	0