- Previous Lecture:
  - Linear search, binary search
  - Insertion sort
  - (Reading: Bubble Sort)

- Today's Lecture:
  - Merge Sort
  - What's next?

- Announcements
  - P6 due Thursday at 11pm
  - Final exam: Dec 17th 7pm, Barton Indoor Track WEST

Announcements

- P6 due Thursday at 11pm
- Final exam:
  - Dec 17th, 7pm, Barton Hall Indoor Track WEST
- Please fill out course evaluation on-line, see "Exercise 16"
- Revised office/consulting hours
- Pick up papers during consulting hours at Carpenter
- Read announcements on course website!

Linear search and binary search

- Linear search
  - "Effort" is linearly proportional to n, the size of the search space (e.g., the length of the vector)
  - Can represent effort by the number of comparisons against the search target done during the search
- Binary search
  - Effort is proportional to $\log_2(n)$ where n is the size of the search space
- Saving of $\log_2(n)$ over n is significant when n is large! But <u>binary search requires sorted vector</u>

Binary search is efficient, but we need to sort the vector in the first place so that we can use binary search

- Many different algorithms out there...
- We saw insertion sort (and read about bubble sort)
- Let's look at **merge sort**
- An example of the "divide and conquer" approach using recursion

Which task is "easier," sort a length 1000 array or merge* two length 500 sorted arrays into one?

A. Sort

B. Merge

*Merge two sorted arrays so that the resultant array is sorted

Motivation: merging is an easier job than sorting!

If I have two helpers, I'd…

- Give each helper half the array to sort
- Then I get back the sorted subarrays and merge them.

What if those two helpers each had two sub-helpers?

