- Previous Lecture:
  - Linear search, binary search
  - Insertion sort
  - (Reading: Bubble Sort)
- Today's Lecture:
  - Merge Sort
  - What's next?
- Announcements
  - P6 due Thursday at 11pm
  - Final exam: Dec 17th 7pm, Barton Indoor Track WEST

Announcements

- P6 due Thursday at 11pm
- Final exam:
  - Dec 17th, 7pm, Barton Hall Indoor Track WEST
- Please fill out course evaluation on-line, see "Exercise 16"
- Revised office/consulting hours
- Pick up papers during consulting hours at Carpenter
- Read announcements on course website!

Linear search and binary search

- Linear search
  - "Effort" is linearly proportional to n, the size of the search space (e.g., the length of the vector)
  - Can represent effort by the number of comparisons against the search target done during the search
- Binary search
  - Effort is proportional to $\log_2(n)$ where n is the size of the search space
- Saving of $\log_2(n)$ over n is significant when n is large! But binary search requires sorted vector

Binary search is efficient, but we need to sort the vector in the first place so that we can use binary search

- Many different algorithms out there...
- We saw insertion sort (and read about bubble sort)
- Let's look at **merge sort**
- An example of the "divide and conquer" approach using recursion

Motivation

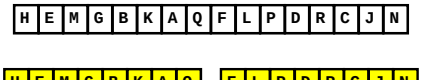
If I have two helpers, I'd...

- Give each helper half the array to sort
- Then I get back the sorted subarrays and merge them.

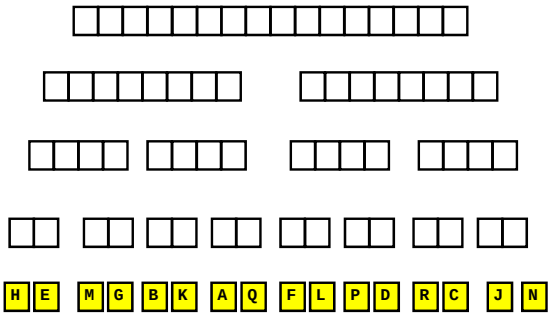
What if those two helpers each had two sub-helpers?

And the sub-helpers each had two sub-sub-helpers? And...

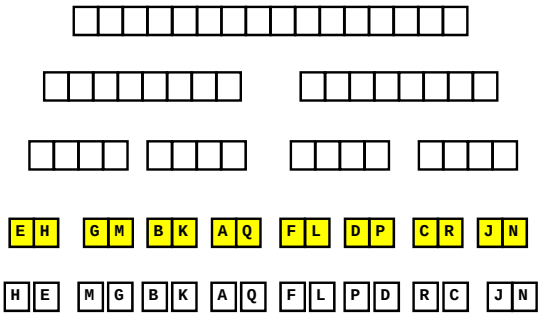
Subdivide the sorting task



And one last time



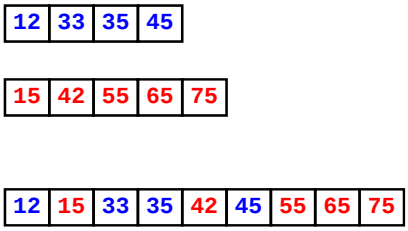
Now merge



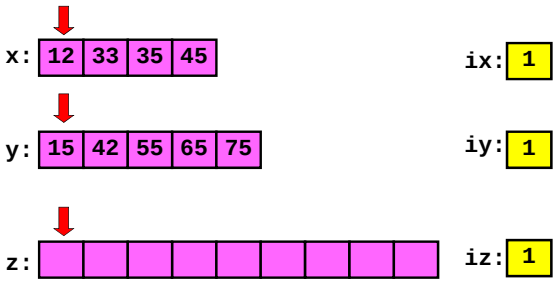

```
function y = mergeSort(x)
% x is a vector.  y is a vector
% consisting of the values in x
% sorted from smallest to largest.

n = length(x);
if n==1
      y = x;
else
      m  = floor(n/2);
      yL = mergeSort(x(1:m));
      yR = mergeSort(x(m+1:n));
      y  = merge(yL,yR);
end
```


