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Lecture 23: More Algorithms for Sorting

CS 1110

Introduction to Computing Using Python

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Announcements

Next Tuesday:

- Lecture is a review session.
- There will be no post-lecture office hours.

Course Staff also hosting additional review sessions (possibly during study days). Announcements forthcoming.

Search Algorithms

Recall from last lecture:

- Searching for data is a common task
  - Linear search: on the order of n
    - input doubles? → work **doubles**!
  - Binary search: on the order of log2 n
    - input doubles? → work **increases by just 1 unit**!
    - BUT data needs to be sorted…

- **Sorting** data now suddenly interesting…

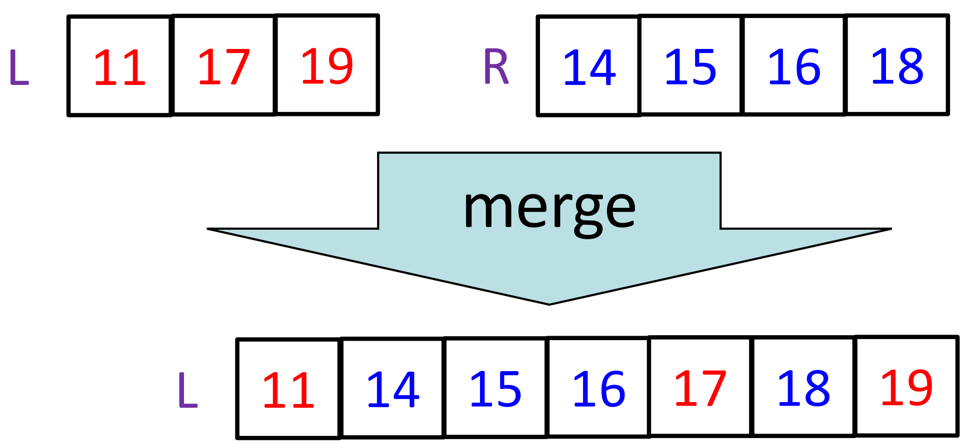
Sorting Algorithms

- Sorting data is a common task
  - Insertion sort: on the order of n^2
    - input doubles? → work **quadruples**! (yikes)

- Today's topic:
  - Merge sort: *can we do better than Insertion Sort?*

Which algorithm does Python's `sort` use?

- Recursive algorithm that runs much faster than insertion sort for the same size list (when the size is big)!
- A variant of an algorithm called "merge sort"
- Based on the idea that sorting is hard, but *"merging" two already sorted lists is easy*.



Merge sort: Motivation

Since merging is easier than sorting, if I had two helpers, I'd...

- Give each helper half the array to sort
- Then I get back their sorted subarrays and merge them.

What if those two helpers each had two sub-helpers?

And the sub-helpers each had two sub-sub-helpers? And...