And the sub-helpers each had two sub-sub-helpers? And…

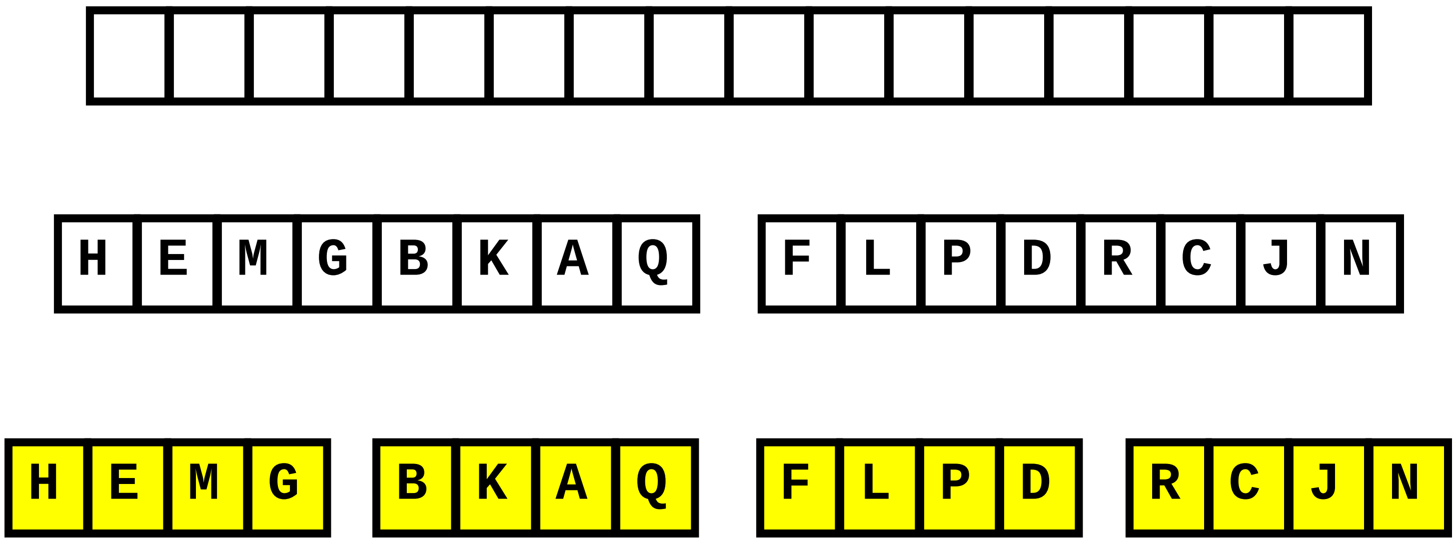
Subdivide the sorting task

H	E	M	G	B	K	A	Q	F	L	P	D	R	C	J	N

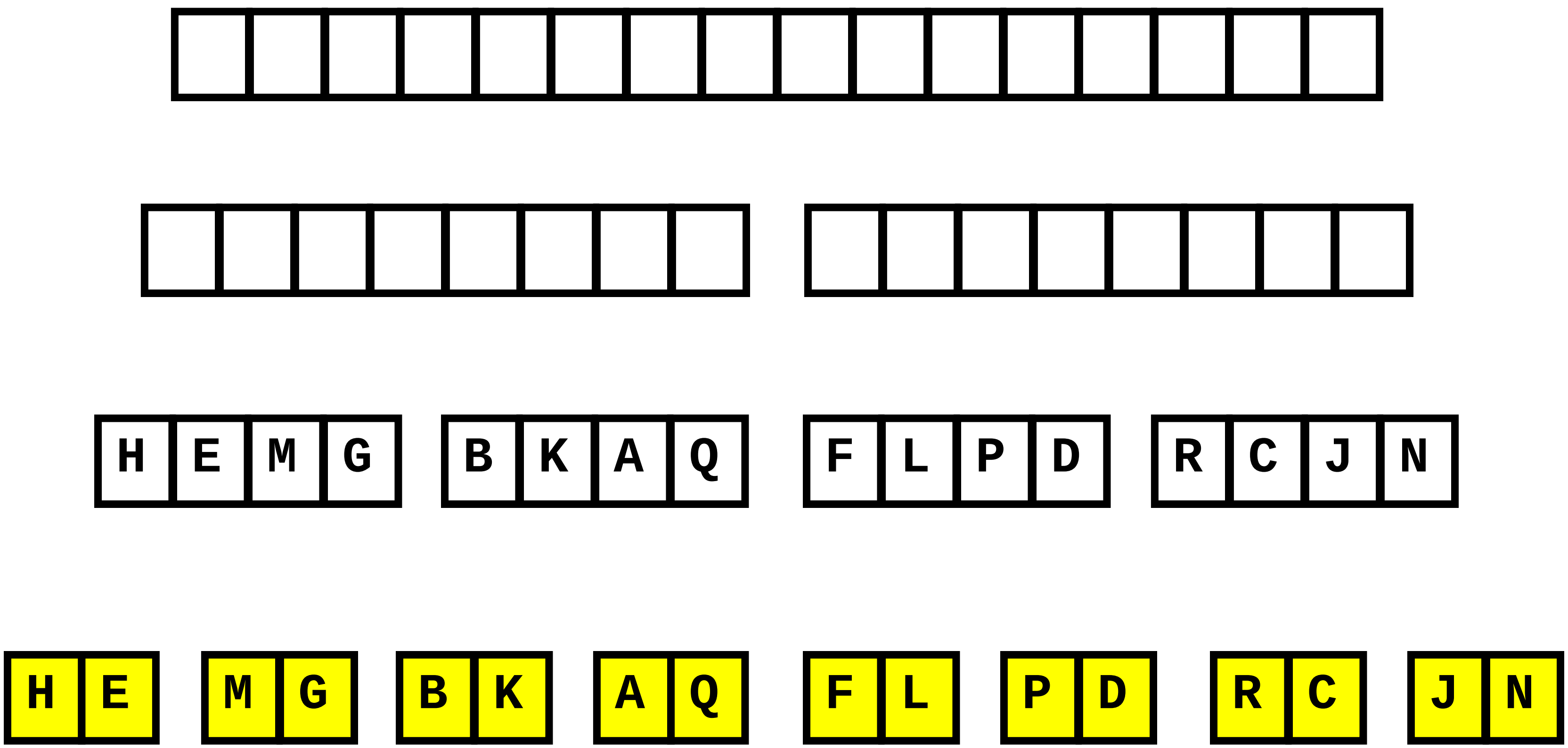
H	E	M	G	B	K	A	Q

F	L	P	D	R	C	J	N

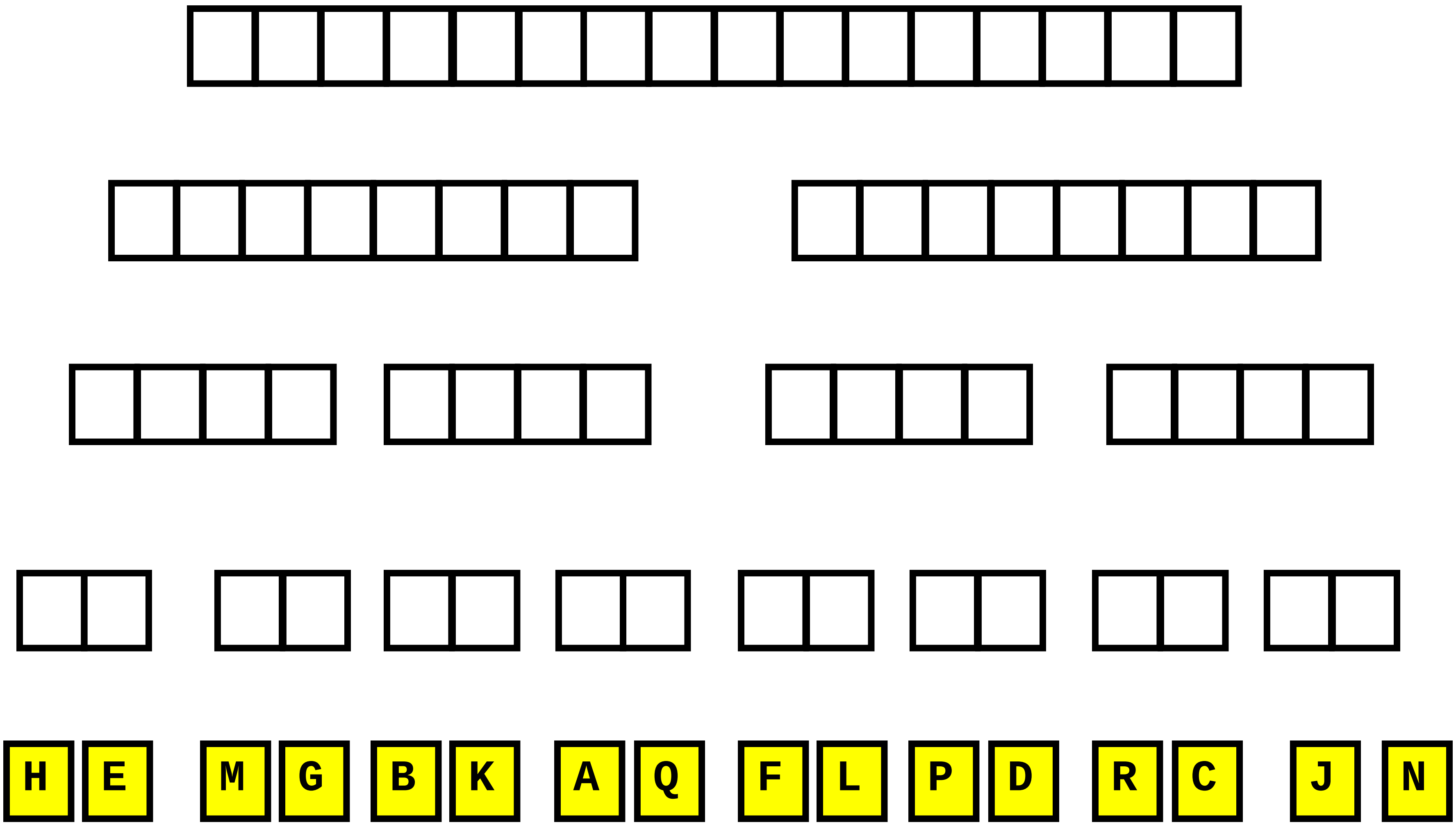
Subdivide again



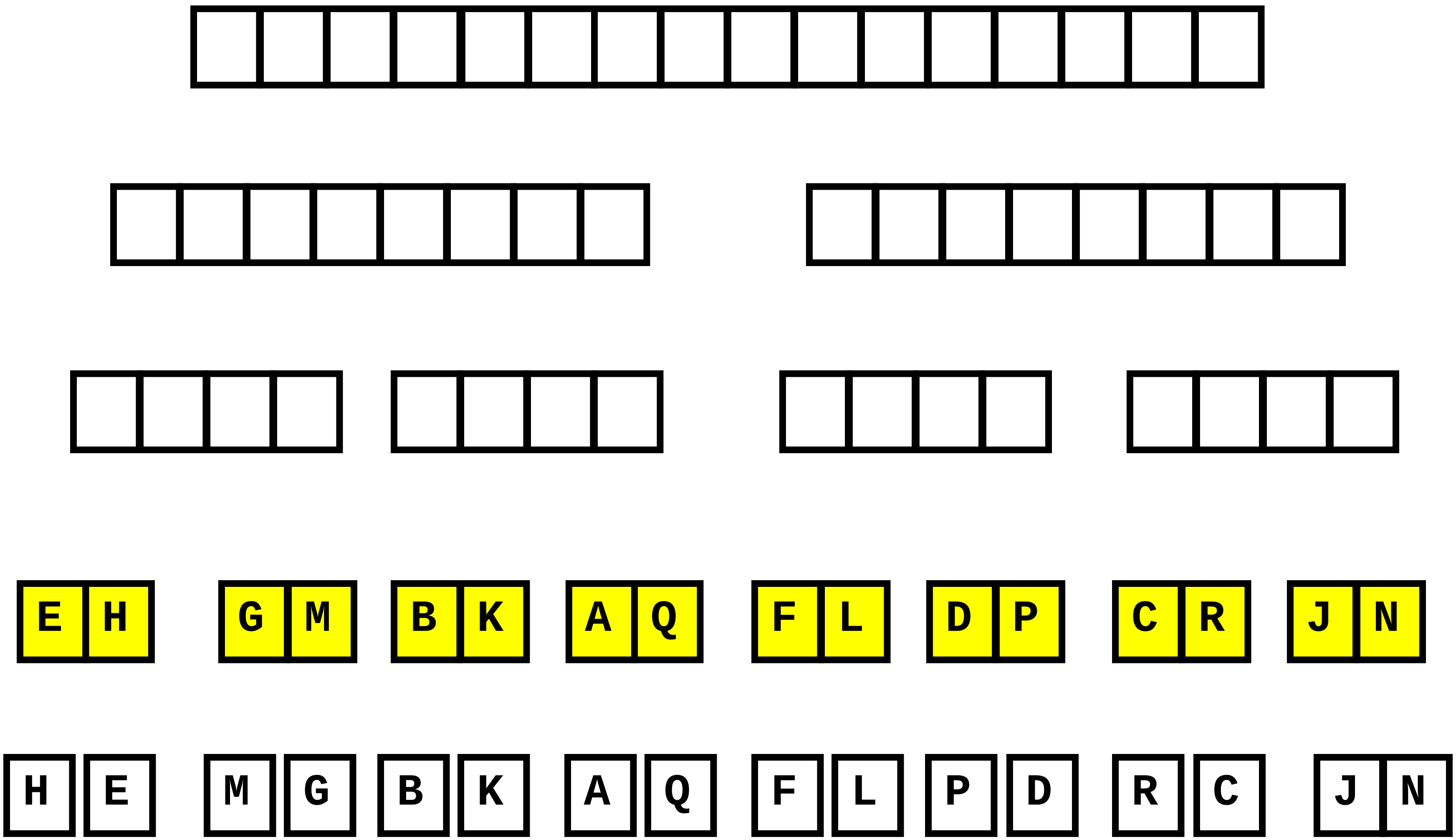
And again



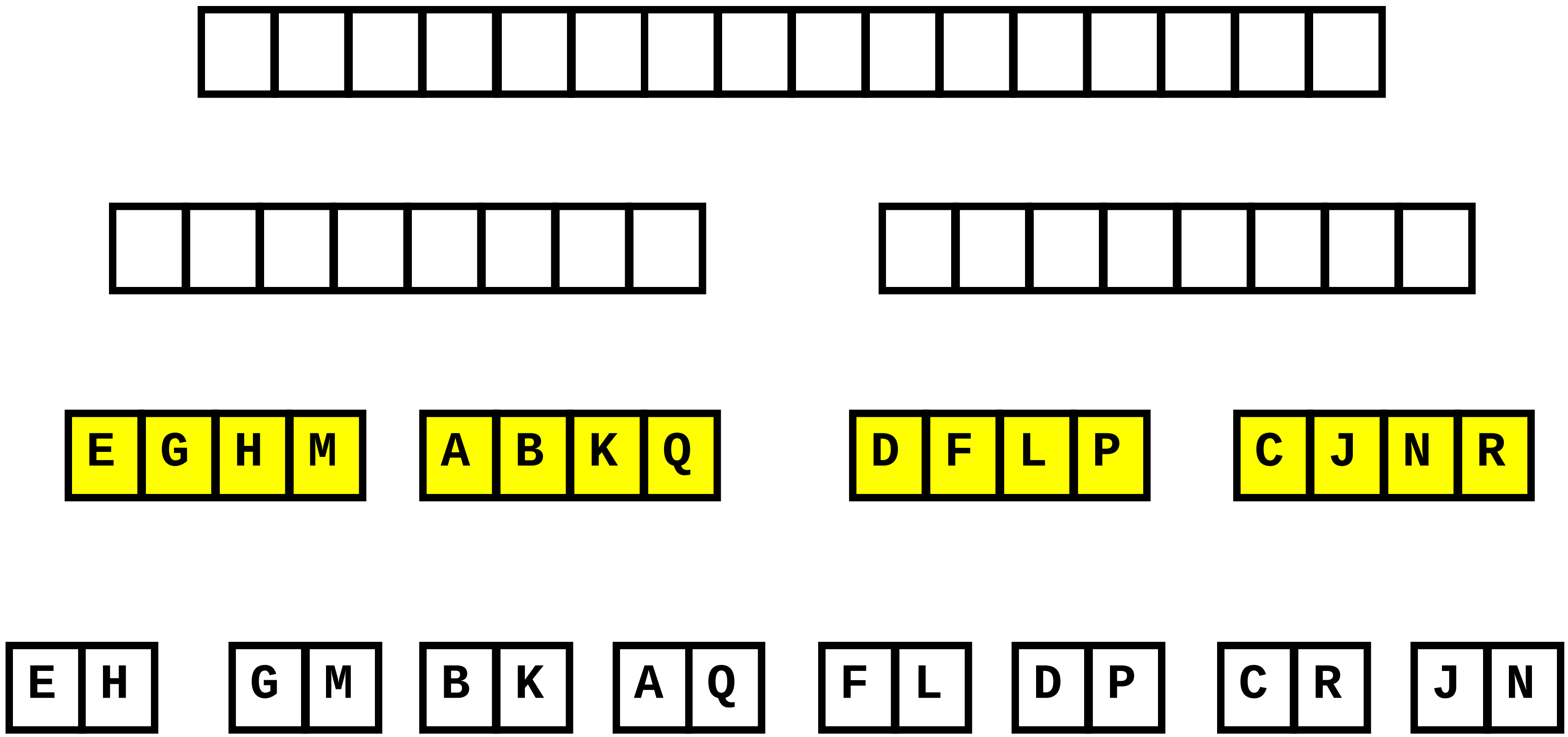
And one last time



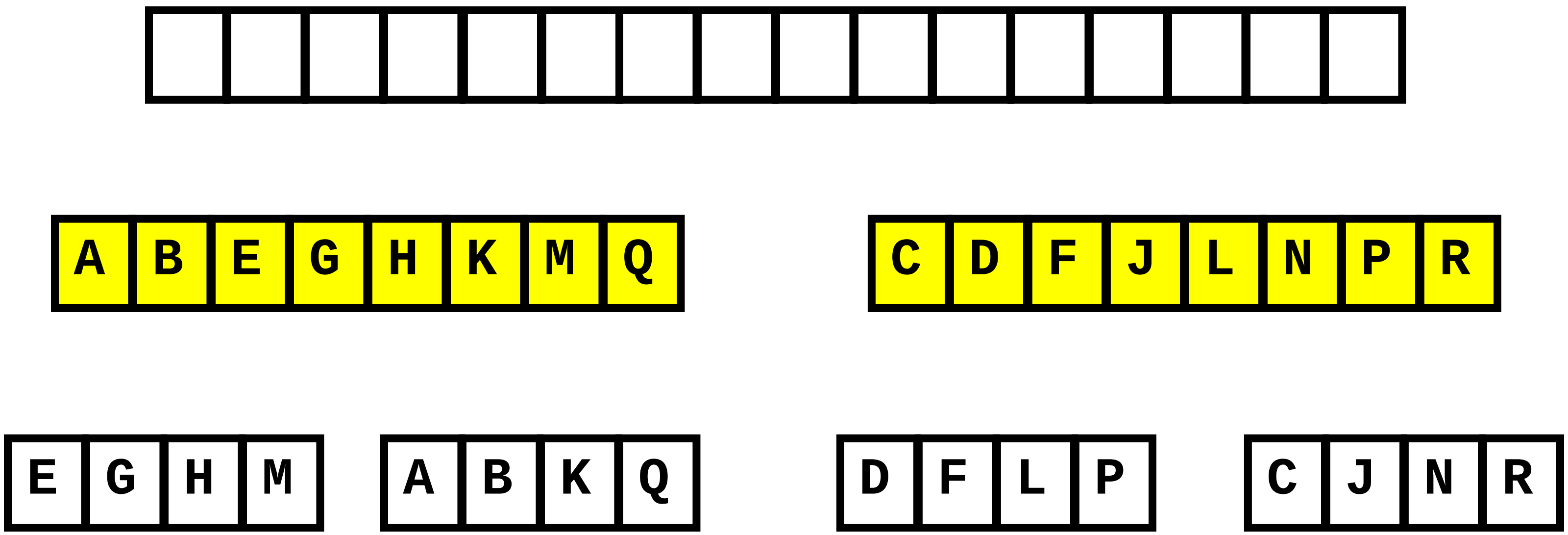
Now merge



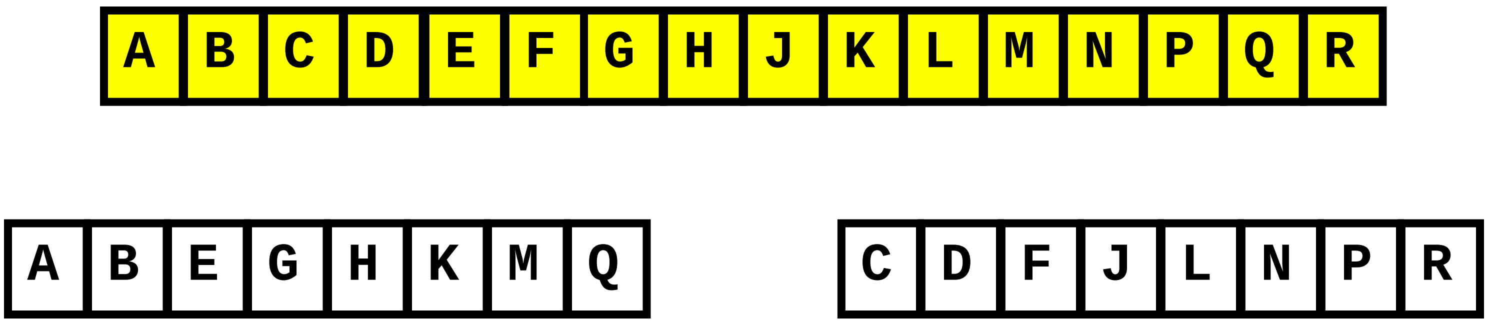
And merge again



And again



And one last time



Done!

A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q	R

```
function y = mergeSort(x)
% x is a vector.  y is a vector
% consisting of the values in x
% sorted from smallest to largest.

n = length(x);
if n==1
      y = x;
else
      m  = floor(n/2);
      yL = mergeSortL(x(1:m));
      yR = mergeSortR(x(m+1:n));
      y  = merge(yL,yR);
end
```

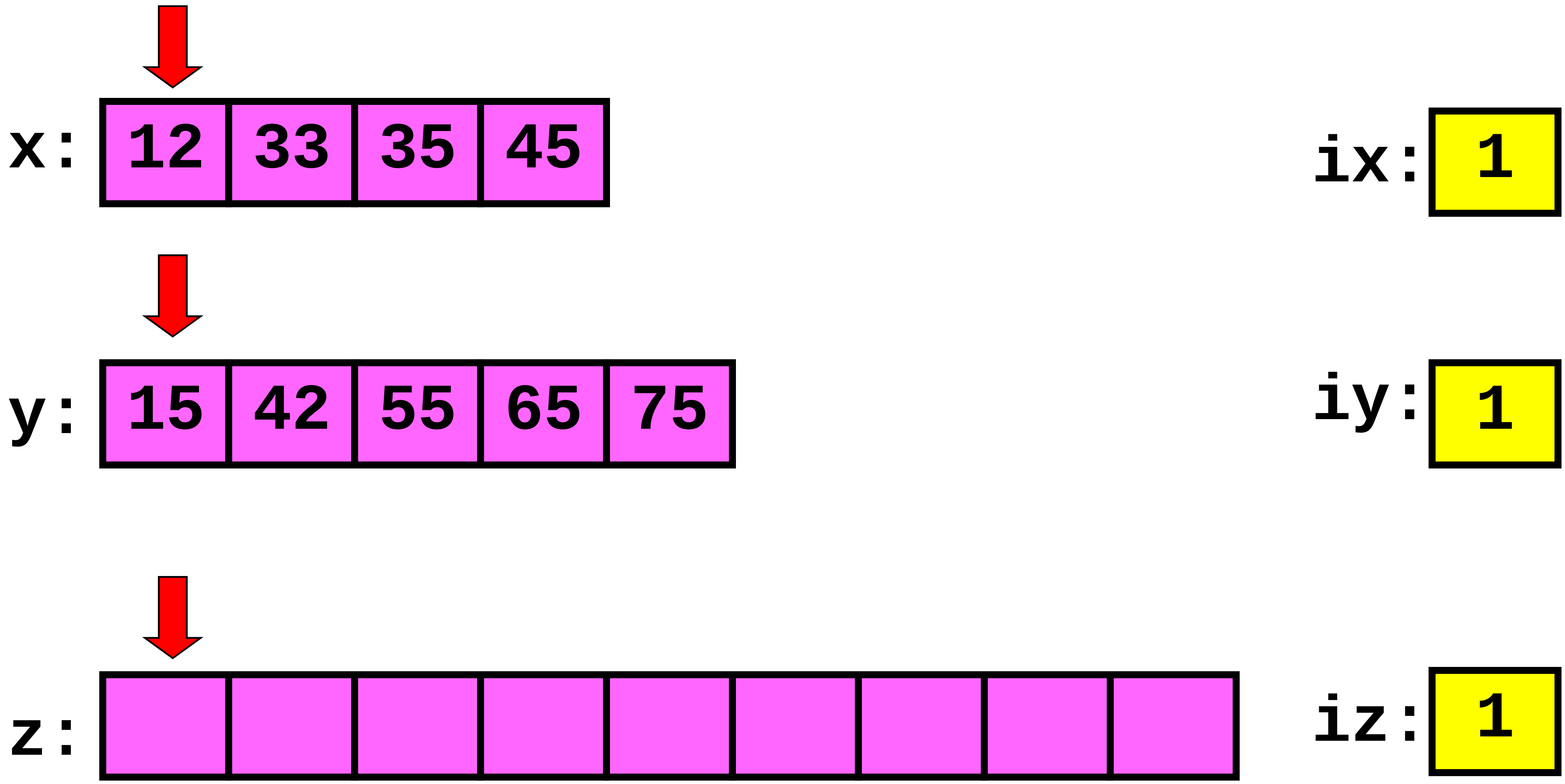
The central sub-problem is the merging of two sorted arrays into one single sorted array

