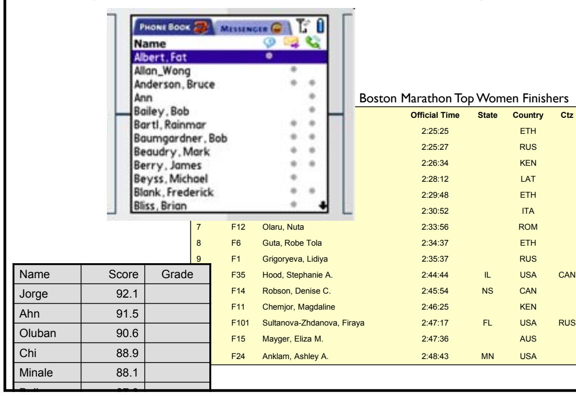
- Previous Lecture:
  - Recursion
- Today's Lecture:
  - Sorting and searching
    - Insertion sort, linear search
    - Read about *Bubble Sort* in Insight
  - "Divide and conquer" strategies
    - Binary search, merge sort
- Announcements
  - Discussion in Upson B7 lab this week
  - P6 due Thursday at 11pm
  - Final exam: Dec 17th 7pm, Barton Indoor Track WEST

Sorting data allows us to search more easily

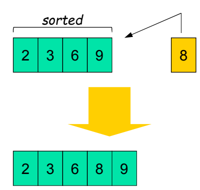


There are many algorithms for sorting

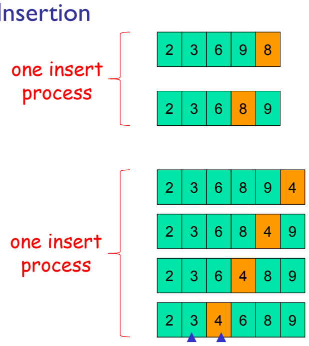
- Insertion Sort (to be discussed today)
- Bubble Sort (read *Insight* §8.2)
- Merge Sort (to be discussed Thursday)
- Quick Sort (a variant used by Matlab's built-in `sort` function)
- Each has advantages and disadvantages. Some algorithms are faster (time-efficient) while others are memory-efficient
- *Great opportunity for learning how to analyze programs and algorithms!*

The Insertion Process

- Given a sorted array x, insert a number y such that the result is sorted



Insertion



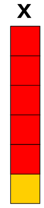
Compare adjacent components:
DONE! No more swaps.

See `Insert.m` for the insert process

Sort vector **x** using the Insertion Sort algorithm

Need to start with a *sorted* subvector. How do you find one?

x



Length 1 subvector is "sorted"

Insert `x(2)`: `[x(1:2),C,S] = Insert(x(1:2))`

Insert `x(3)`: `[x(1:3),C,S] = Insert(x(1:3))`

Insert `x(4)`: `[x(1:4),C,S] = Insert(x(1:4))`

Insert `x(5)`: `[x(1:5),C,S] = Insert(x(1:5))`

Insert `x(6)`: `[x(1:6),C,S] = Insert(x(1:6))`

`InsertionSort.m`

Insertion Sort vs. Bubble Sort

- Read about Bubble Sort in *Insight* §8.2
- Both algorithms involve the repeated comparison of adjacent values and swaps
- Find out which algorithm is more efficient on average

Other efficiency considerations

- Worst case, best case, average case
- Use of subfunction incurs an "overhead"
- Memory use and access

- Example: Rather than directing the *insert* process to a subfunction, have it done "in-line."
- Also, Insertion sort can be done "in-place," i.e., using "only" the memory space of the original vector.


```
function x = InsertionSortInplace(x)
% Sort vector x in ascending order with insertion sort

n = length(x);
for i= 1:n-1
      % Sort x(1:i+1) given that x(1:i) is sorted
      j= i;

      while

          % swap x(j+1) and x(j)



          j= j-1;

      end
end
```


Sort an array of objects

- Given x, a 1-d array of Interval references, sort x according to the widths of the Intervals from narrowest to widest
- Use the insertion sort algorithm
- How much of our code needs to be changed?

A. No change
B. One statement
C. About half the code
D. Most of the code

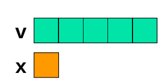
Searching for an item in a collection

Is the collection organized?
What is the organizing scheme?

Indiana Jones and the Raiders of the Lost Ark

Searching for an item in an unorganized collection?