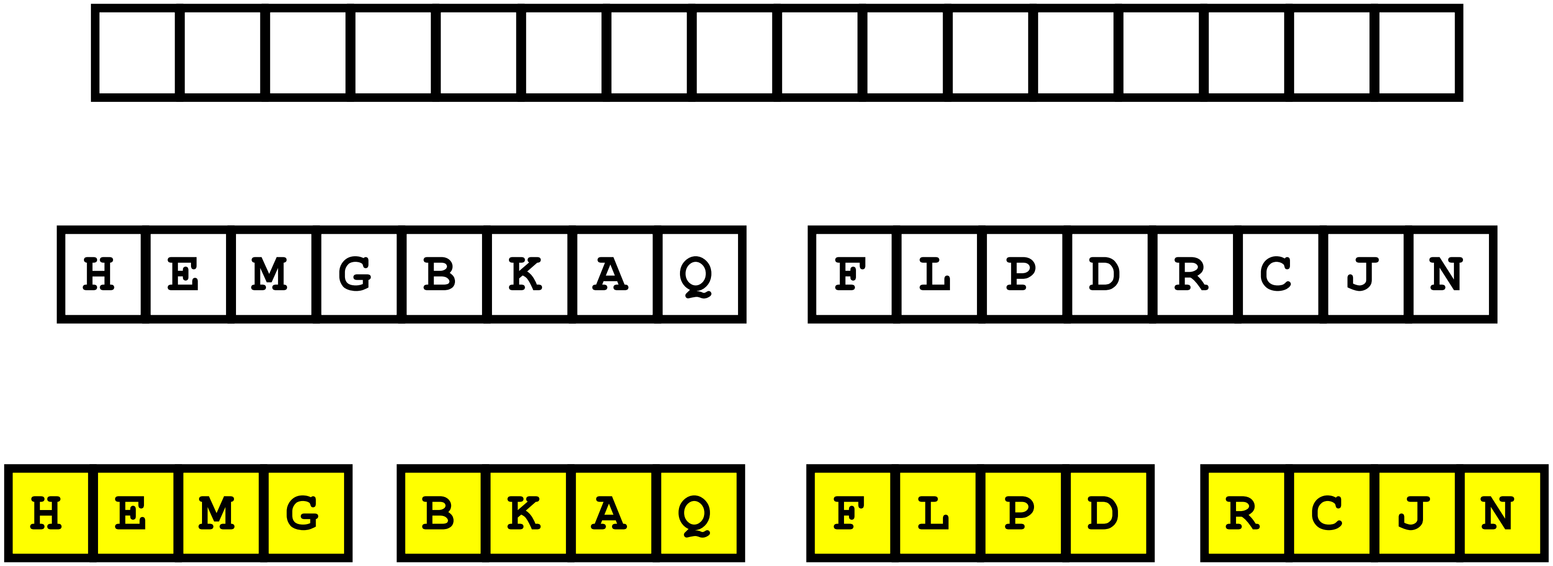
Subdivide the sorting task

H	E	M	G	B	K	A	Q	F	L	P	D	R	C	J	N

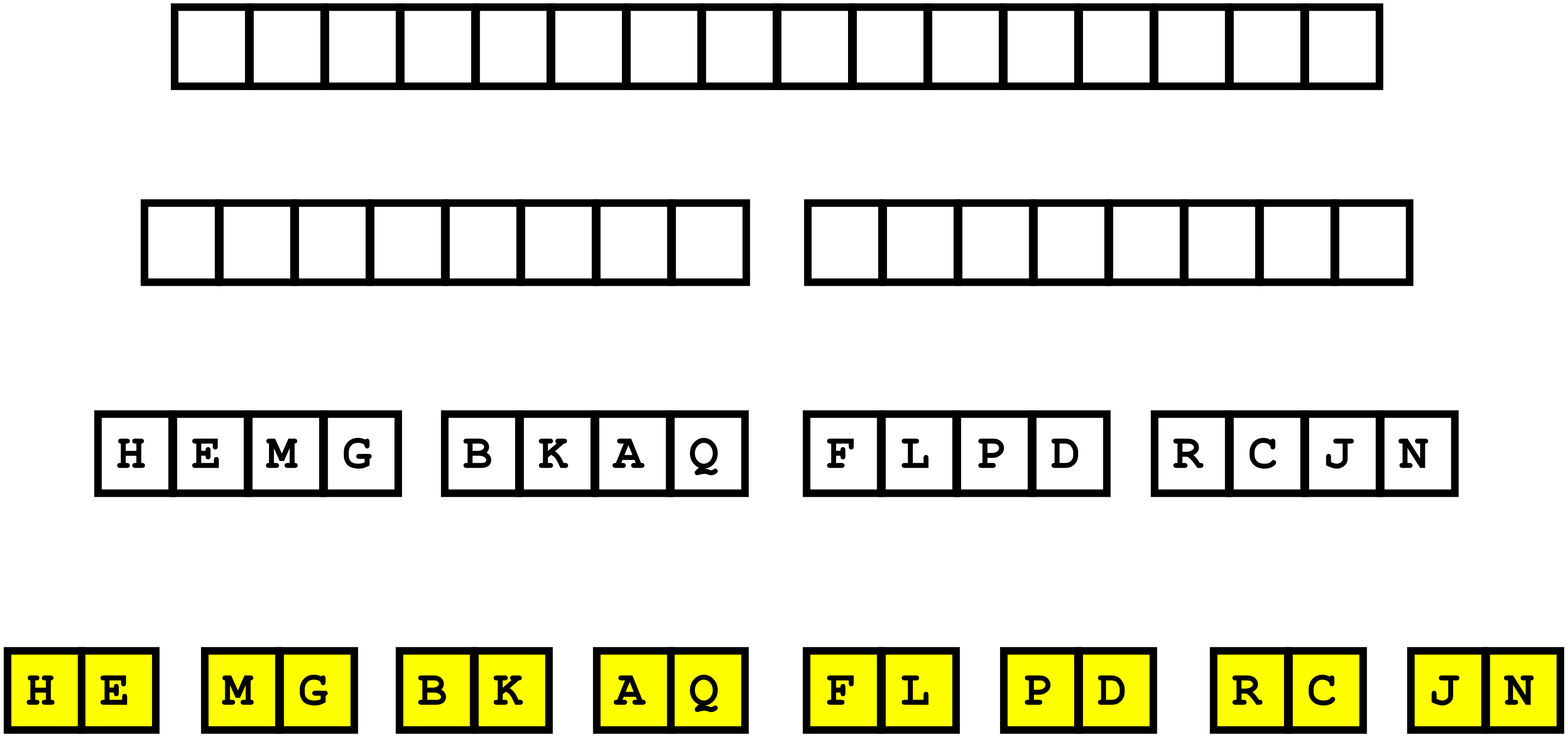
H	E	M	G	B	K	A	Q

F	L	P	D	R	C	J	N

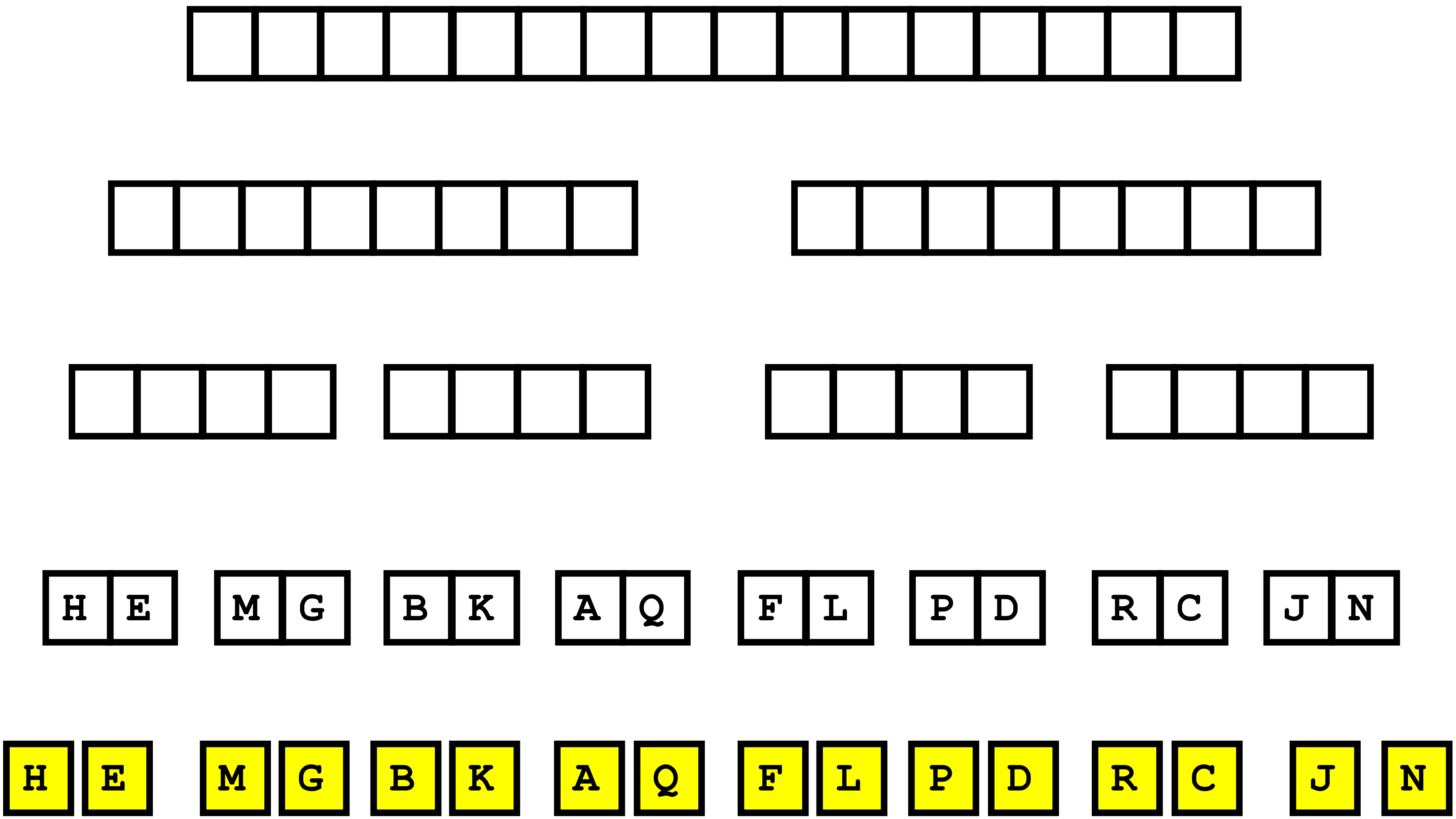
Subdivide again



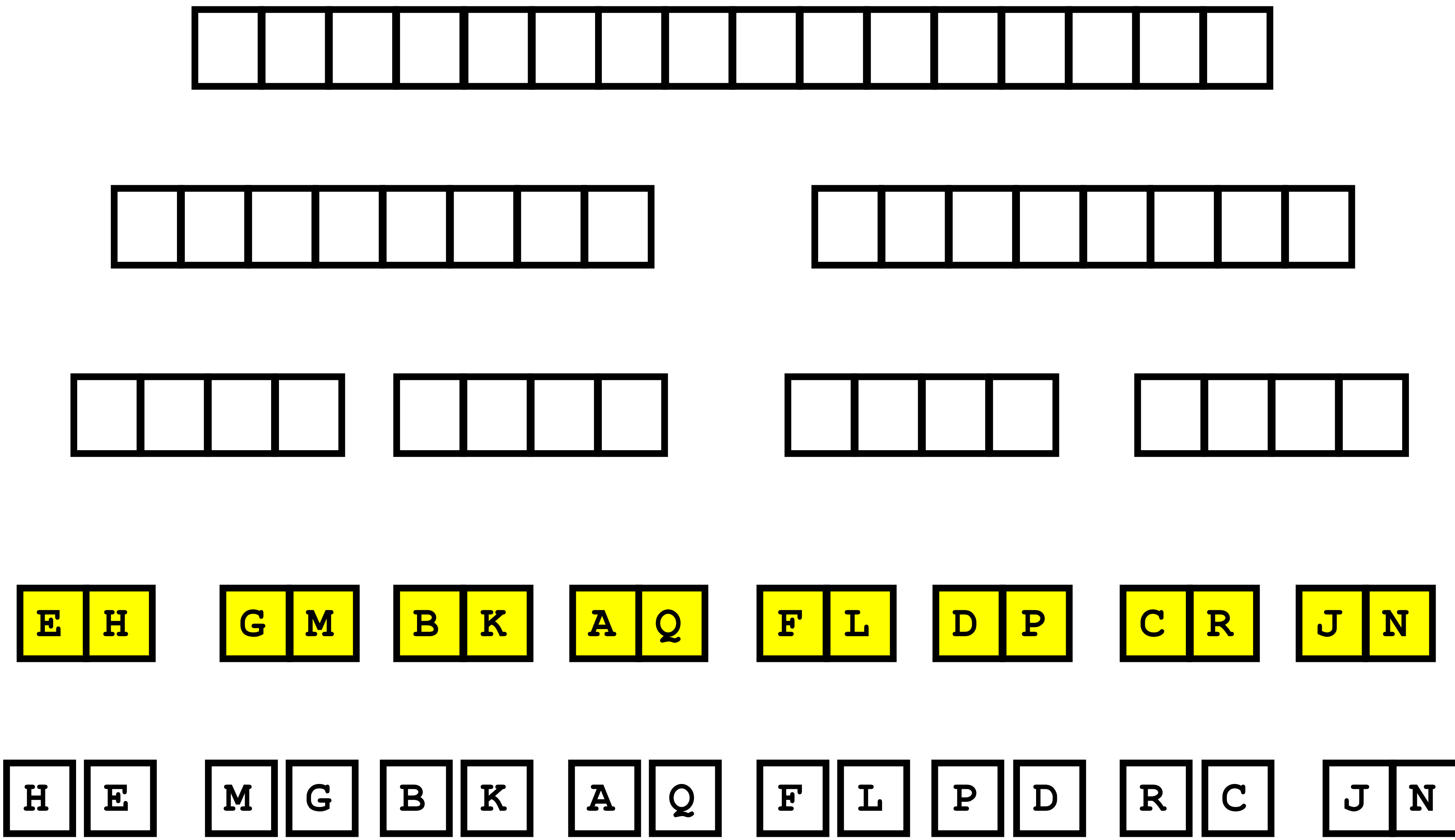
And again



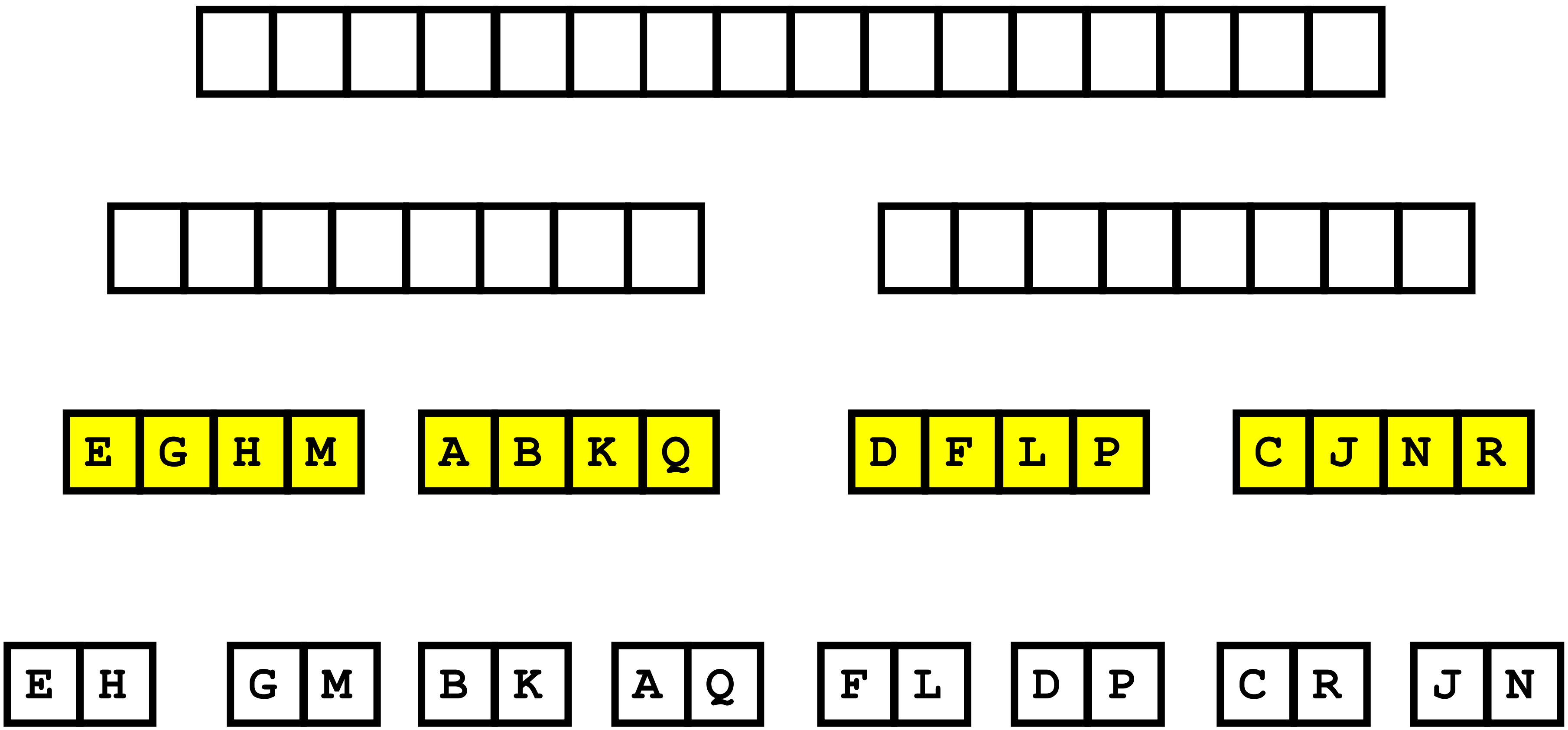
And one last time



Now merge



And merge again



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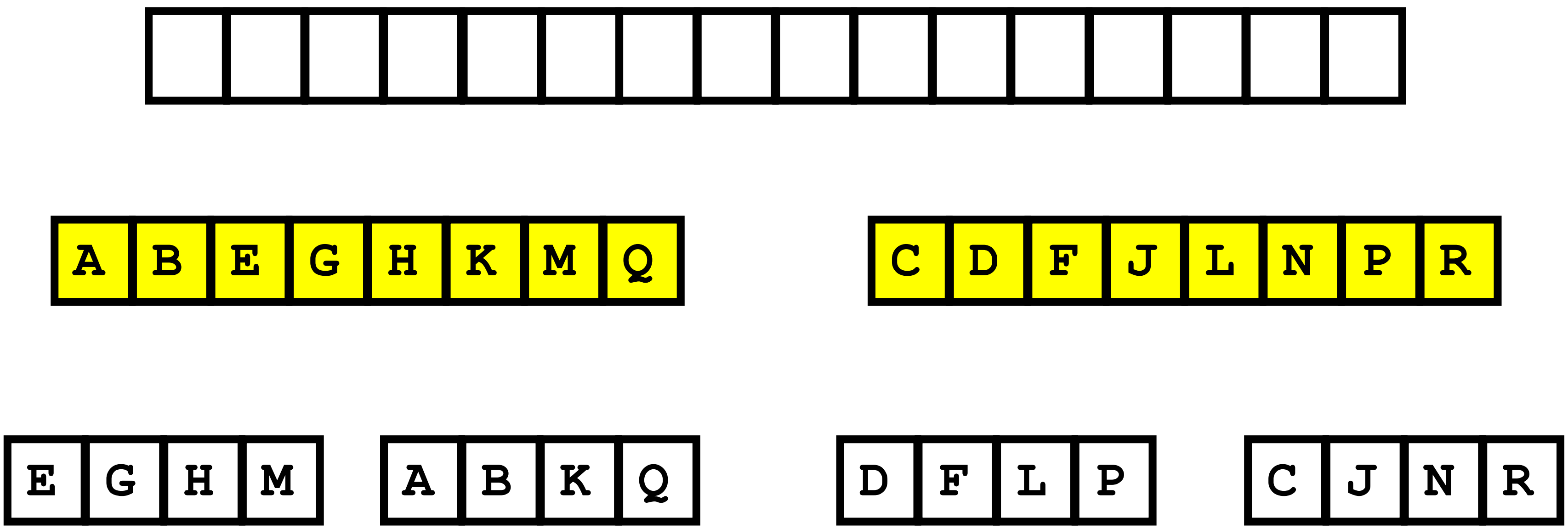


And one last time

A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q	R

A	B	E	G	H	K	M	Q

C	D	F	J	L	N	P	R

Done!