The central sub-problem is the merging of two sorted arrays into one single sorted array

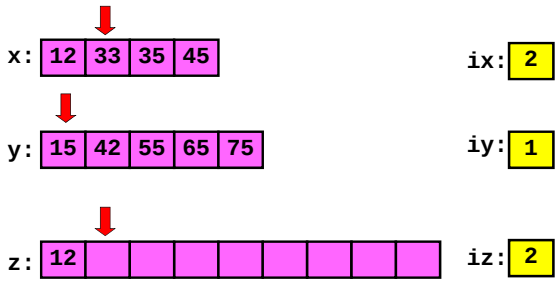


Merge



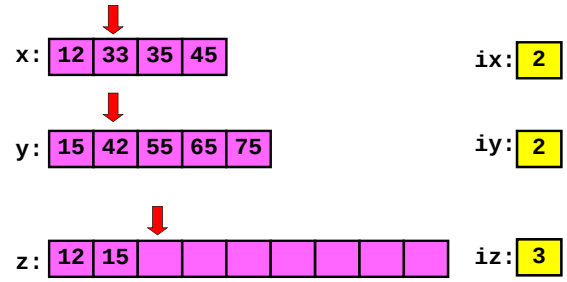
ix<=4 and iy<=5: x(ix) <= y(iy) ???

Merge



ix<=4 and iy<=5: x(ix) <= y(iy) ???

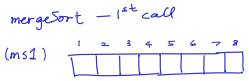
Merge



`ix<=4 and iy<=5:  x(ix) <= y(iy)  ???`

```
function z = merge(x,y)
nx = length(x); ny = length(y);
z = zeros(1, nx+ny);
ix = 1; iy = 1; iz = 1;
while ix<=nx && iy<=ny
    if  x(ix) <= y(iy)
        z(iz)= x(ix);  ix=ix+1;  iz=iz+1;
    else
        z(iz)= y(iy);  iy=iy+1;  iz=iz+1;
    end
end
while ix<=nx  % copy remaining x-values
  z(iz)= x(ix);  ix=ix+1;  iz=iz+1;
end
while iy<=ny  % copy remaining y-values
  z(iz)= y(iy);  iy=iy+1;  iz=iz+1;
end
```

```
function y=mergeSort(x)
n=length(x);
if n==1
      y=x;
else
      m=floor(n/2);
      yL=mergeSort(x(1:m));
      yR=mergeSort(x(m+1:n));
      y=merge(yL,yR);
end
```



How do merge sort, insertion sort, and bubble sort compare?

- Insertion sort and bubble sort are similar
  - Both involve a series of comparisons and swaps
  - Both involve nested loops
- Merge sort uses recursion

See **`InsertionSort.m`**

How do merge sort and insertion sort compare?

- Insertion sort: (worst case) makes k comparisons to insert an element in a sorted array of k elements. For an array of length N:

  _____ for big N

- Merge sort: _____
- Insertion sort is done *in-place*; merge sort (recursion) requires much more memory

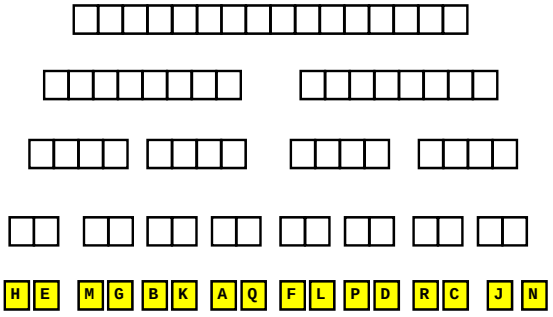

```
function y = mergeSort(x)
% x is a vector.  y is a vector
% consisting of the values in x
% sorted from smallest to largest.

n = length(x);
if n==1
      y = x;
else
      m  = floor(n/2);
      yL = mergeSort(x(1:m));
      yR = mergeSort(x(m+1:n));
      y  = merge(yL,yR);
end
```