12	33	35	45

15	42	55	65	75

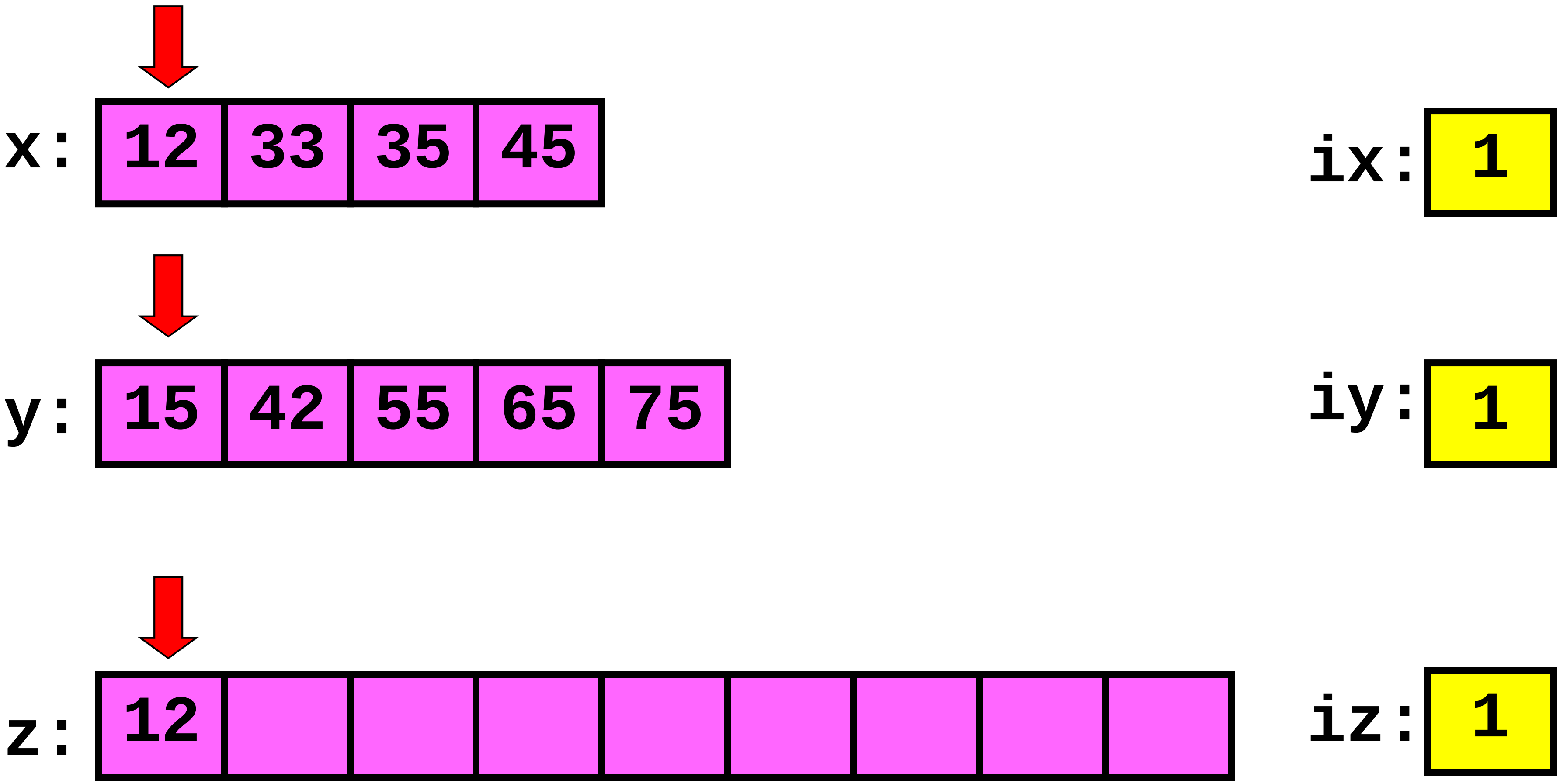
12	15	33	35	42	45	55	65	75

Merge



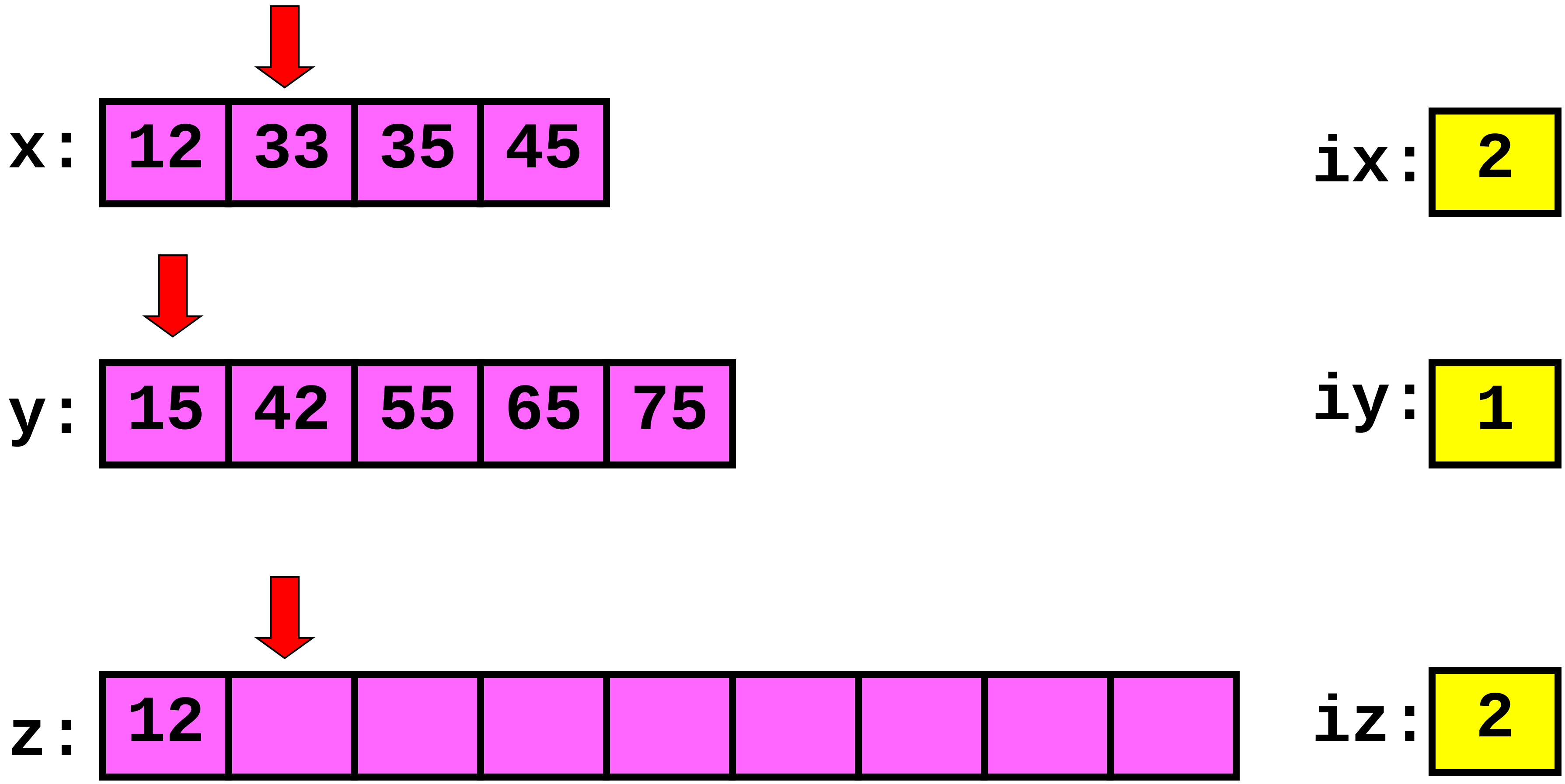
ix<=4 and iy<=5: x(ix) <= y(iy) ???

Merge



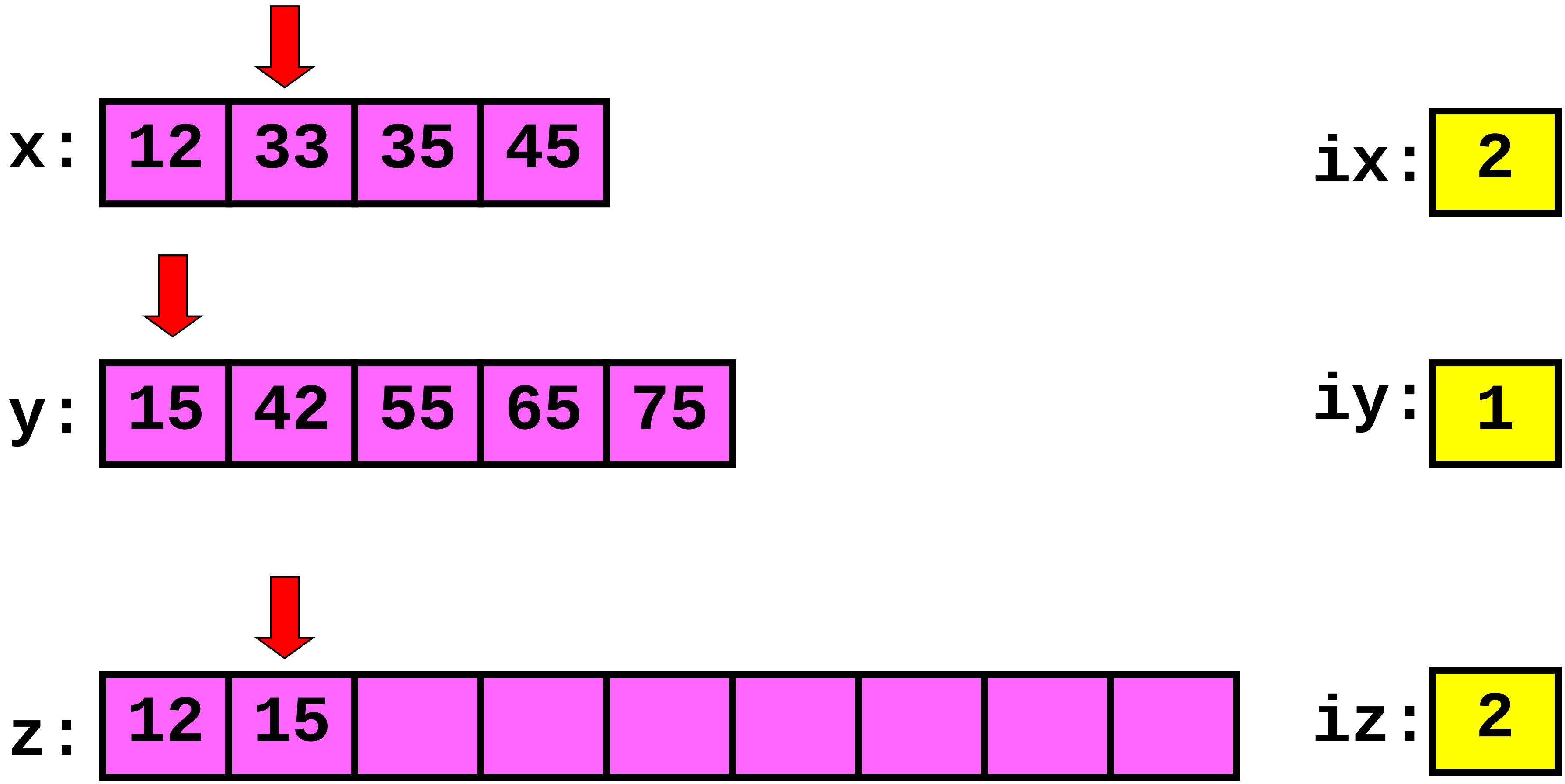
ix<=4 and iy<=5: x(ix) <= y(iy) YES

Merge



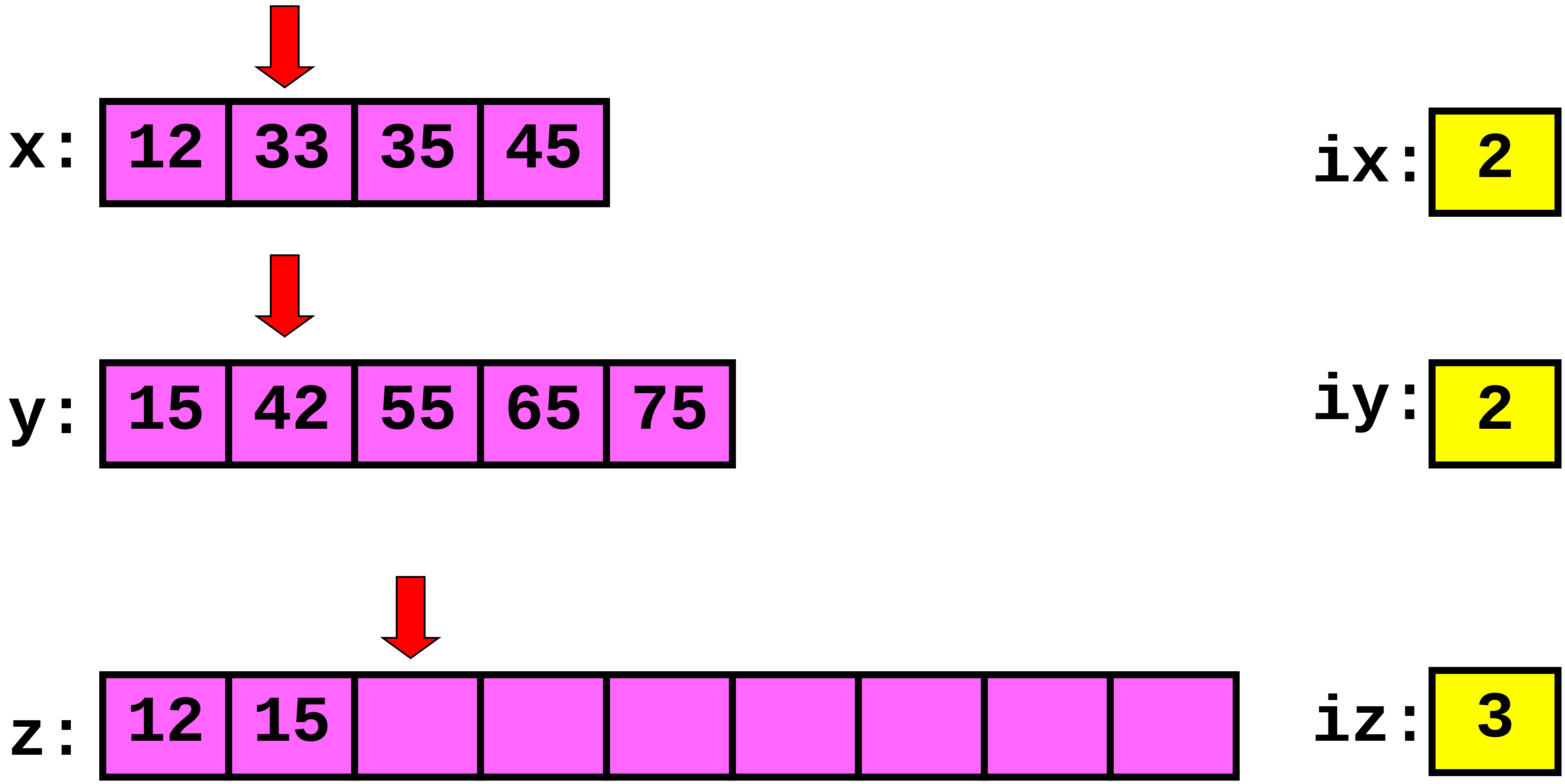
ix<=4 and iy<=5: x(ix) <= y(iy)   ???