A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q	R

```python
def mergeSort(li):
    """Sort list li using Merge Sort"""
    if len(li) > 1:
        # Divide into two parts
        mid= len(li)//2
        left= li[:mid]
        right= li[mid:]

        # Recursive calls
        mergeSort(left)
        mergeSort(right)

        # Merge left & right back to li
        ???
    # base case does nothing!
    # a list with len 0 or 1 is sorted!
```

The central sub-problem is the **merging** of two sorted lists into one single sorted list

12	33	35	45

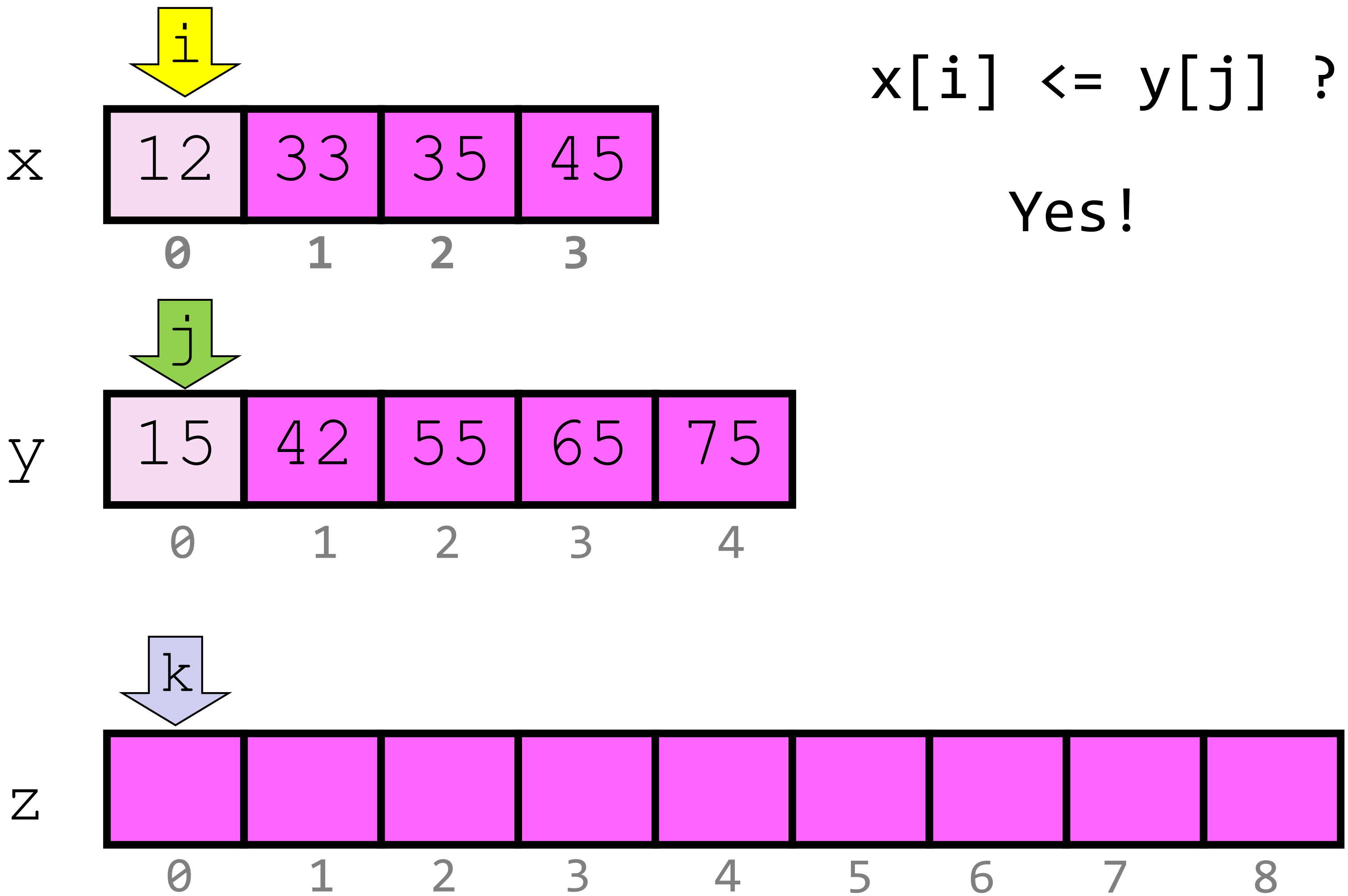
15	42	55	65	75

Approach:
keep comparing the smallest element of first list with smallest element of second list.

12	15	33	35	42	45	55	65	75

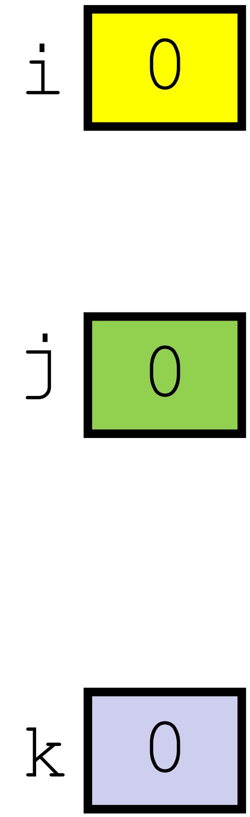
How to Merge

as long as both x and y
have unprocessed elements



x[i] <= y[j] ?

Yes!



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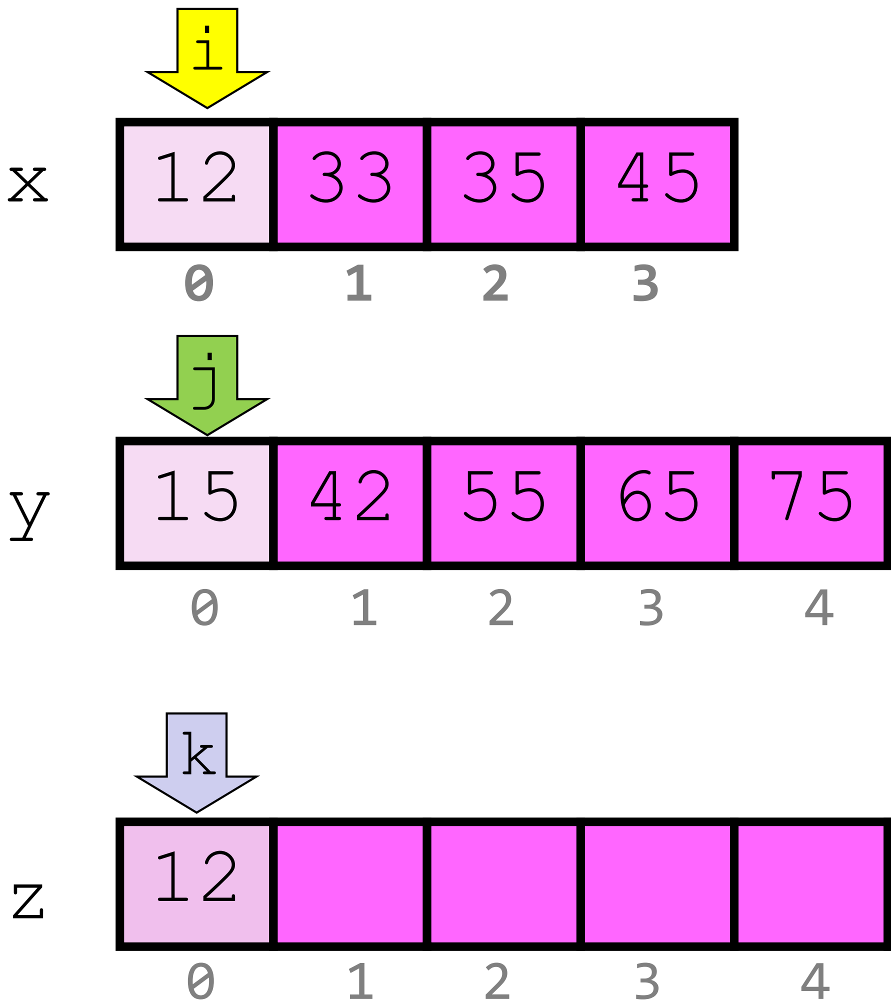
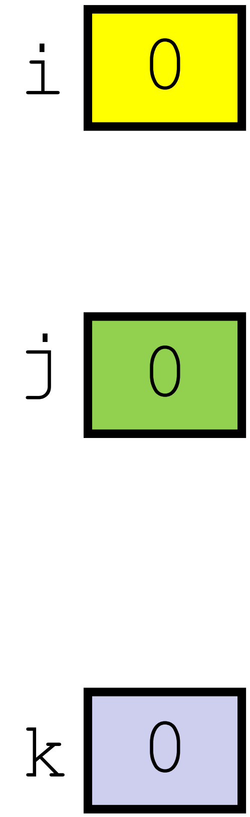


x[i] <= y[j] ?