- May need to look through the whole collection to find the target item
- E.g., find value x in vector v



- Linear search

```
% Linear Search
% f is index of first occurrence
%   of value x in vector v.
% f is -1 if x not found.
k= 1;
while  k<=length(v) && v(k)~=x
    k= k + 1;
end
if  k>length(v)
    f= -1; % signal for x not found
else
    f= k;
end
```

Searching in a sorted list should require less work

v	12	15	33	35	42	45

x	31

What if **v** is sorted?

An ordered (sorted) list

The Manhattan phone book has 1,000,000+ entries.

How is it possible to locate a name by examining just a tiny, tiny fraction of those entries?

Key idea of "phone book search": repeated halving

To find the page containing Pat Reed's number…

```
while  (Phone book is longer than 1 page)
    Open to the middle page.
    if  "Reed" comes before the first entry,
        Rip and throw away the 2nd half.
    else
        Rip and throw away the 1st half.
    end
end
```


What happens to the phone book length?

```
Original:      3000 pages
After 1 rip:   1500 pages
After 2 rips:   750 pages
After 3 rips:   375 pages
After 4 rips:   188 pages
After 5 rips:    94 pages
      :
After 12 rips:    1 page
```


Binary Search

Repeatedly halving the size of the "search space" is the main idea behind the method of binary search.

An item in a sorted array of length n can be located with just $\log_2 n$ comparisons.


```
% Linear Search
% f is index of first occurrence of value x in vector v.
% f is -1 if x not found.
k= 1;
while  k<=length(v) && v(k)~=x
    k= k + 1;
end
if  k>length(v)
    f= -1; % signal for x not found
else
    f= k;
end
```

n comparisons against the target are needed in worst case, n=length(v).

Binary Search

Repeatedly halving the size of the "search space" is the main idea behind the method of binary search.

An item in a sorted array of length n can be located with just $\log_2 n$ comparisons.

"Savings" is significant!

n	log2(n)
100	7
1000	10
10000	13

Binary search: target x = 70

	1	2	3	4	5	6	7	8	9	10	11	12
v	12	15	33	35	42	45	51	62	73	75	86	98

L: 1

Mid: 6

R: 12

v(Mid) <= x

So throw away the left half…

Binary search: target x = 70

	1	2	3	4	5	6	7	8	9	10	11	12
v	12	15	33	35	42	45	51	62	73	75	86	98

L: 6

Mid: 9

R: 12

x < v(Mid)

So throw away the right half…

Binary search: target x = 70

	1	2	3	4	5	6	7	8	9	10	11	12
v	12	15	33	35	42	45	51	62	73	75	86	98

L: 8

Mid: 8

R: 9

Done because R-L = 1


```
function L = binarySearch(x, v)
% Find position after which to insert x. v(1)<…<v(end).
% L is the index such that v(L) <= x < v(L+1);
% L=0 if x<v(1).  If x>v(end), L=length(v) but x~=v(L).

% Maintain a search window [L..R] such that v(L)<=x<v(R).
% Since x may not be in v, initially set ...
L=0;  R=length(v)+1;

% Keep halving [L..R] until R-L is 1,
%   always keeping  v(L) <= x < v(R)
while  R ~= L+1
    m= floor((L+R)/2);  % middle of search window
    if


    else


    end
end
```

Binary search is efficient, but we need to sort the vector in the first place so that we can use binary search

- Many different algorithms out there…
- We saw insertion sort (and read about bubble sort)
- Let's look at **merge sort**
- An example of the "divide and conquer" approach using recursion

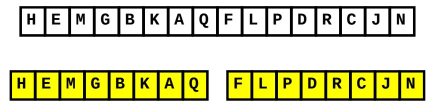
Merge sort: Motivation

If I have two helpers, I'd…
- Give each helper half the array to sort
- Then I get back the sorted subarrays and merge them.

What if those two helpers each had two sub-helpers?

And the sub-helpers each had two sub-sub-helpers? And…

Subdivide the sorting task



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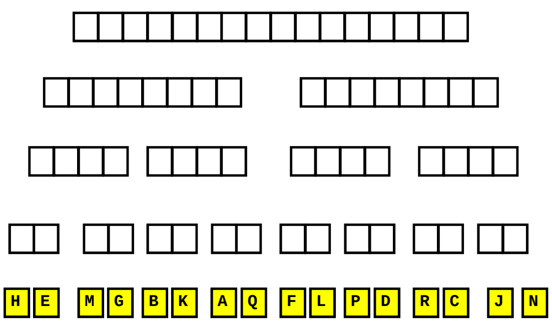
And one last time