All the comparisons between vector values are done in **`merge`**

```
function z = merge(x,y)
nx = length(x); ny = length(y);
z = zeros(1, nx+ny);
ix = 1; iy = 1; iz = 1;
while ix<=nx && iy<=ny
    if x(ix) <= y(iy)
        z(iz)= x(ix);  ix=ix+1;  iz=iz+1;
    else
        z(iz)= y(iy);  iy=iy+1;  iz=iz+1;
    end
end
while ix<=nx  % copy remaining x-values
  z(iz)= x(ix);  ix=ix+1;  iz=iz+1;
end
while iy<=ny  % copy remaining y-values
  z(iz)= y(iy);  iy=iy+1;  iz=iz+1;
end
```


Merge sort: $\log_2(N)$ "levels"; N comparisons each level



How to choose??

- Depends on application
- Merge sort is especially good for sorting large data set (but watch out for memory usage)
- Insertion sort is "order N^2" at worst case, but what about an average case? If the application requires that you *maintain* a sorted array, insertion sort may be a good choice

Why not just use Matlab's sort function?

- Flexibility
- E.g., to maintain a sorted list, just write the code for insertion sort
- E.g., sort strings or other complicated structures
- Sort according to some criterion set out in a function file
  - Observe that we have the comparison `x(j+1)<x(j)`
  - The comparison can be a function that returns a boolean value
- Can combine different sort/search algorithms for specific problem

ENGRG/CS 2110 OOP and Data Structures

- Learn new programming concepts and further explores those you've seen in CS1112
  - OOP, program design and development
  - Recursion
  - Complex data structures and related algorithms
- Taught in Java
- Optional CS 2111 meets 1 hr/week; additional practice with OOP, Java, and other course topics
- During break, check out this website: http://www.cs.cornell.edu/courses/CS1130/2014sp/

We've reached the end of CS1112… now what?

- Continue practicing your problem solving—problem decomposition—skills, in programming and other arenas!
- Interested in further study?
  - ENGRD/CS 2110 Object-oriented programming and data structure
  - Short courses in Python (CS 1133), C++ (CS 2024), …, etc.
  - More general CS courses: CS 2800 Discrete structures, CS 2850 Networks

What we learned…

- Develop/implement algorithms for problems
- Develop programming skills
  - Design, implement, document, test, and debug
- Programming "tool bag"
  - Functions for reducing redundancy
  - Control flow (if-else; loops)
  - Recursion
  - Data structures
  - Graphics
  - File handling

What we learned… (cont'd)

- Applications and concepts
  - Image processing
  - Object-oriented programming
  - Sorting and searching—you should know the algorithms covered
  - Divide-and-conquer strategies
  - Approximation and error
  - Simulation
  - Computational effort and efficiency

Computing gives us *insight* into a problem

- Computing is not about getting one answer!
- We build models and write programs so that we can "play" with the models and programs, learning—gaining insights—as we vary the parameters and assumptions
- Good models require domain-specific knowledge (and experience)
- Good programs …
  - are modular and cleanly organized
  - are well-documented
  - use appropriate data structures and algorithms
  - are reasonably efficient in time and memory

Final Exam

- Dec 17, 7-9:30pm, Barton Hall indoor tracks WEST
- Covers entire course; some emphasis on material after Prelim 2
- Closed-book exam, no calculators
- Bring student ID card

- Check for announcements on webpage:
  - Study break office/consulting hours
  - Review session time and location
  - Review questions
  - List of potentially useful functions

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Best wishes
and
good luck with all your exams!

- List of potentially useful functions