Yes!

copy x[i] to z



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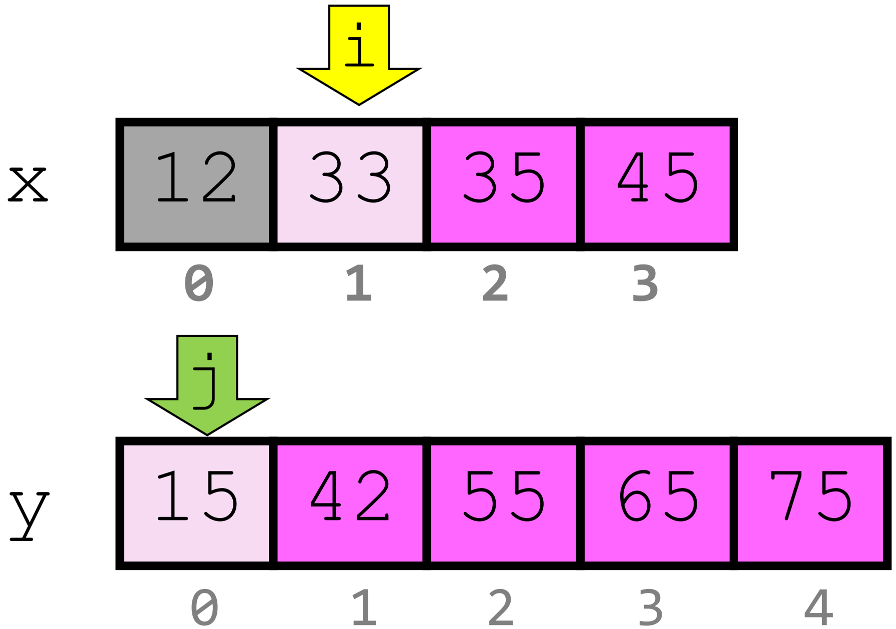

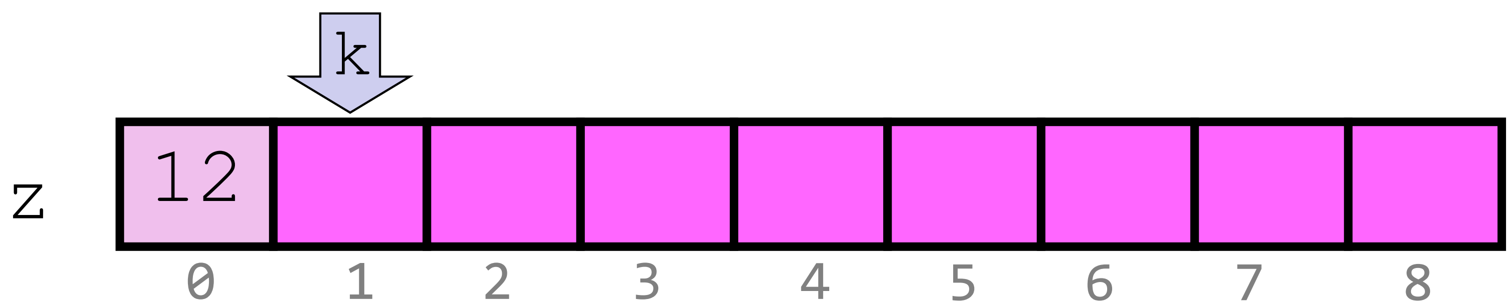

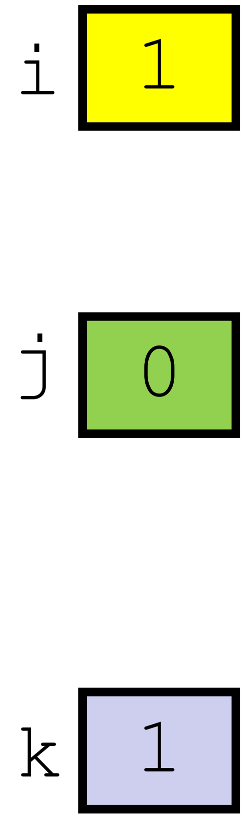

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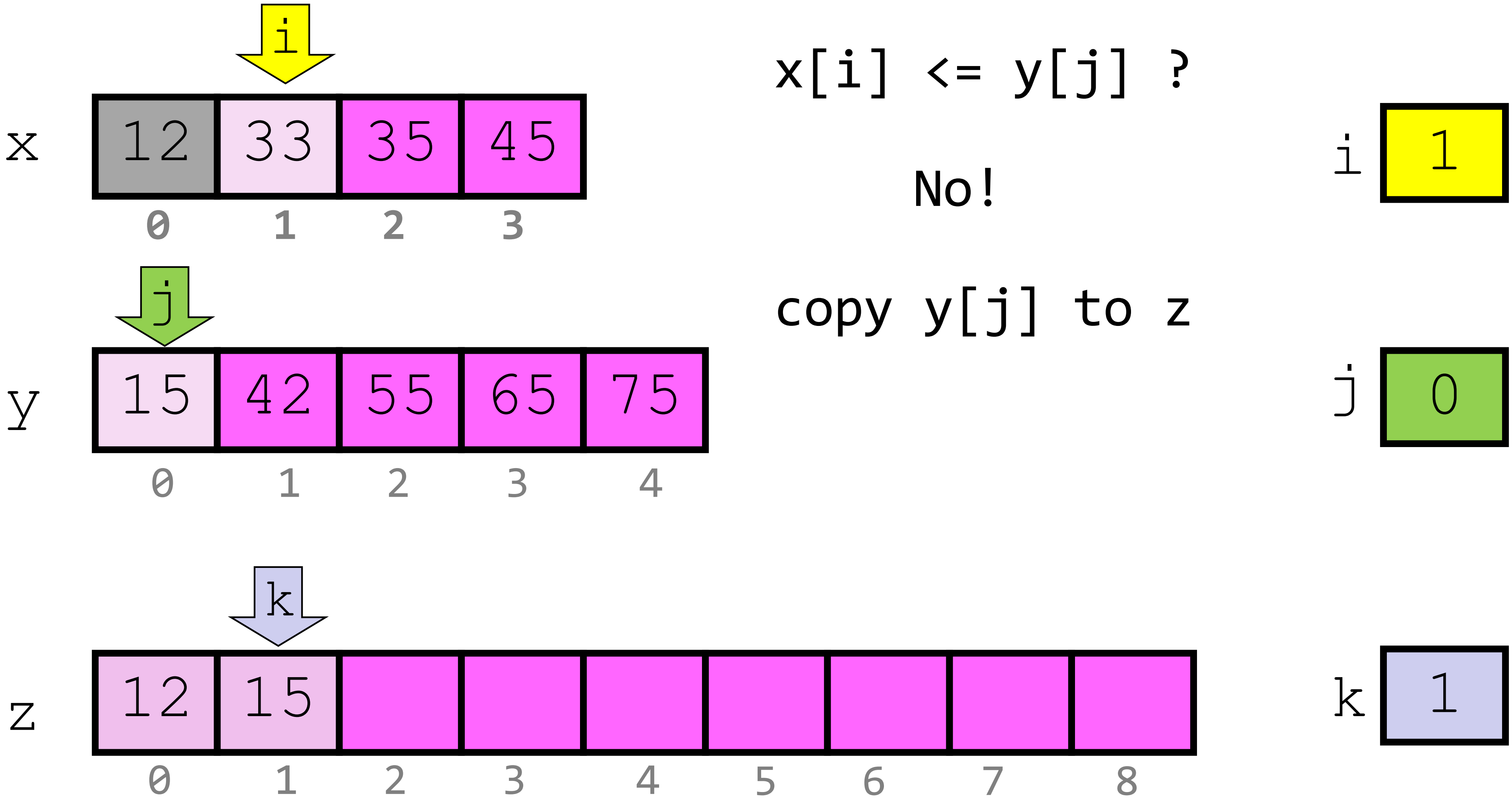

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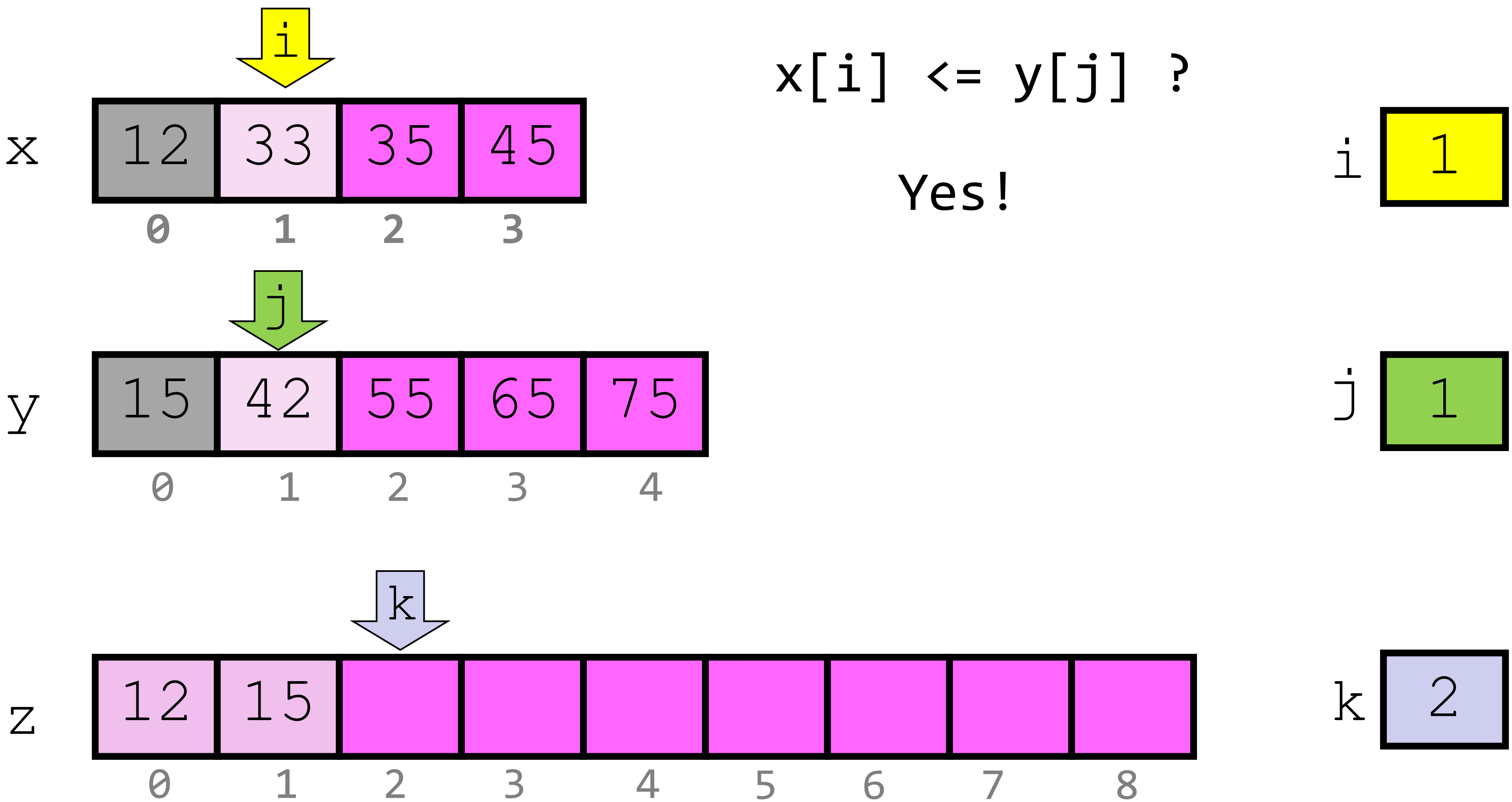