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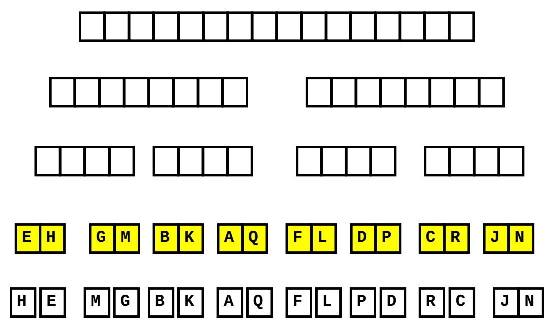
Now merge



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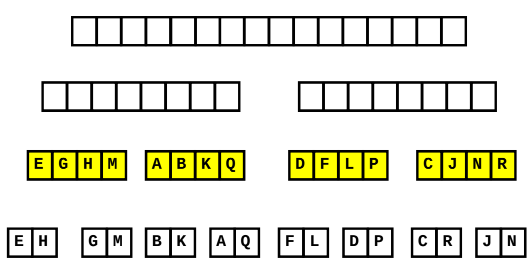
And merge again



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```
function y = mergeSort(x)
% x is a vector.  y is a vector
% consisting of the values in x
% sorted from smallest to largest.

n = length(x);
if n==1
      y = x;
else
      m  = floor(n/2);
      yL = mergeSort(x(1:m));
      yR = mergeSort(x(m+1:n));
      y  = merge(yL,yR);
end
```

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The central sub-problem is the merging of two sorted arrays into one single sorted array

12	33	35	45	
15	42	55	65	75

12	15	33	35	42	45	55	65	75

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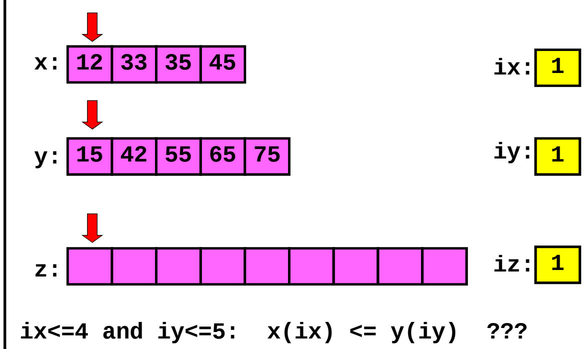
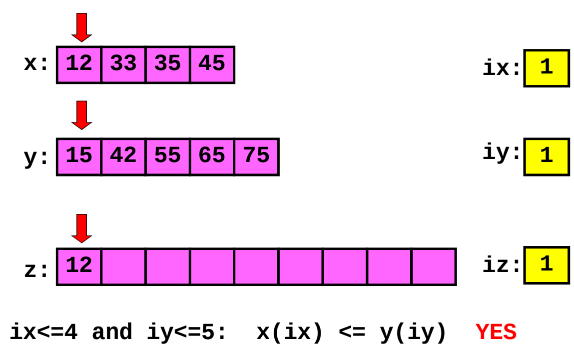
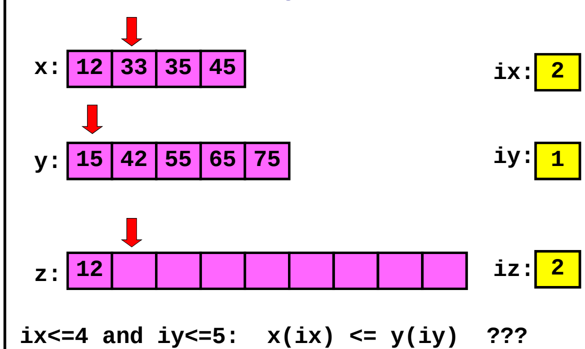
Merge



Merge



Merge

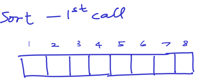



```
function z = merge(x,y)
nx = length(x); ny = length(y);
z = zeros(1, nx+ny);
ix = 1; iy = 1; iz = 1;
while ix<=nx && iy<=ny
    if  x(ix) <= y(iy)
        z(iz)= x(ix);  ix=ix+1;  iz=iz+1;
    else
        z(iz)= y(iy);  iy=iy+1;  iz=iz+1;
    end
end
while ix<=nx  % copy remaining x-values
  z(iz)= x(ix);  ix=ix+1;  iz=iz+1;
end
while iy<=ny  % copy remaining y-values
  z(iz)= y(iy);  iy=iy+1;  iz=iz+1;
end
```

```
function y=mergeSort(x)
n=length(x);
if n==1
      y=x;
else
      m=floor(n/2);
      yL=mergeSort(x(1:m));
      yR=mergeSort(x(m+1:n));
      y=merge(yL,yR);
end
```

mergeSort — 1st call

(ms1)



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