Merge



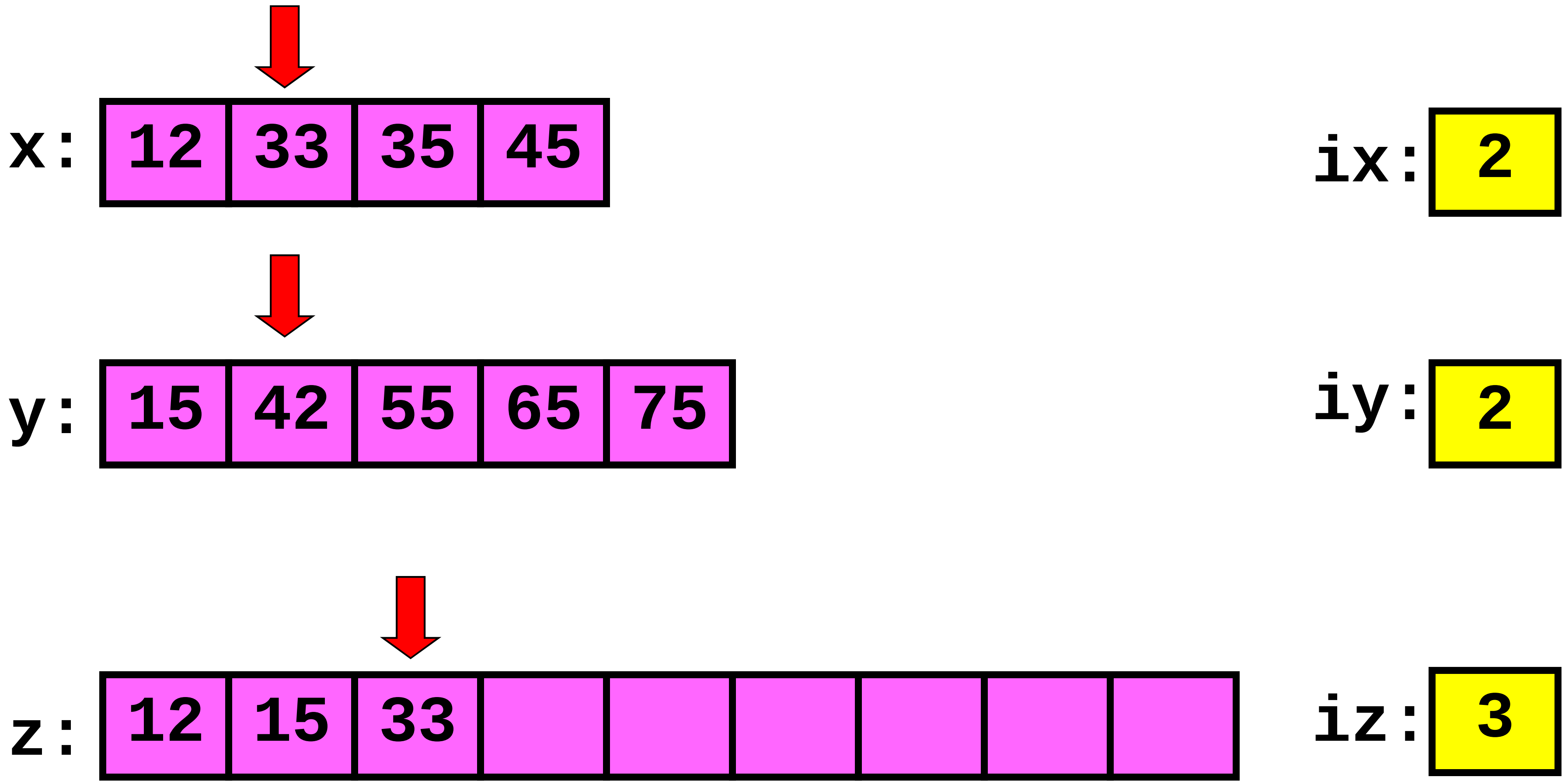
ix<=4 and iy<=5: x(ix) <= y(iy) NO

Merge



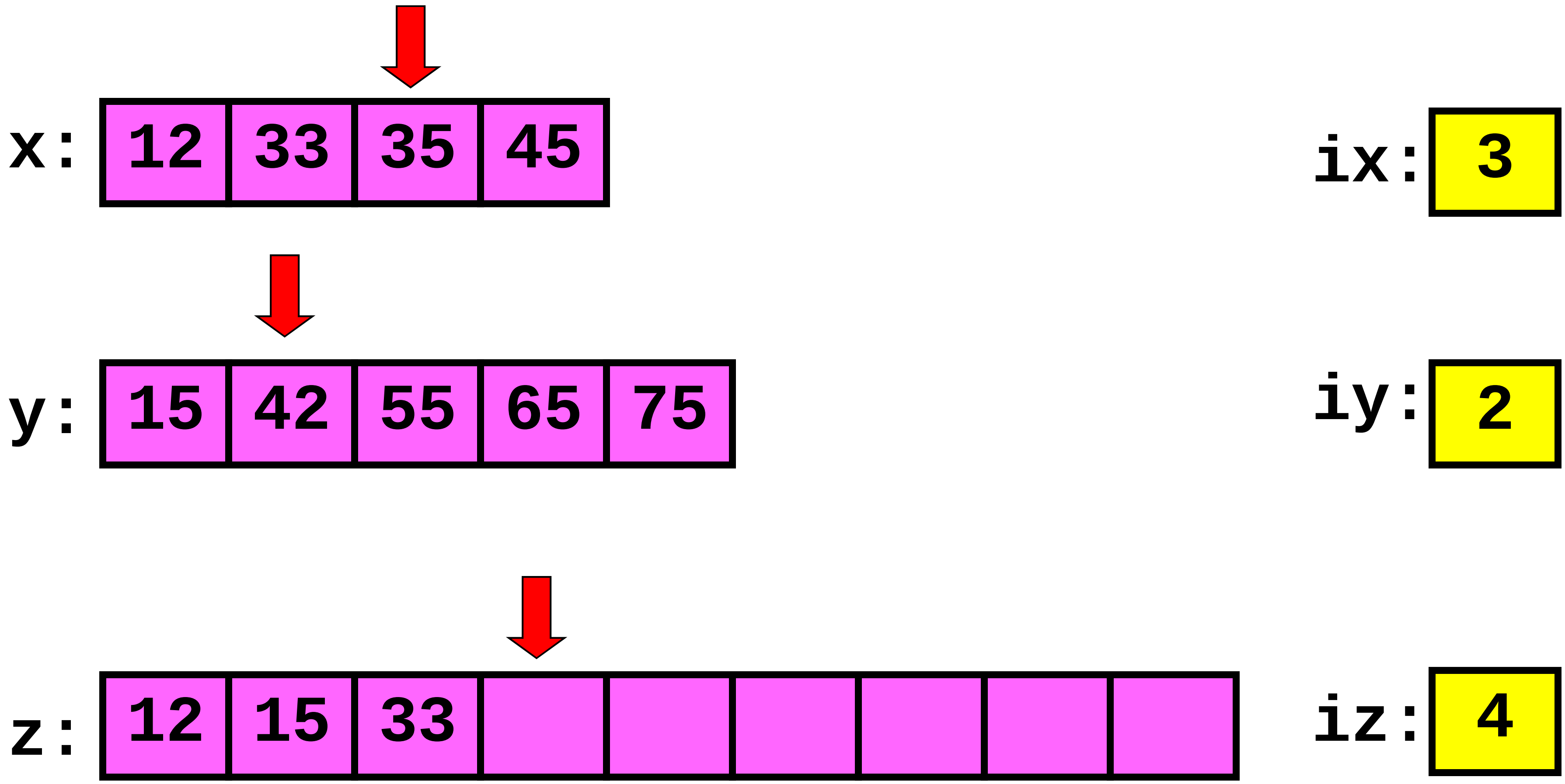
ix<=4 and iy<=5: x(ix) <= y(iy) ???

Merge



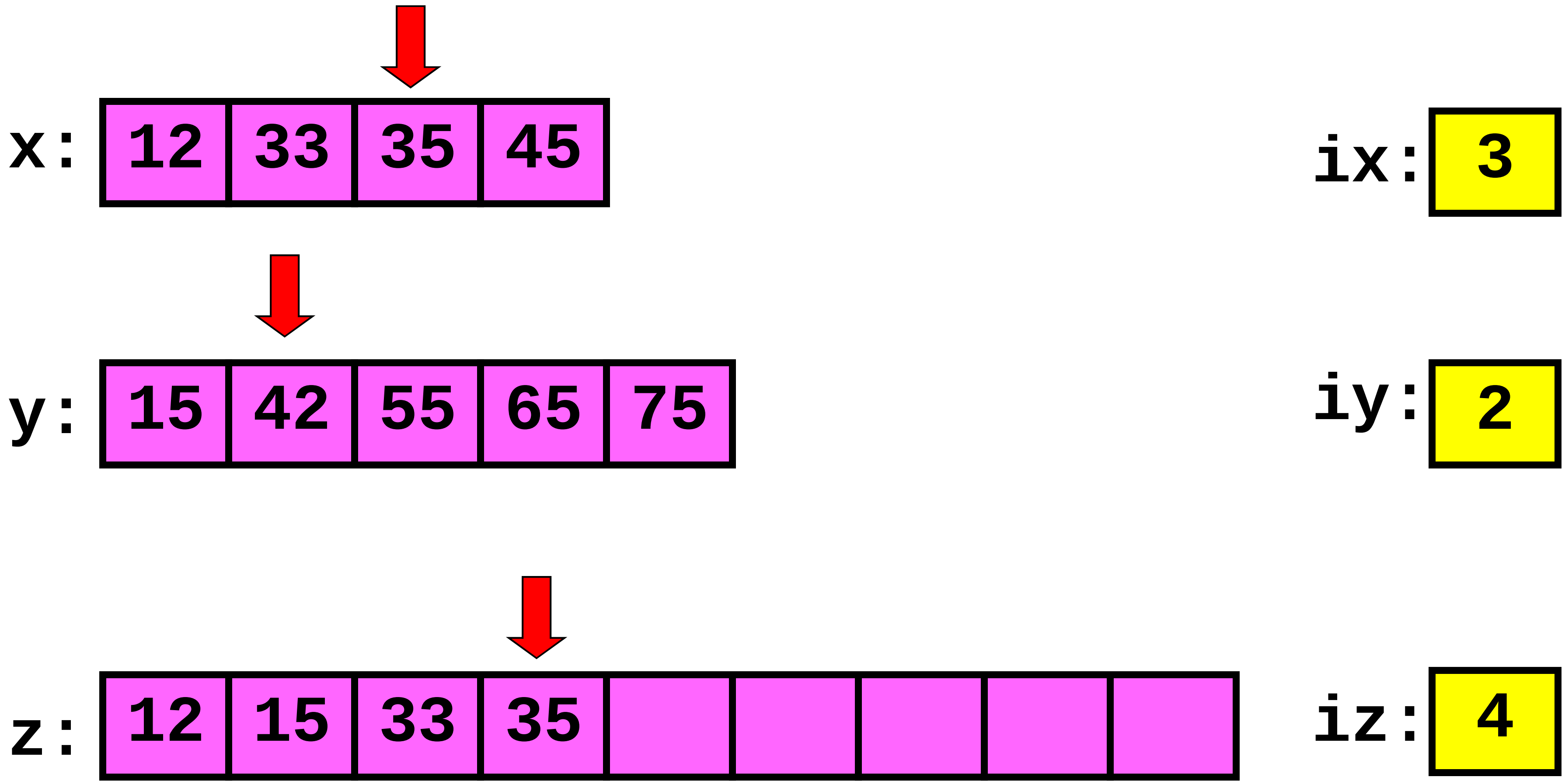
`ix<=4 and iy<=5:  x(ix) <= y(iy)` **YES**

Merge



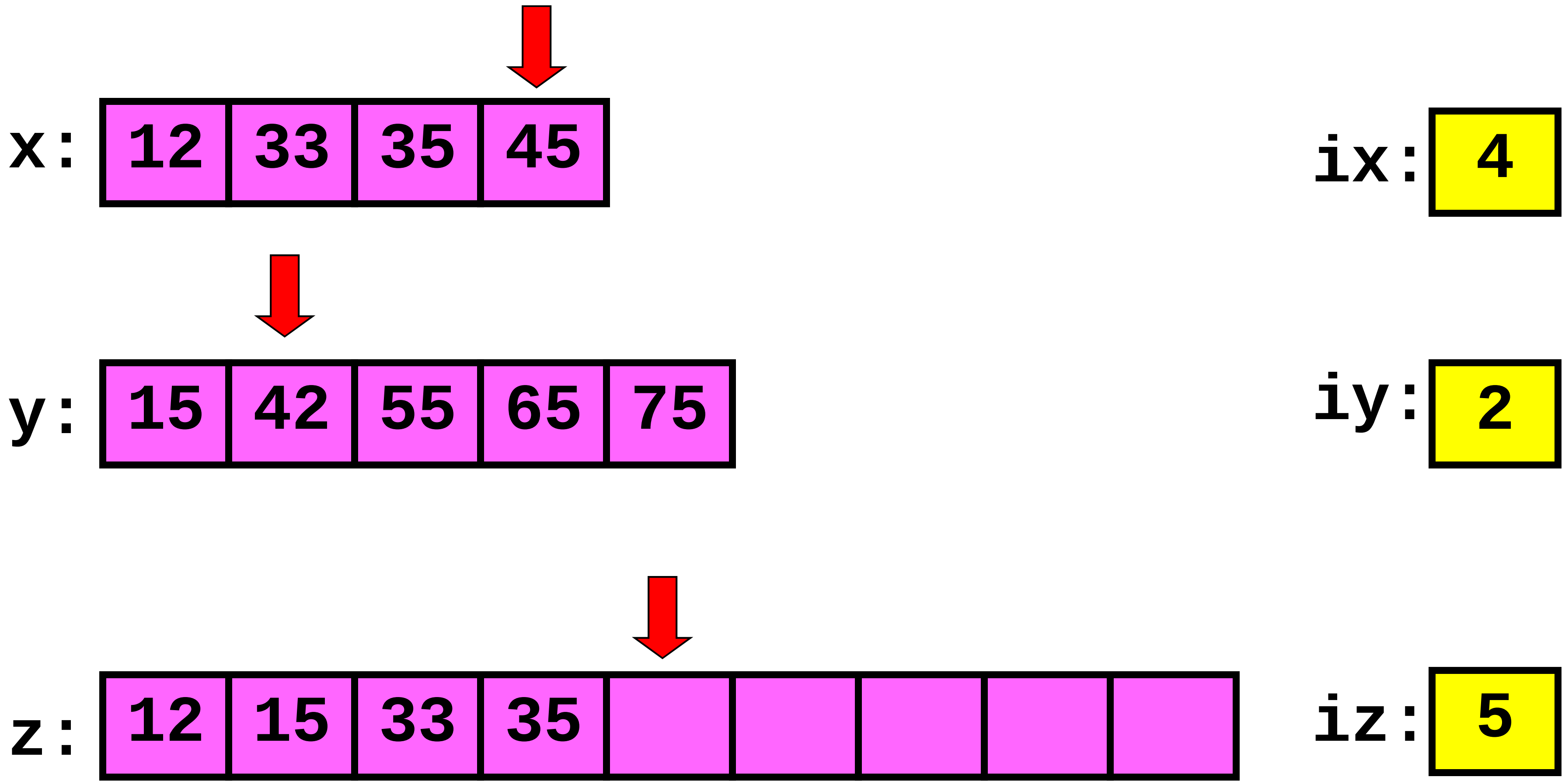
ix<=4 and iy<=5: x(ix) <= y(iy) ???