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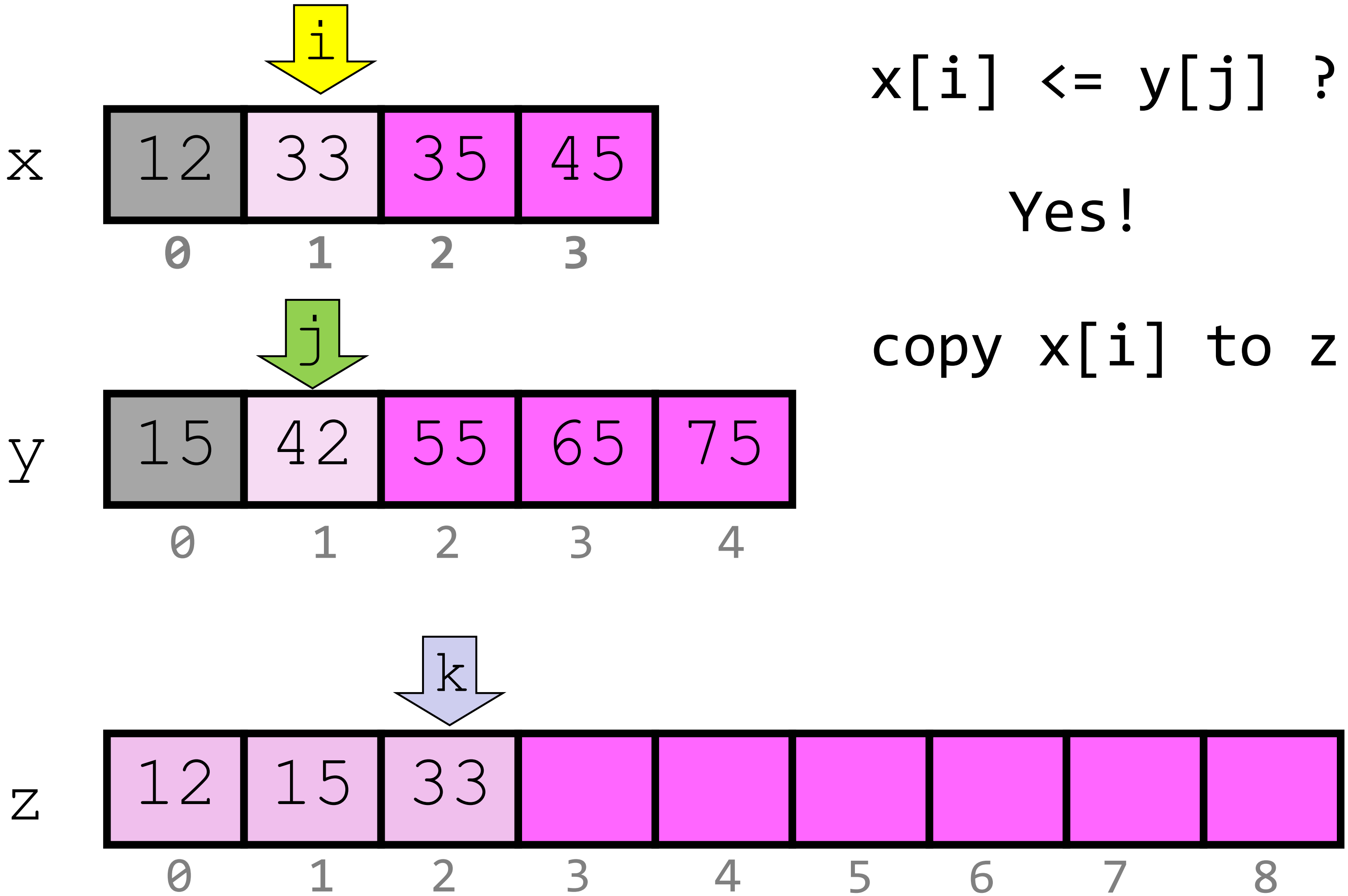

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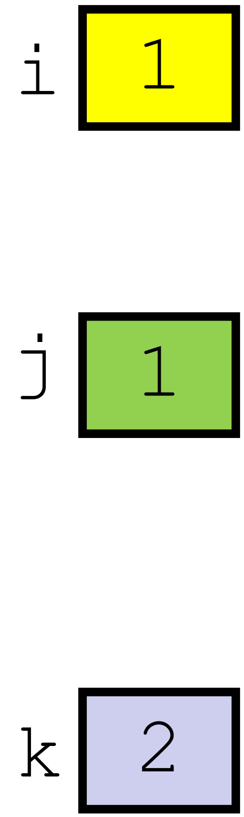

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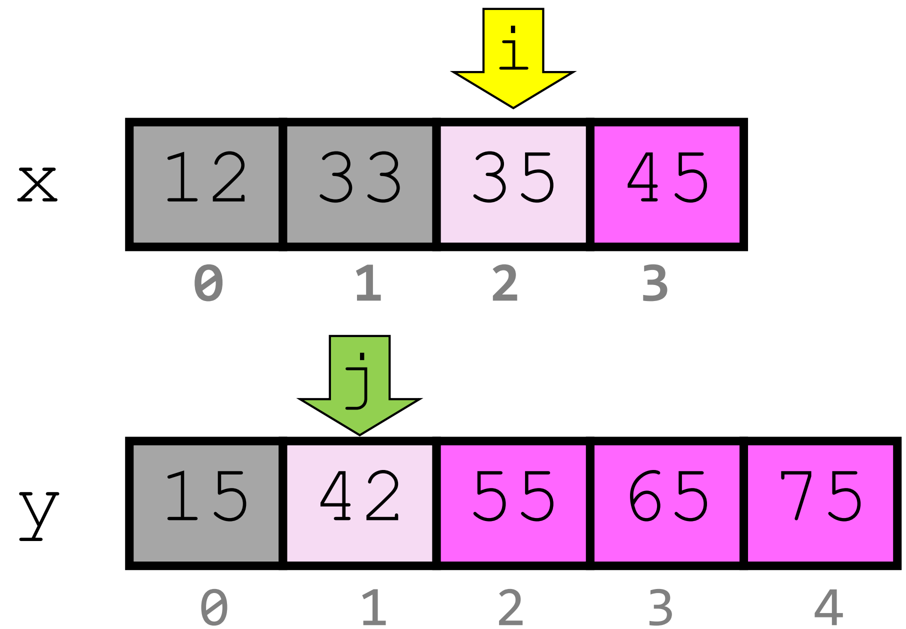

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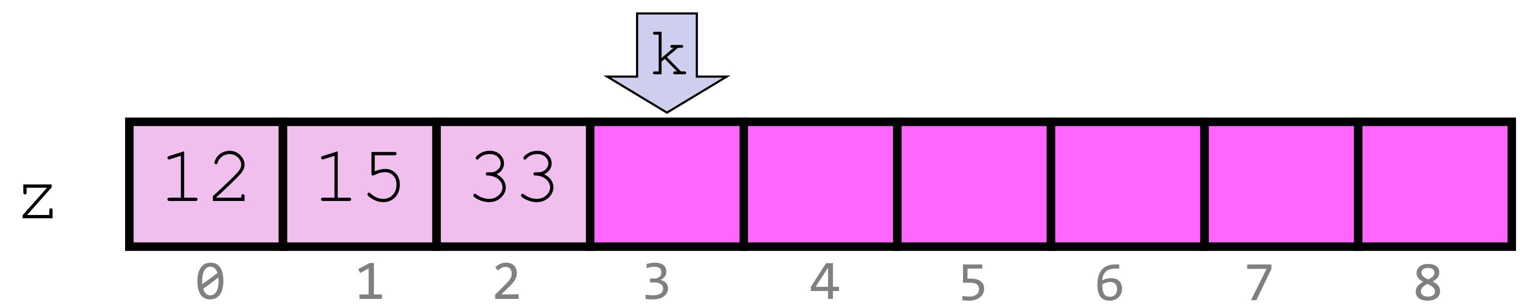

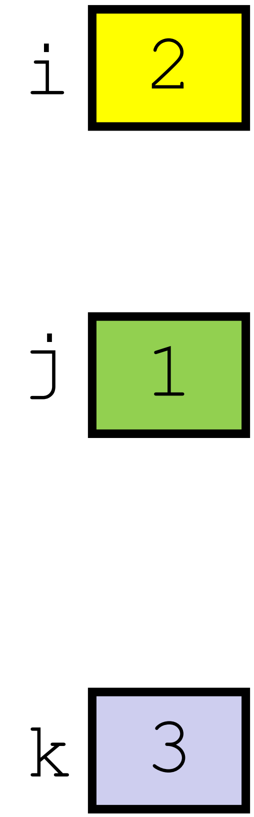

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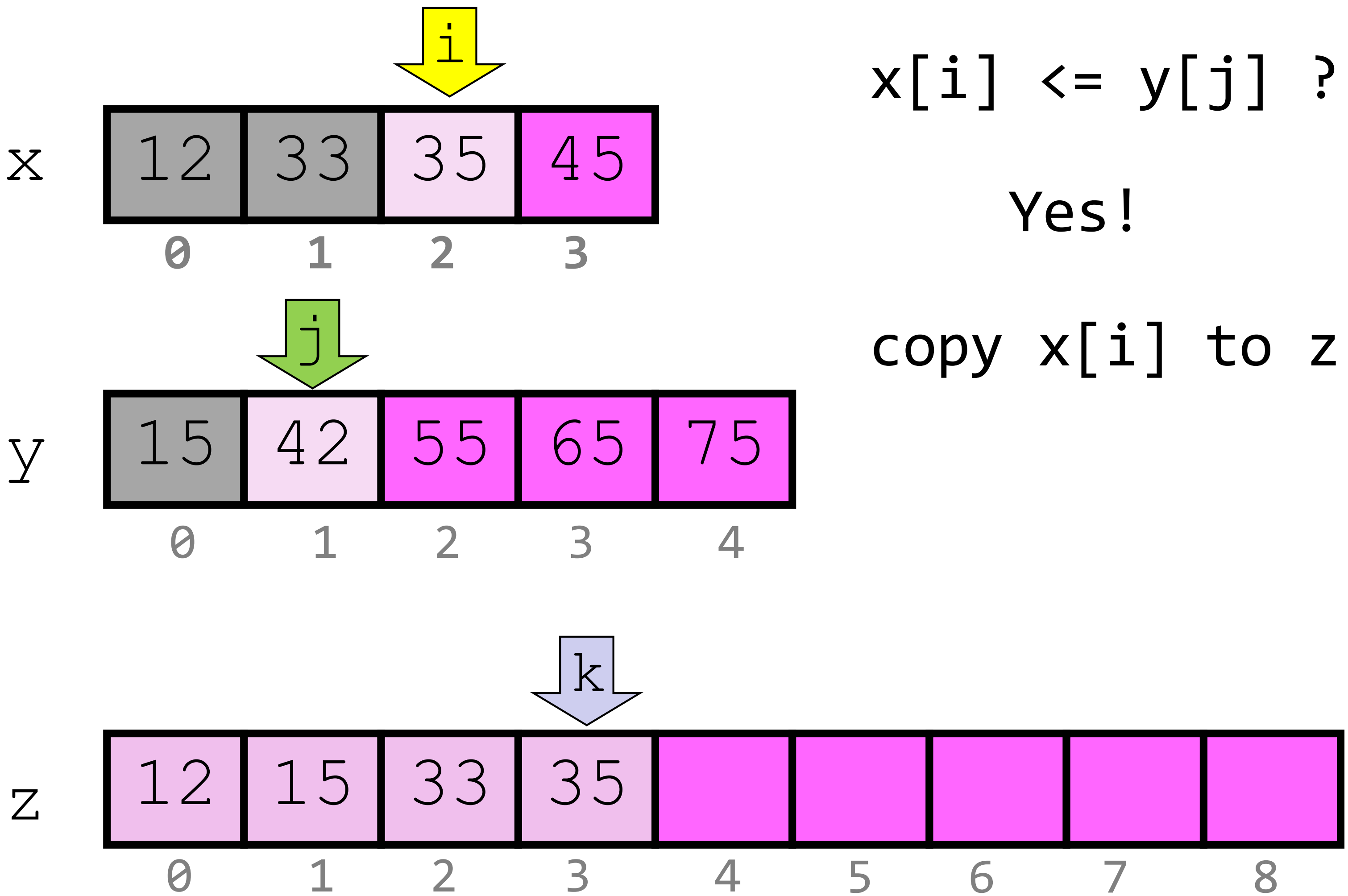