Merge



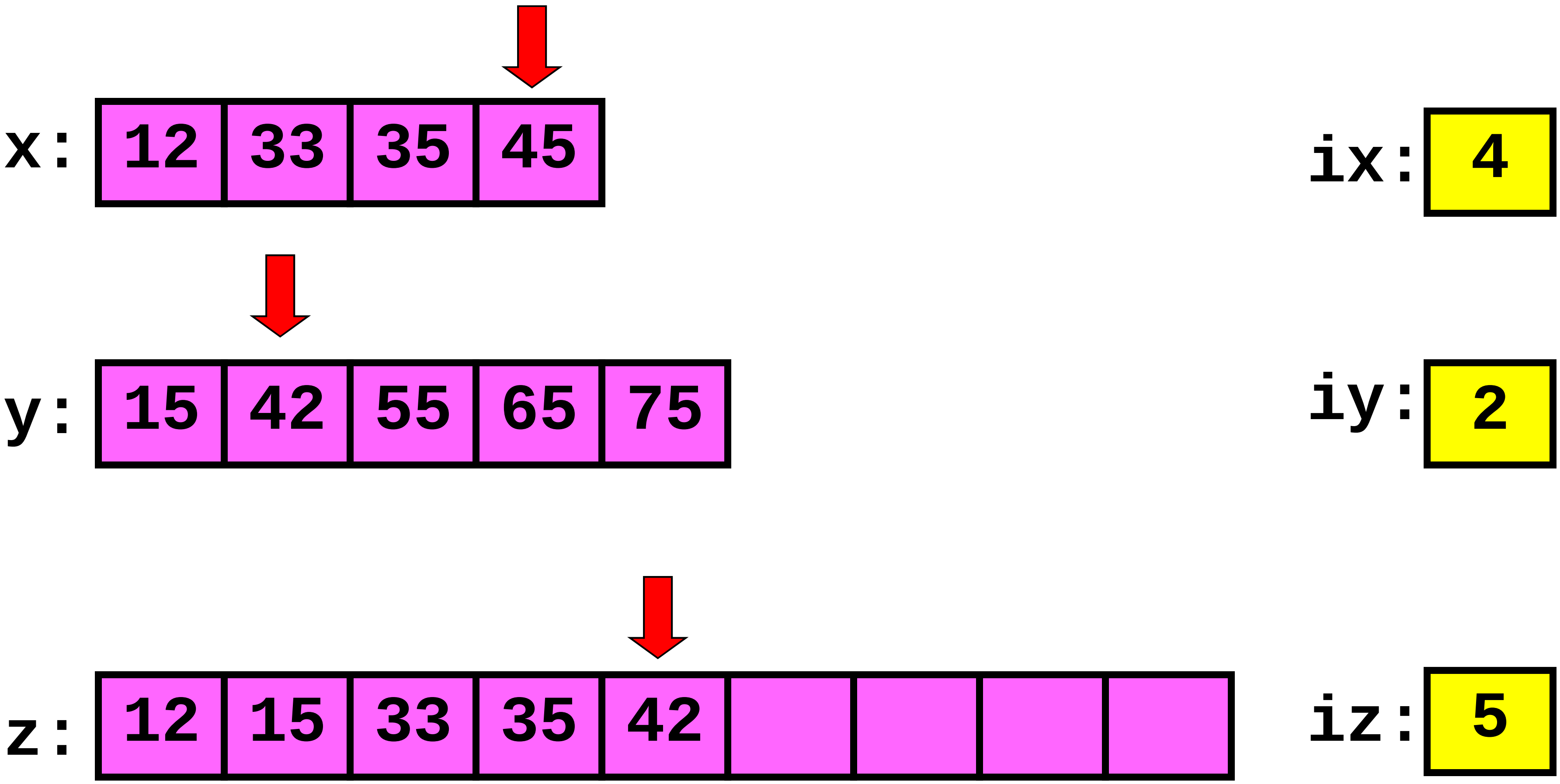
ix<=4 and iy<=5: x(ix) <= y(iy) YES

Merge



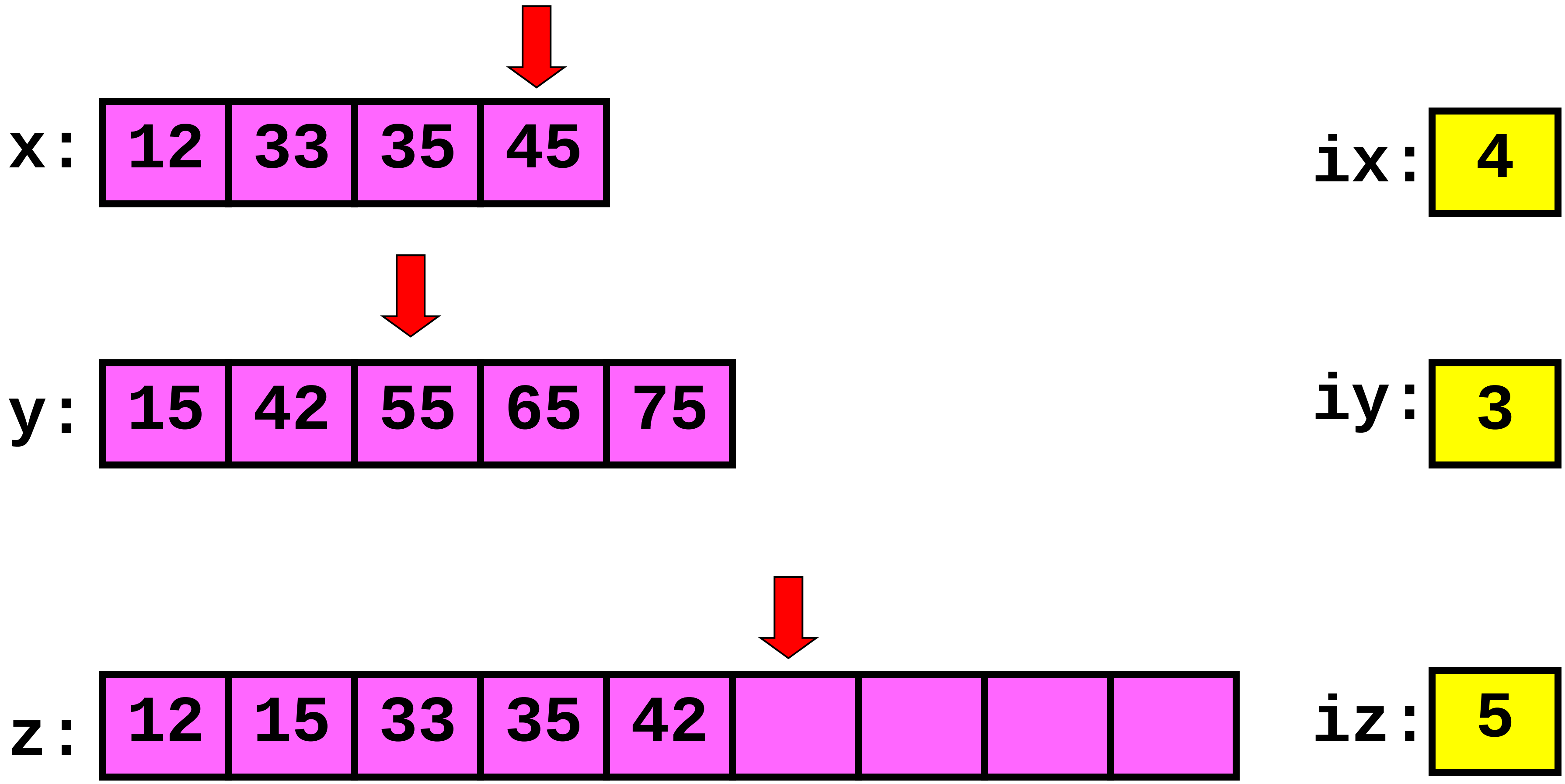
ix<=4 and iy<=5: x(ix) <= y(iy) ???