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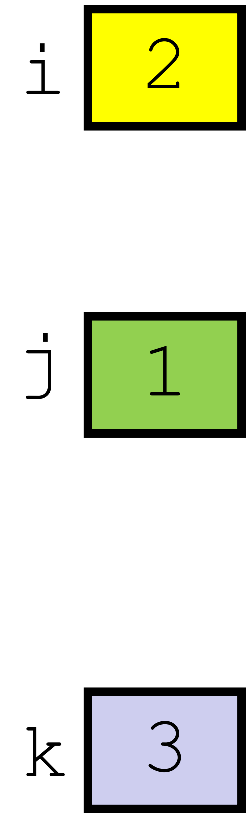

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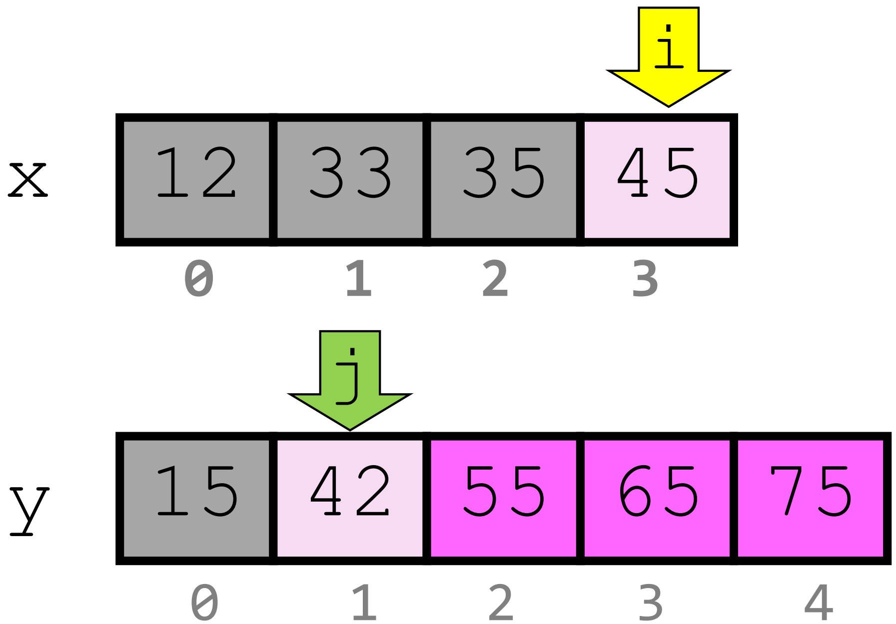

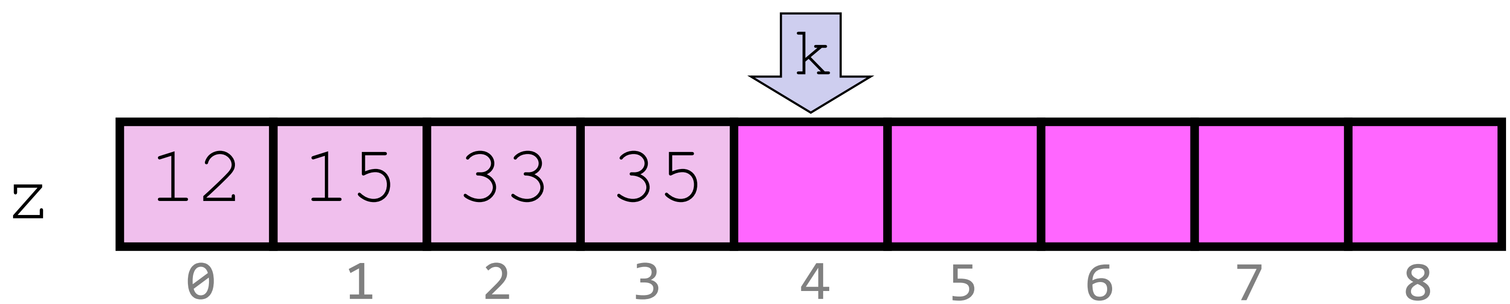

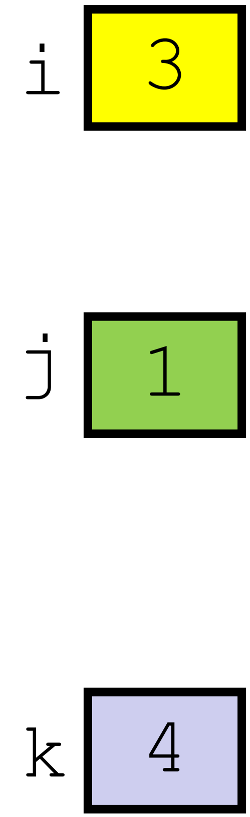

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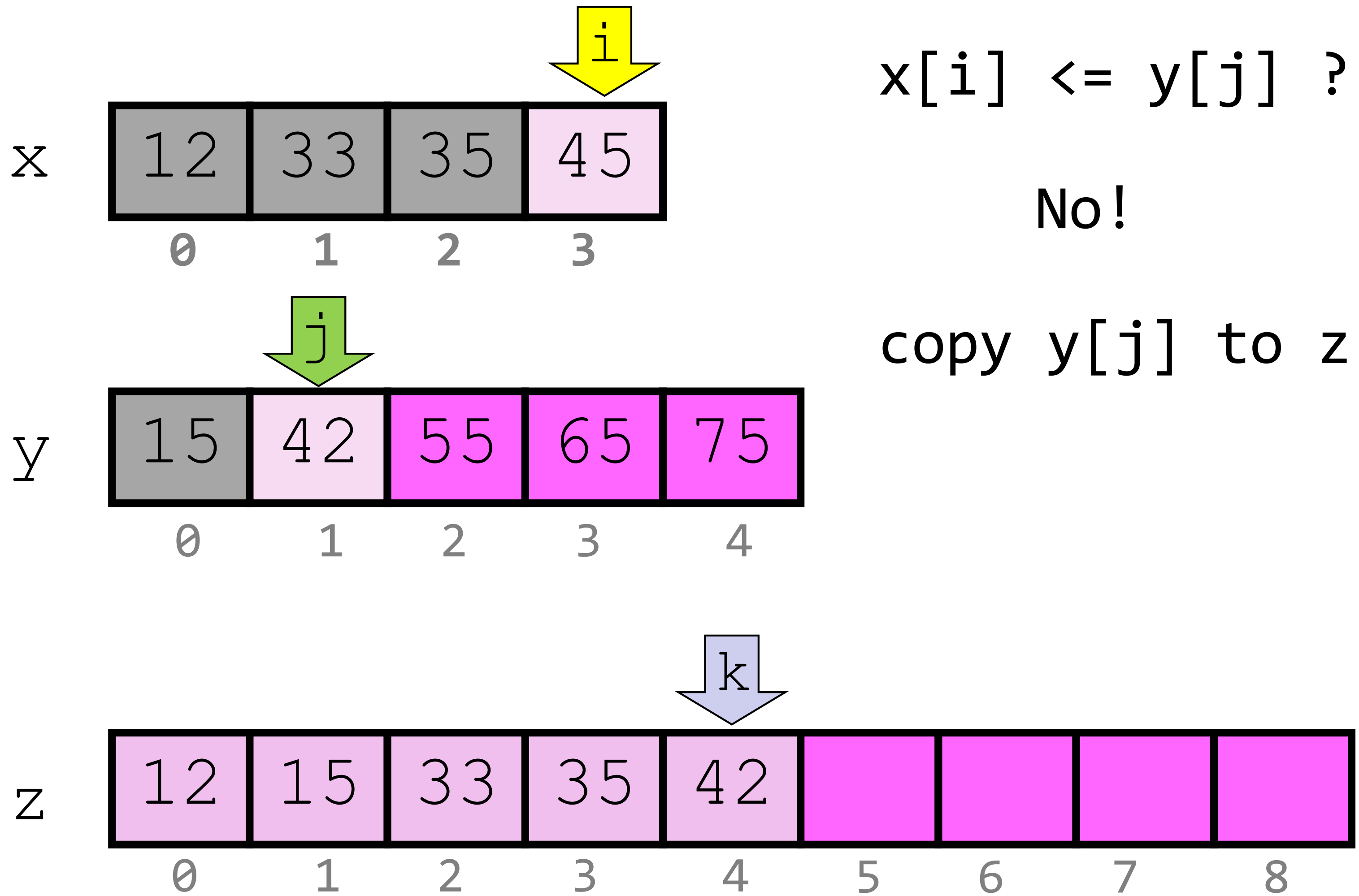
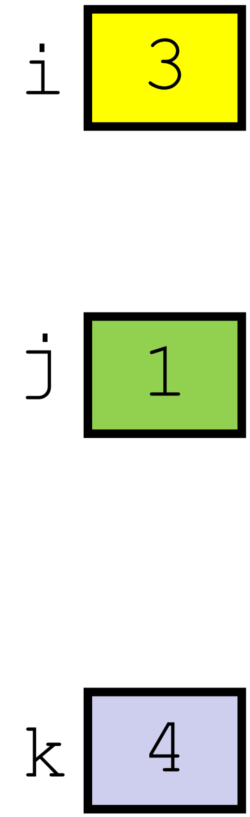


x[i] <= y[j] ?

No!

copy y[j] to z



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```

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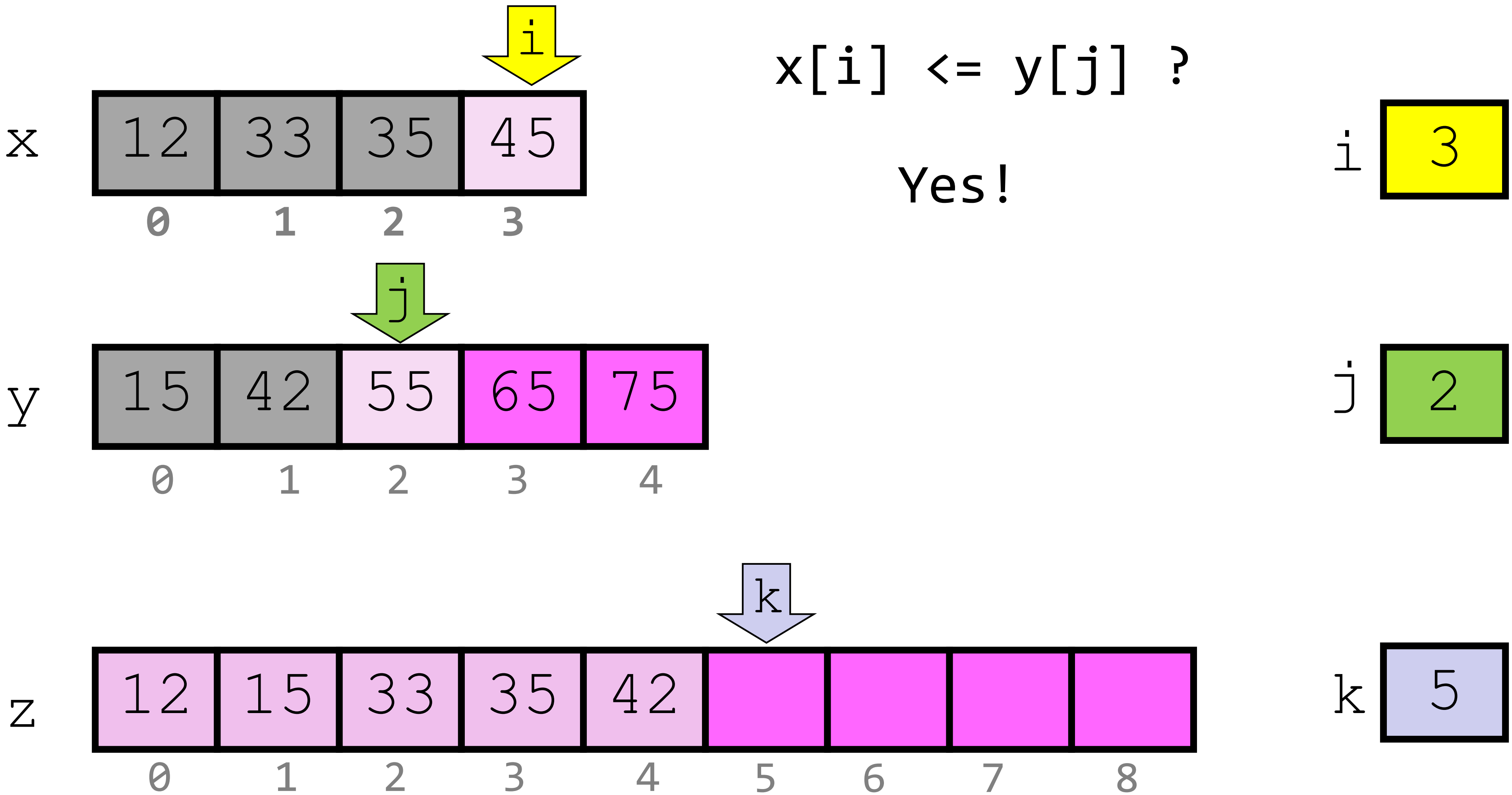

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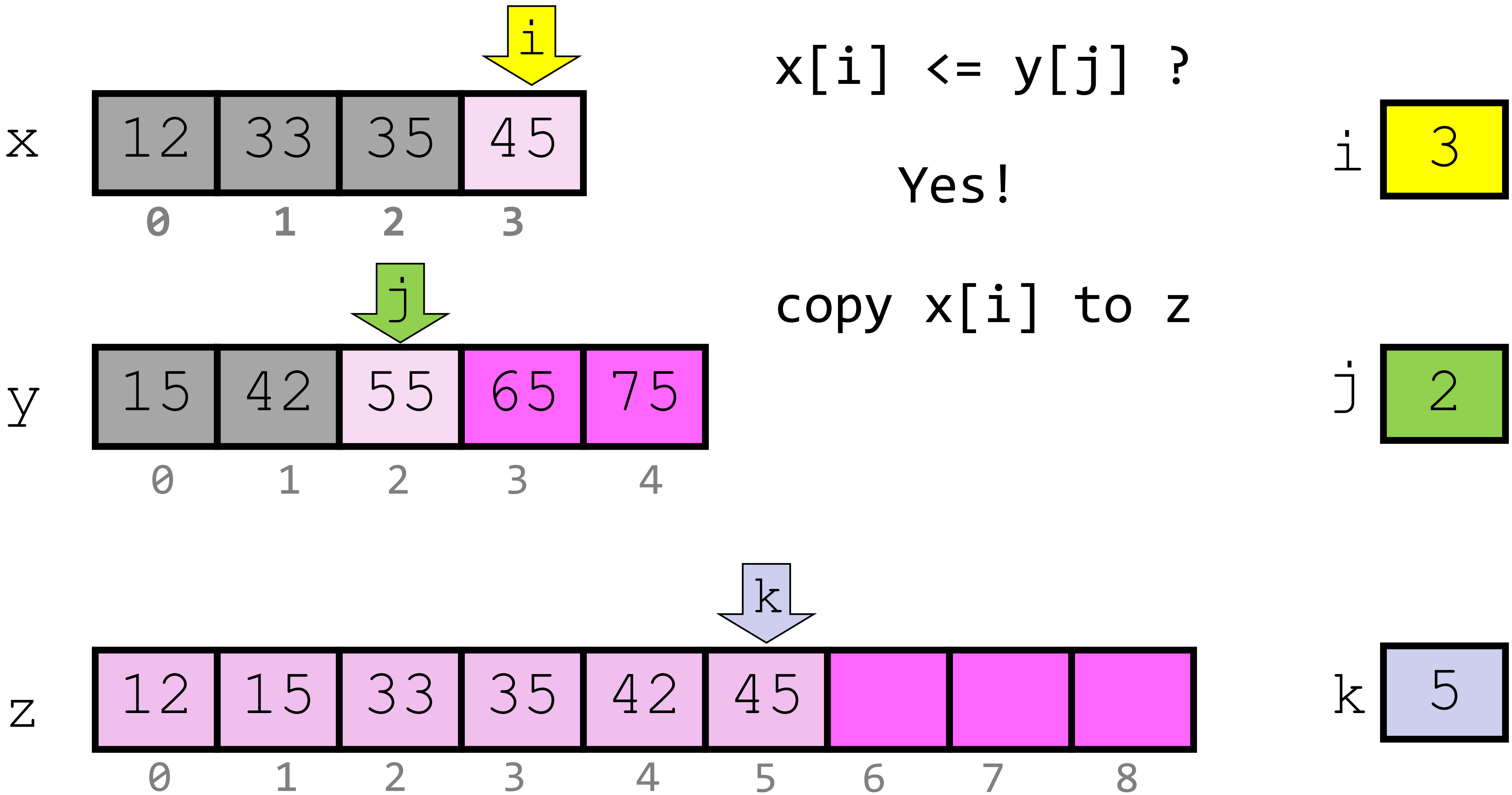

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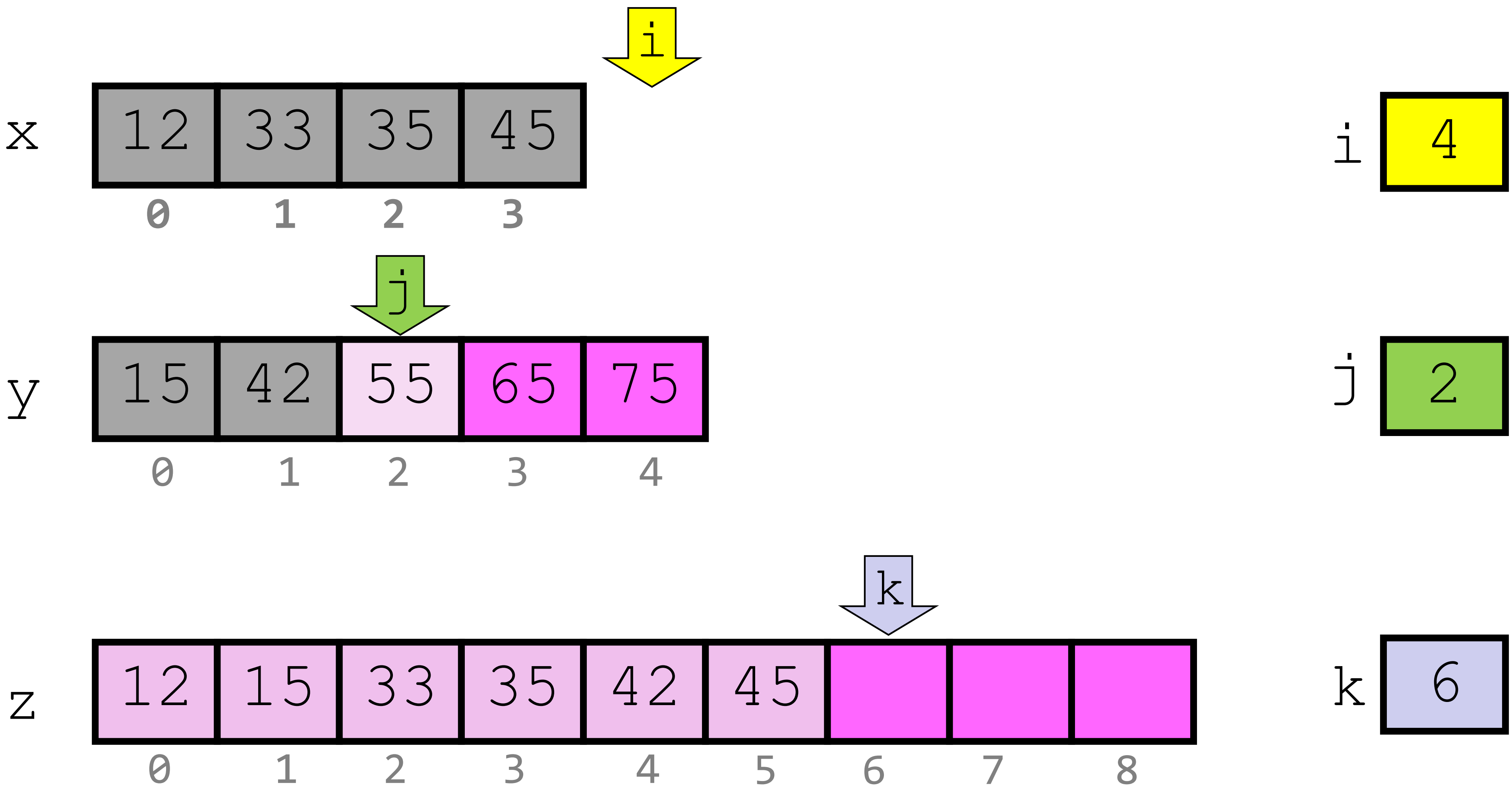