Merge



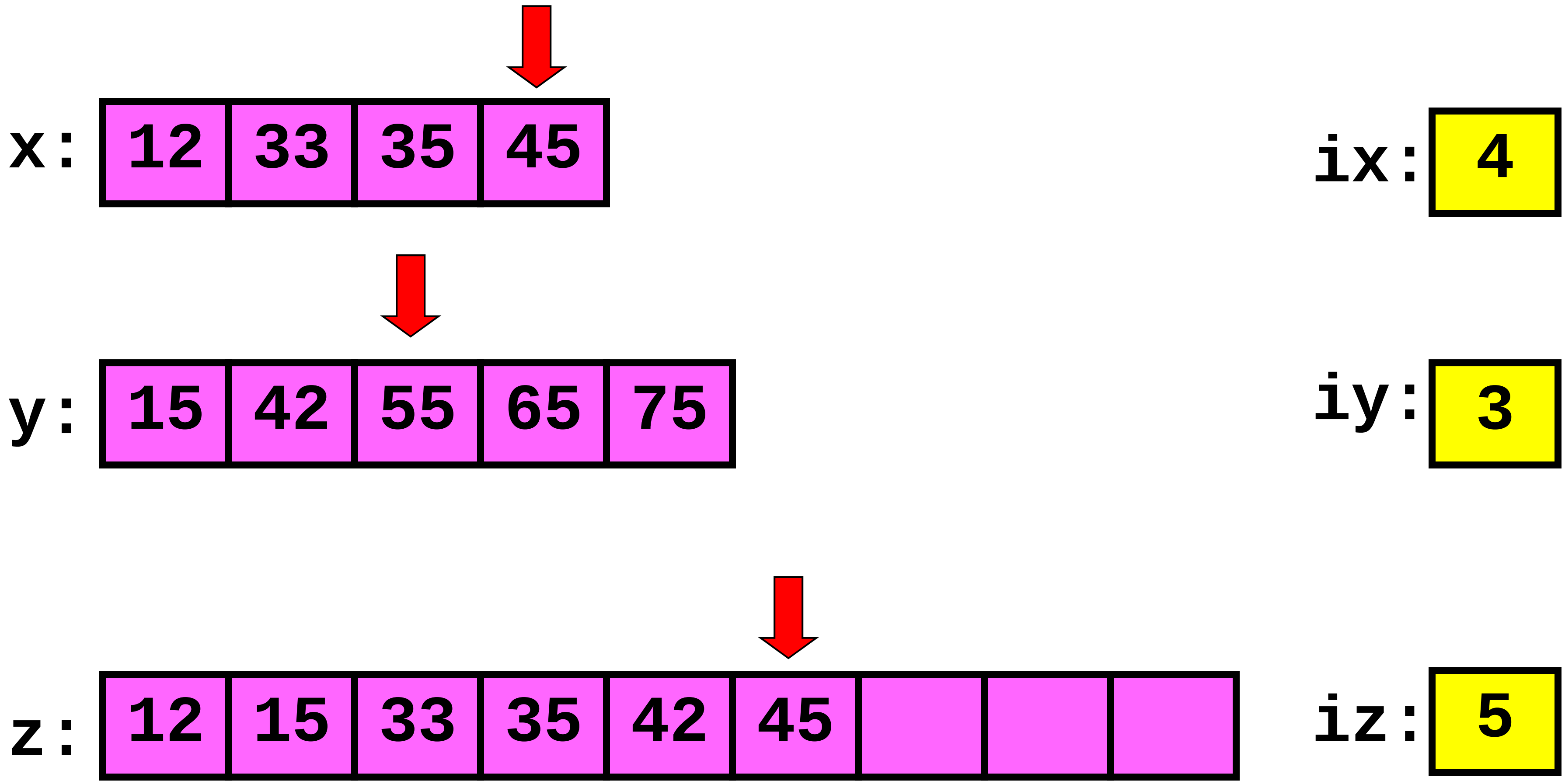
ix<=4 and iy<=5: x(ix) <= y(iy) NO

Merge



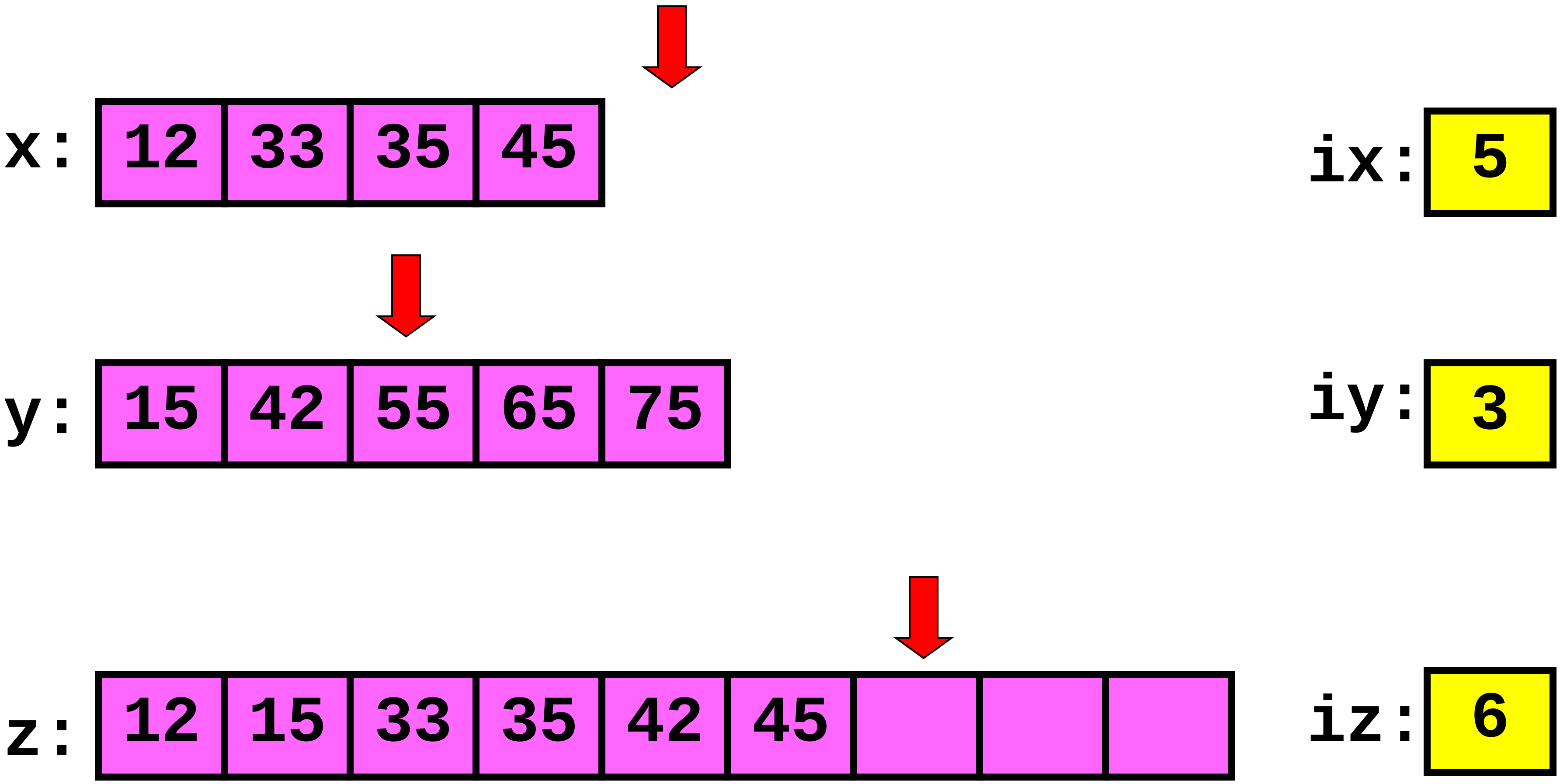

```
ix<=4 and iy<=5:  x(ix) <= y(iy)  ???
```

Merge



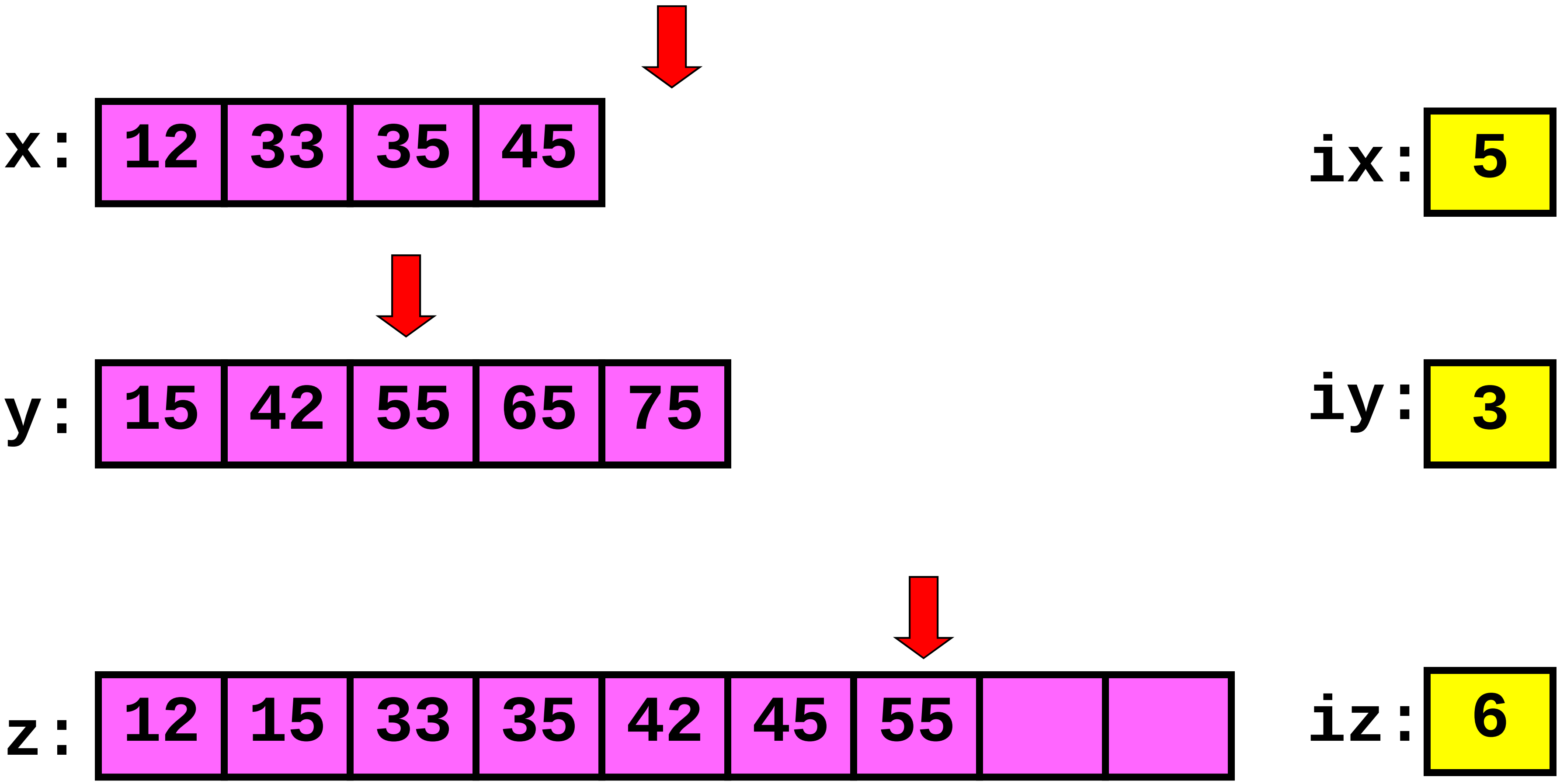
ix<=4 and iy<=5: x(ix) <= y(iy) YES

Merge



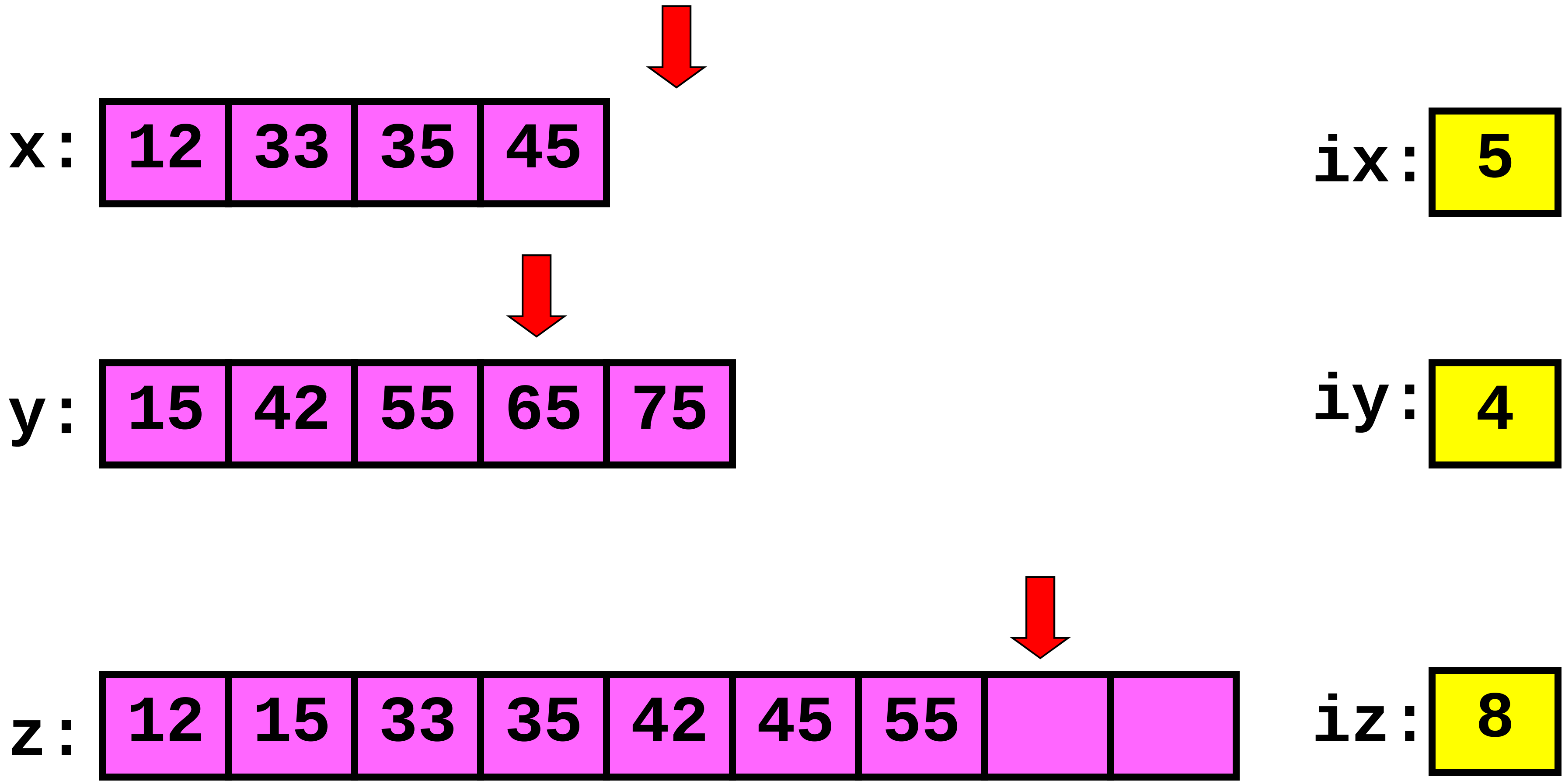
ix > 4

Merge



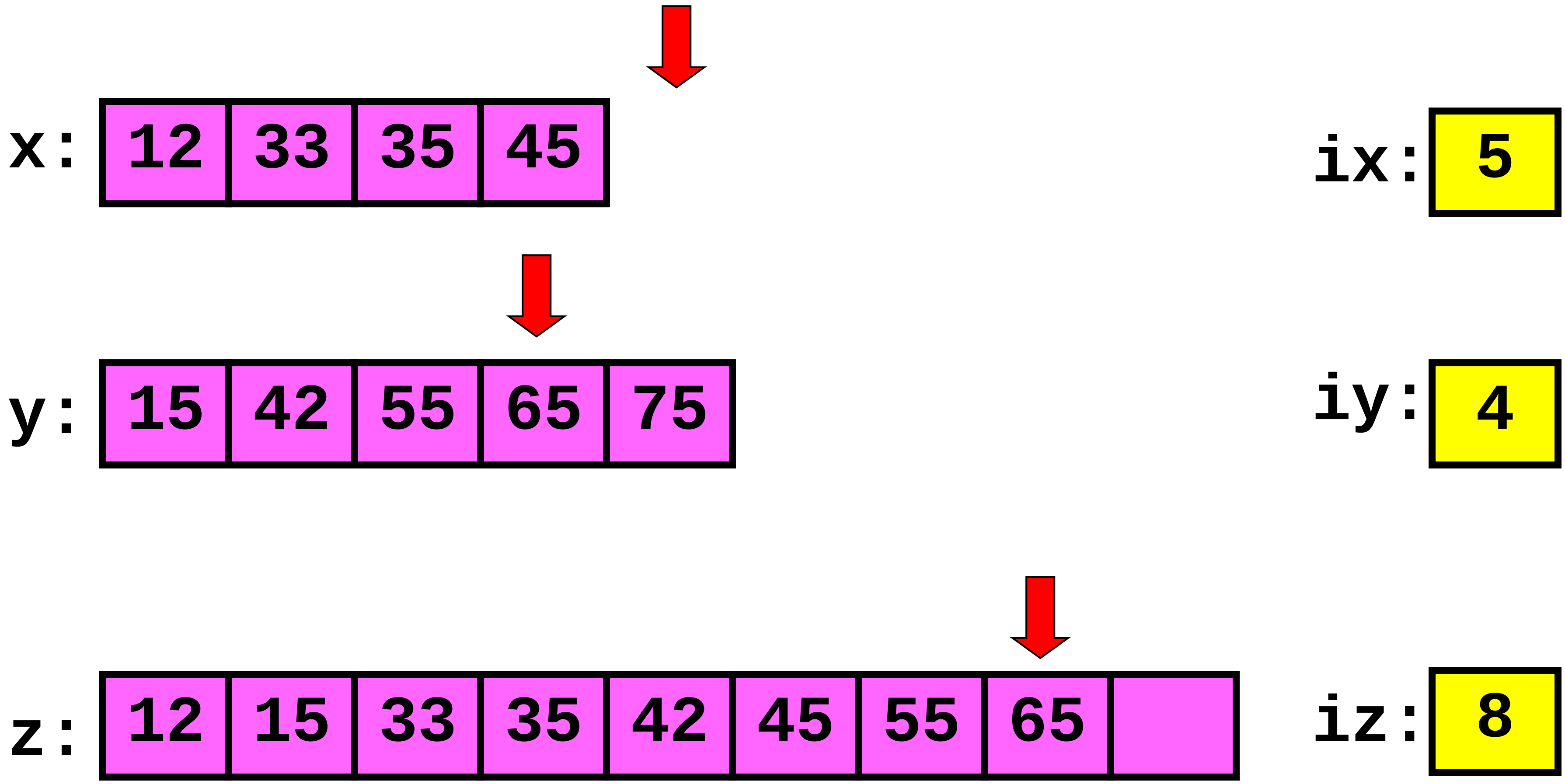
ix > 4: take y(iy)

Merge



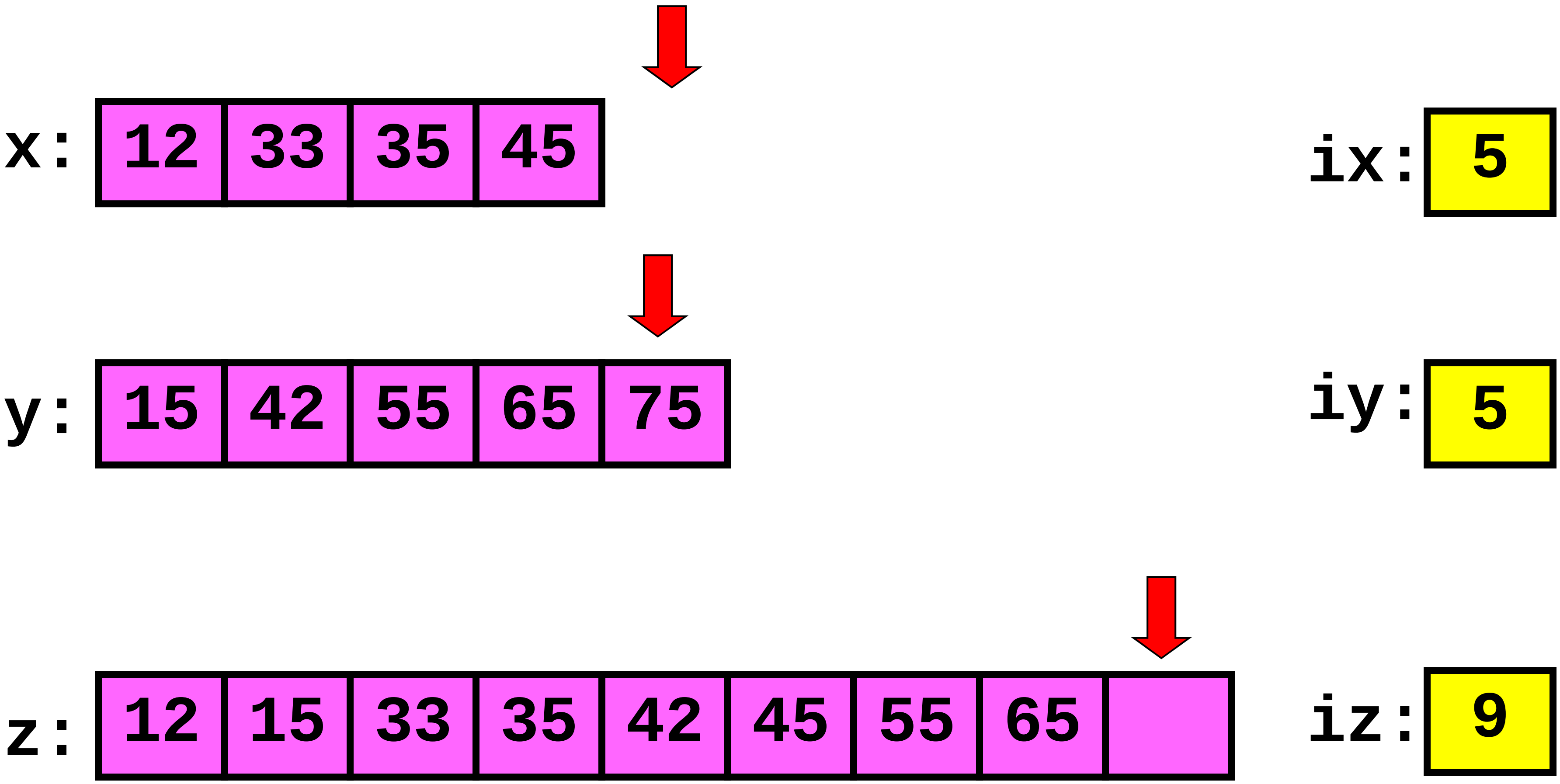
iy <= 5

Merge



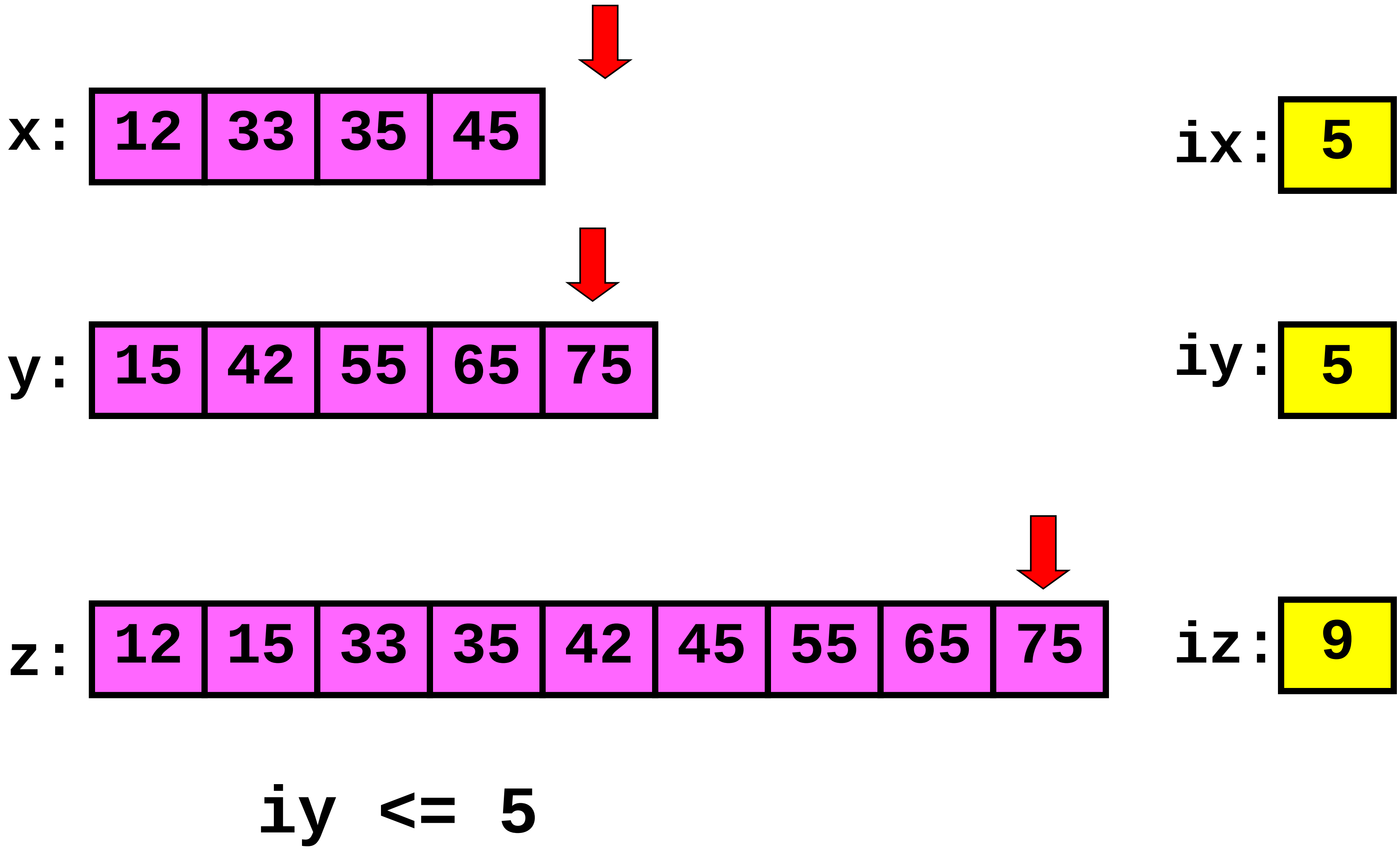
iy <= 5

Merge




```
iy <= 5
```

Merge



x:
12
33
35
45
ix:
5
y:
15
42
55
65
75
iy:
5
z:
12
15
33
35
42
45
55
65
75
iz:
9
iy <= 5

```
function z = merge(x,y)
nx = length(x); ny = length(y);
z = zeros(1, nx+ny);
ix = 1; iy = 1; iz = 1;
```

```
function z = merge(x,y)
nx = length(x); ny = length(y);
z = zeros(1, nx+ny);
ix = 1; iy = 1; iz = 1;
while ix<=nx && iy<=ny








end
% Deal with remaining values in x or y
```

```
function z = merge(x,y)
nx = length(x); ny = length(y);
z = zeros(1, nx+ny);
ix = 1; iy = 1; iz = 1;
while ix<=nx && iy<=ny
    if  x(ix) <= y(iy)
        z(iz)= x(ix);  ix=ix+1;  iz=iz+1;
    else
        z(iz)= y(iy);  iy=iy+1;  iz=iz+1;
    end
end
% Deal with remaining values in x or y
```

```
function z = merge(x,y)
nx = length(x); ny = length(y);
z = zeros(1, nx+ny);
ix = 1; iy = 1; iz = 1;
while ix<=nx && iy<=ny
    if  x(ix) <= y(iy)
        z(iz)= x(ix);  ix=ix+1;  iz=iz+1;
    else
        z(iz)= y(iy);  iy=iy+1;  iz=iz+1;
    end
end
while ix<=nx  % copy remaining x-values
  z(iz)= x(ix);  ix=ix+1;  iz=iz+1;
end
while iy<=ny  % copy remaining y-values
  z(iz)= y(iy);  iy=iy+1;  iz=iz+1;
end
```

```
function y = mergeSort(x)
% x is a vector.  y is a vector
% consisting of the values in x
% sorted from smallest to largest.

n = length(x);
if n==1
      y = x;
else
      m  = floor(n/2);
      yL = mergeSortL(x(1:m));
      yR = mergeSortR(x(m+1:n));
      y  = merge(yL,yR);
end
```

```
function y = mergeSortL(x)
% x is a vector.  y is a vector
% consisting of the values in x
% sorted from smallest to largest.

n = length(x);
if n==1
      y = x;
else
      m  = floor(n/2);
      yL = mergeSortL_L(x(1:m));
      yR = mergeSortL_R(x(m+1:n));
      y  = merge(yL,yR);
end
```

```
function y = mergeSortL_L(x)
% x is a vector.  y is a vector
% consisting of the values in x
% sorted from smallest to largest.

n = length(x);
if n==1
      y = x;
else
      m  = floor(n/2);
      yL = mergeSortL_L_L(x(1:m));
      yR = mergeSortL_L_R(x(m+1:n));
      y  = merge(yL,yR);
end
```

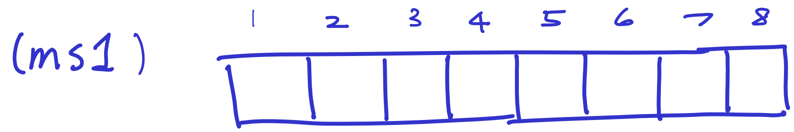
```
function y = mergeSort(x)
% x is a vector.  y is a vector
% consisting of the values in x
% sorted from smallest to largest.

n = length(x);
if n==1
      y = x;
else
      m  = floor(n/2);
      yL = mergeSort(x(1:m));
      yR = mergeSort(x(m+1:n));
      y  = merge(yL,yR);
end
```

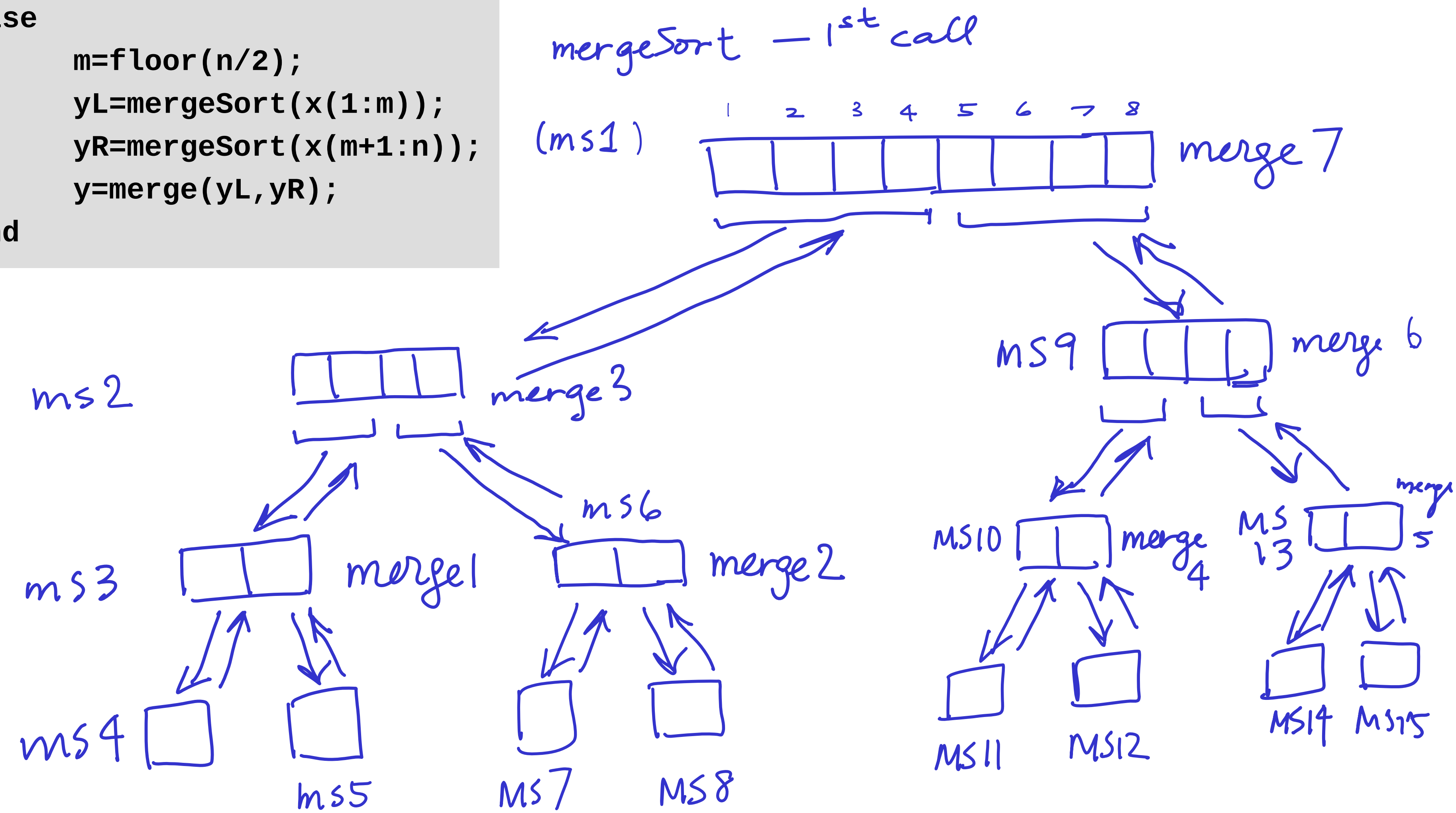
```
function y=mergeSort(x)
n=length(x);
if n==1
      y=x;
else
      m=floor(n/2);
      yL=mergeSort(x(1:m));
      yR=mergeSort(x(m+1:n));
      y=merge(yL,yR);
end
```

```
function y=mergeSort(x)
n=length(x);
if n==1
      y=x;
else
      m=floor(n/2);
      yL=mergeSort(x(1:m));
      yR=mergeSort(x(m+1:n));
      y=merge(yL,yR);
end
```

mergeSort — 1st call



```
function y=mergeSort(x)
n=length(x);
if n==1
      y=x;
else
      m=floor(n/2);
      yL=mergeSort(x(1:m));
      yR=mergeSort(x(m+1:n));
      y=merge(yL,yR);
end
```



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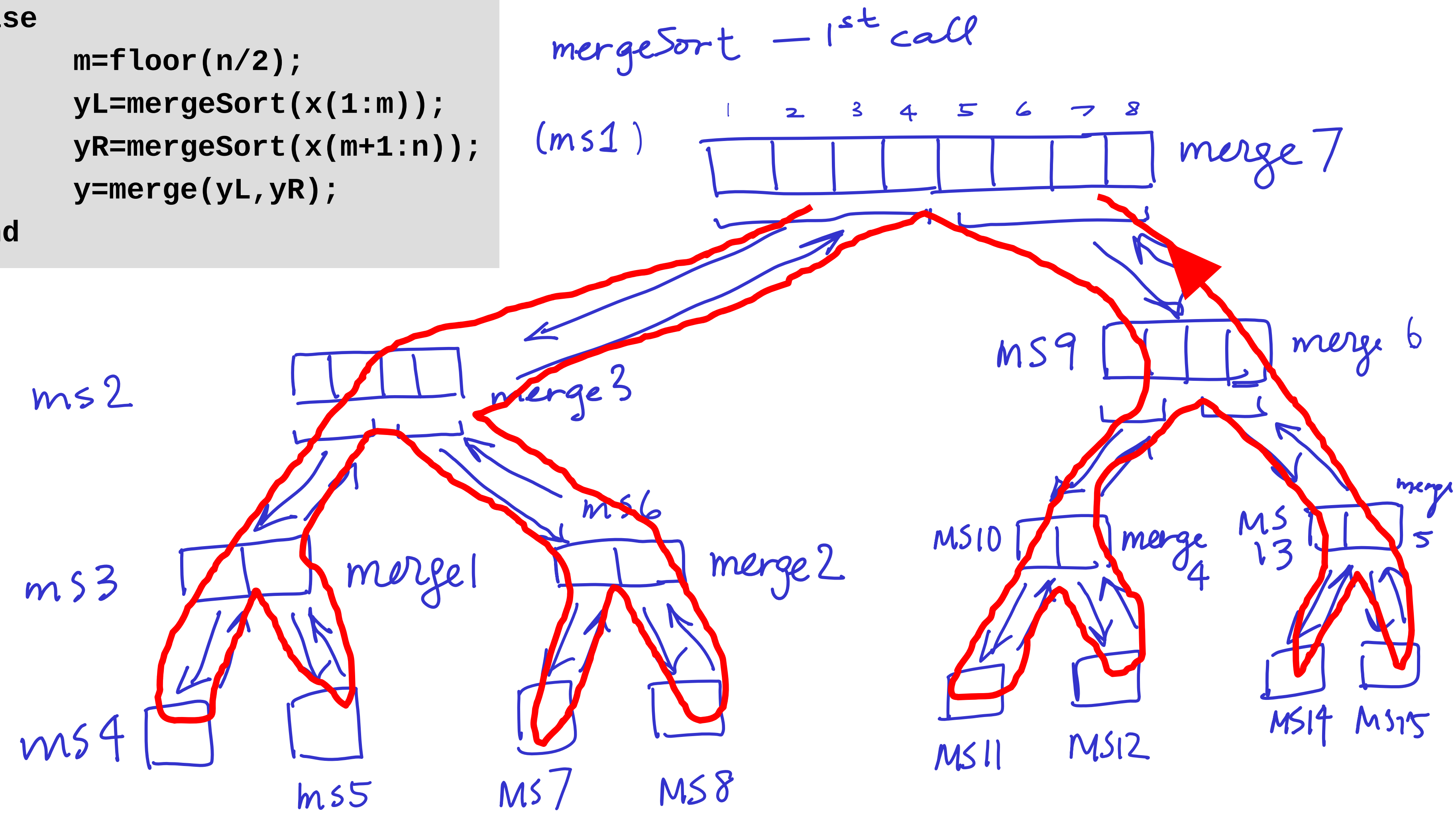
function y=mergeSort(x)
n=length(x);
if n==1
      y=x;
else
      m=floor(n/2);
      yL=mergeSort(x(1:m));
      yR=mergeSort(x(m+1:n));
      y=merge(yL,yR);
end
```



How do merge sort, insertion sort, and bubble sort compare?

- Insertion sort and bubble sort are similar
  - Both involve a series of comparisons and swaps
  - Both involve nested loops
- Merge sort uses recursion

See **`InsertionSort.m`**

```
function x = insertSort(x)
% Sort vector x in ascending order with insertion sort

n = length(x);
for i= 1:n-1

  % Sort x(1:i+1) given that x(1:i) is sorted
  j= i;
  while  j>0 && x(j+1)<x(j)

     % swap x(j+1) and x(j)
     temp= x(j);
     x(j)= x(j+1);
     x(j+1)= temp;

     j= j-1;

  end
end
```

How do merge sort and insertion sort compare?

- Insertion sort: (worst case) makes k comparisons to insert an element in a sorted array of k elements. For an array of length N:

  $1+2+\ldots+(N-1) = N(N-1)/2$, say N^2 for big N

- Merge sort:

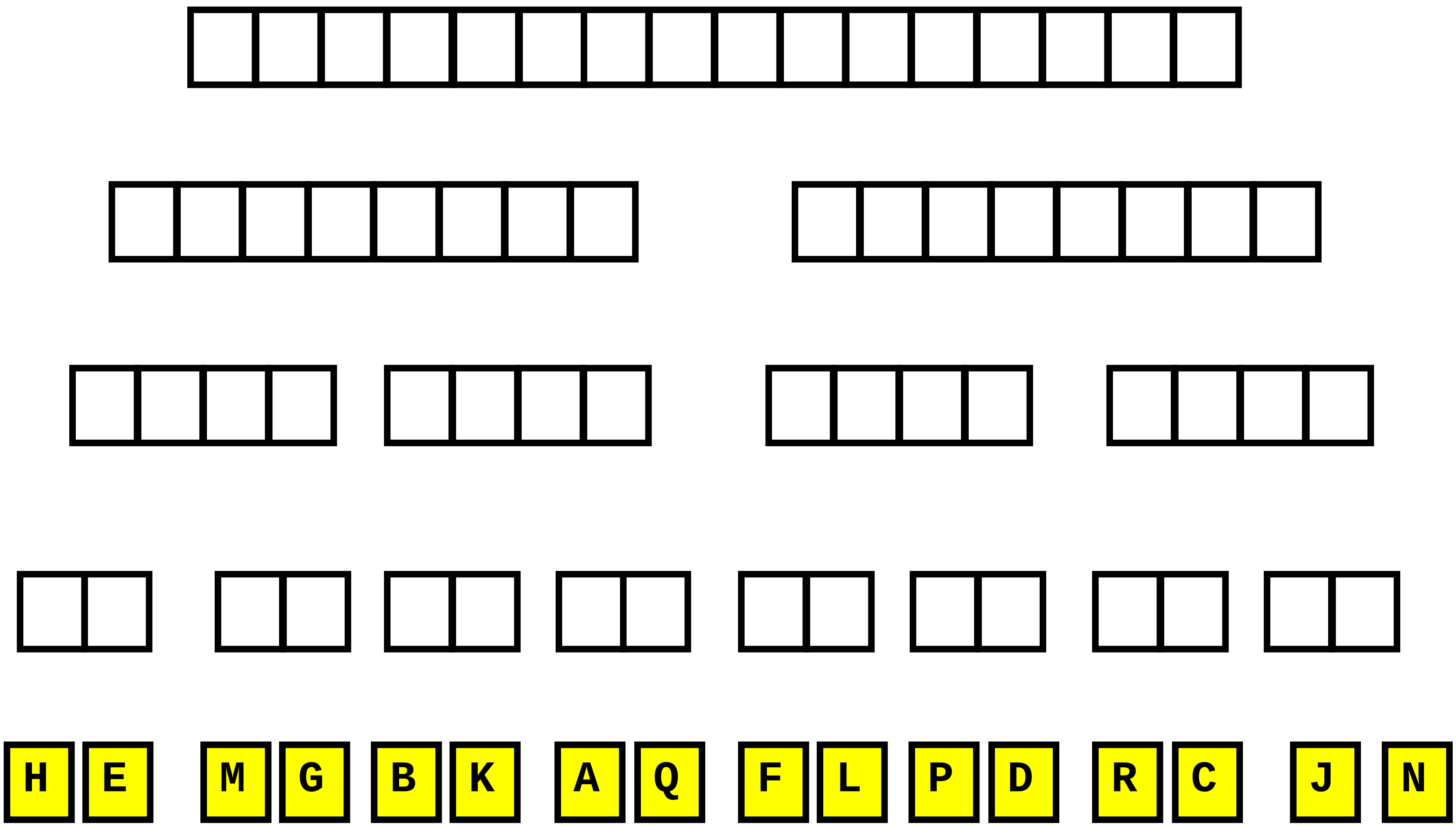
```
function y = mergeSort(x)
% x is a vector.  y is a vector
% consisting of the values in x
% sorted from smallest to largest.

n = length(x);
if n==1
      y = x;
else
      m  = floor(n/2);
      yL = mergeSort(x(1:m));
      yR = mergeSort(x(m+1:n));
      y  = merge(yL,yR);
end
```

All the comparisons between vector values are done in `merge`

```
function z = merge(x,y)
nx = length(x); ny = length(y);
z = zeros(1, nx+ny);
ix = 1; iy = 1; iz = 1;
while ix<=nx && iy<=ny
    if x(ix) <= y(iy)
        z(iz)= x(ix);  ix=ix+1;  iz=iz+1;
    else
        z(iz)= y(iy);  iy=iy+1;  iz=iz+1;
    end
end
while ix<=nx  % copy remaining x-values
  z(iz)= x(ix);  ix=ix+1;  iz=iz+1;
end
while iy<=ny  % copy remaining y-values
  z(iz)= y(iy);  iy=iy+1;  iz=iz+1;
end
```

Merge sort: $\log_2(N)$ "levels"; N comparisons each level



How do merge sort and insertion sort compare?

- Insertion sort: (worst case) makes i comparisons to insert an element in a sorted array of i elements. For an array of length N:

  $1+2+\ldots+(N-1) = N(N-1)/2$, say N^2 for big N

  $O(N^2)$

- Merge sort: $N \cdot \log_2(N)$

  $O(\ N \log_2(N)\ )$

  Order of magnitude

- Insertion sort is done *in-place*; merge sort (recursion) requires much more memory

See `compareInsertMerge.m`

How to choose??

- Depends on application
- Merge sort is especially good for sorting large data set (but watch out for memory usage)
- Insertion sort is "order N^2" at worst case, but what about an average case? If the application requires that you *maintain* a sorted array, insertion sort may be a good choice

Why not just use Matlab's sort function?

- Flexibility
- E.g., to maintain a sorted list, just write the code for insertion sort
- E.g., sort strings or other complicated structures
- Sort according to some criterion set out in a function file
  - Observe that we have the comparison `x(j+1)<x(j)`
  - The comparison can be a function that returns a boolean value
- Can combine different sort/search algorithms for specific problem

We've reached the end of CS1112… now what?

- Continue practicing your problem solving—problem decomposition—skills, in programming and other arenas!
- Interested in further study?
  - ENGRD/CS 2110 Object-oriented programming and data structure

ENGRG/CS 2110 OOP and Data Structures

- Learn new programming concepts and further explores those you've seen in CS1112
  - OOP, program design and development
  - Recursion
  - Complex data structures and related algorithms
- Taught in Java
- Optional CS 2111 meets 1 hr/week; additional practice with OOP, Java, and other course topics
- During break, check out this website: http://www.cs.cornell.edu/courses/CS1130/2014sp/

We've reached the end of CS1112… now what?

- Continue practicing your problem solving—problem decomposition—skills, in programming and other arenas!
- Interested in further study?
  - ENGRD/CS 2110 Object-oriented programming and data structure
  - Short courses in Python (CS 1133), C++ (CS 2024), …, etc.
  - More general CS courses: CS 2800 Discrete structures, CS 2850 Networks

What we learned…

- Develop/implement algorithms for problems
- Develop programming skills
  - Design, implement, document, test, and debug
- Programming "tool bag"
  - Functions for reducing redundancy
  - Control flow (if-else; loops)
  - Recursion
  - Data structures
  - Graphics
  - File handling

What we learned… (cont'd)

- Applications and concepts
  - Image processing
  - Object-oriented programming
  - Sorting and searching—you should know the algorithms covered
  - Divide-and-conquer strategies
  - Approximation and error
  - Simulation
  - Computational effort and efficiency

Computing gives us *insight* into a problem

- Computing is <u>not</u> about getting one answer!
- We build models and write programs so that we can "play" with the models and programs, learning—gaining insights—as we vary the parameters and assumptions
- Good models require domain-specific knowledge (and experience)
- Good programs …
  - are modular and cleanly organized
  - are well-documented
  - use appropriate data structures and algorithms
  - are reasonably efficient in time and memory

Final Exam

- Dec 17, 7-9:30pm, Barton Hall indoor tracks WEST
- Covers entire course; some emphasis on material after Prelim 2
- Closed-book exam, no calculators
- Bring student ID card

- Check for announcements on webpage:
  - Study break office/consulting hours
  - Review session time and location
  - Review questions
  - List of potentially useful functions

Final Exam

- Dec 17, 7-9:30pm, Barton Hall indoor tracks WEST
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  Prelim 2
- Closed-book exam
- Bri

Best wishes
and
good luck with all your exams!

ns
List of potentially useful functions