How to Merge

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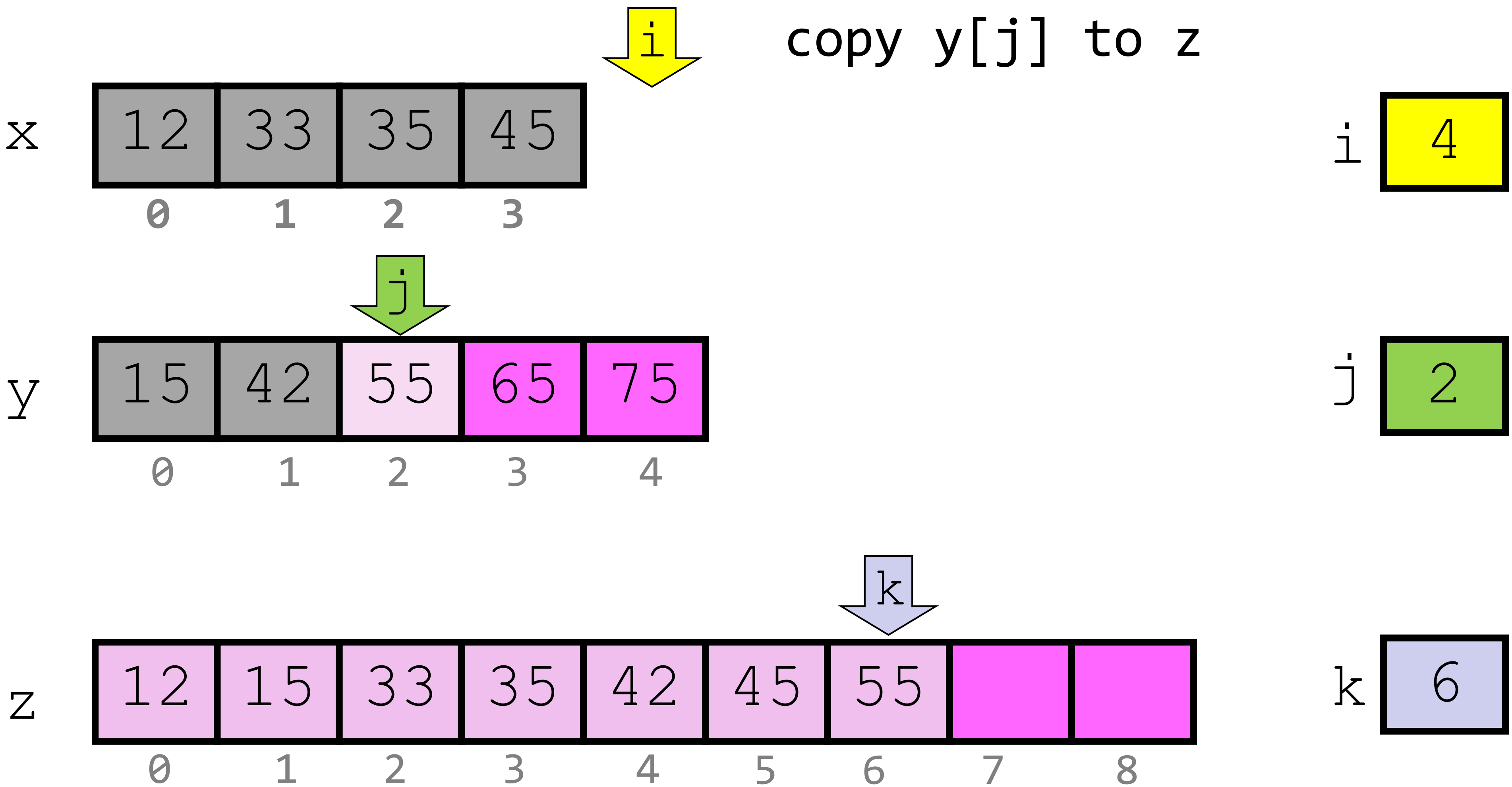
copy y[j] to z



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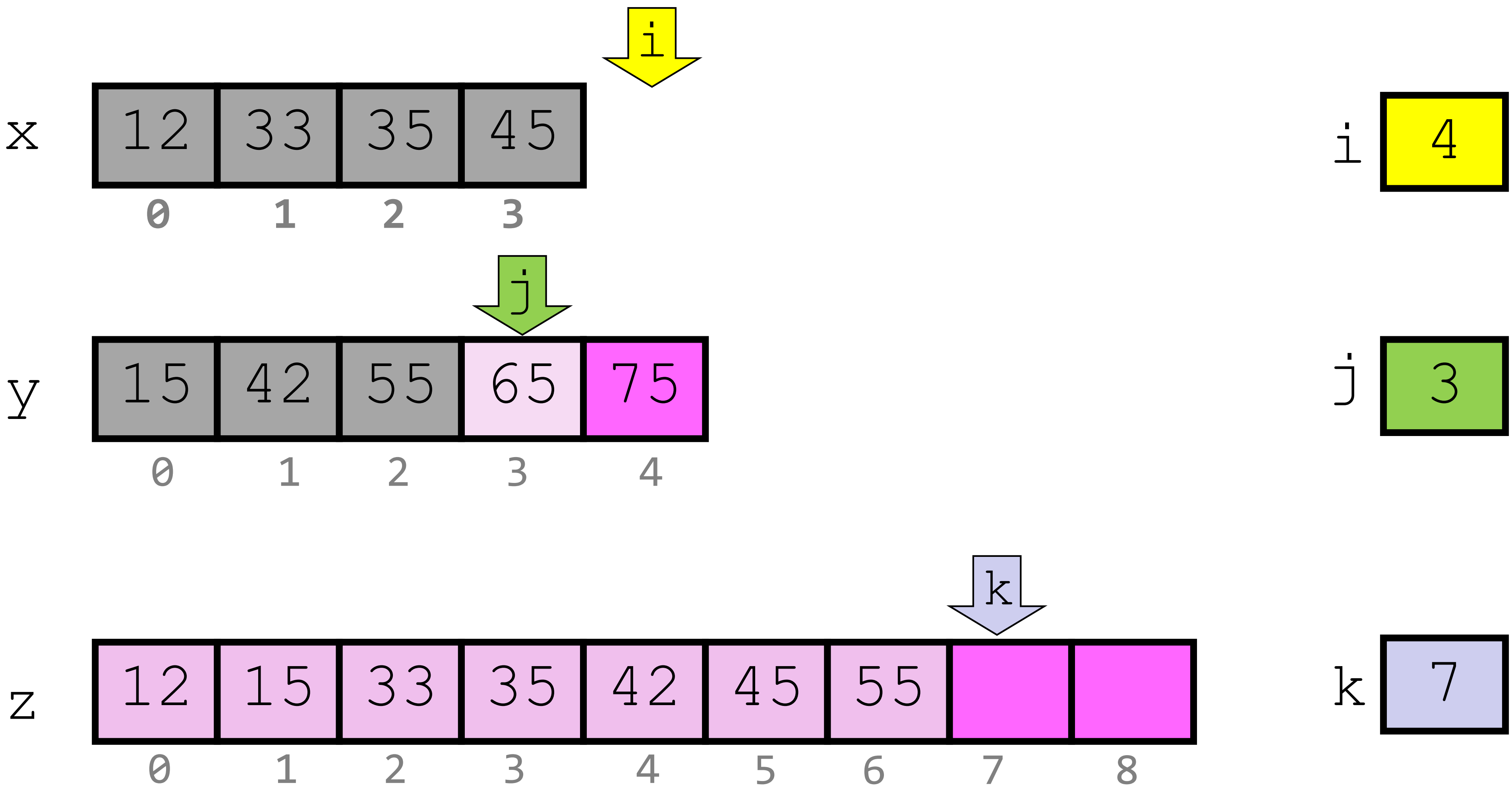

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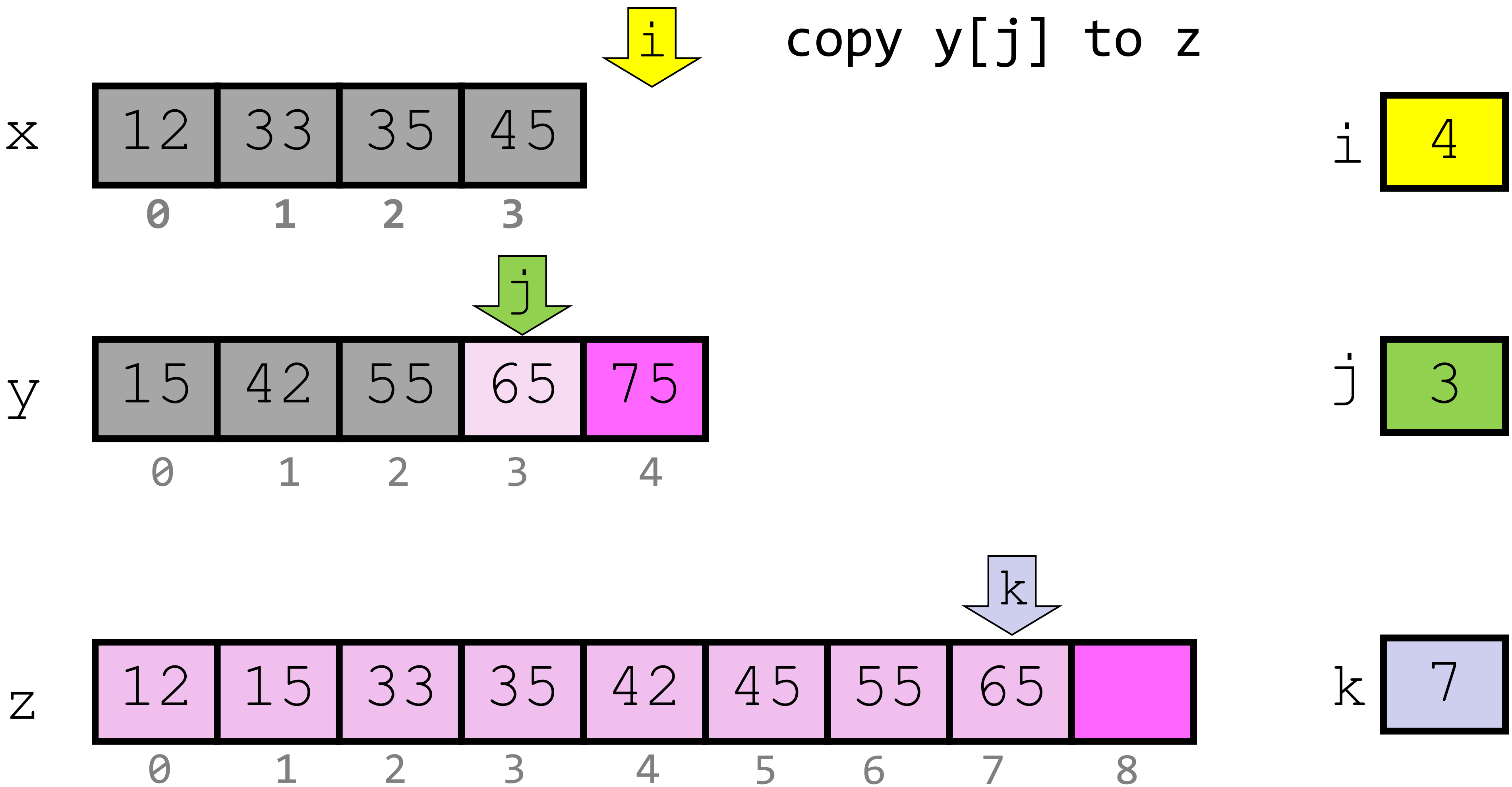

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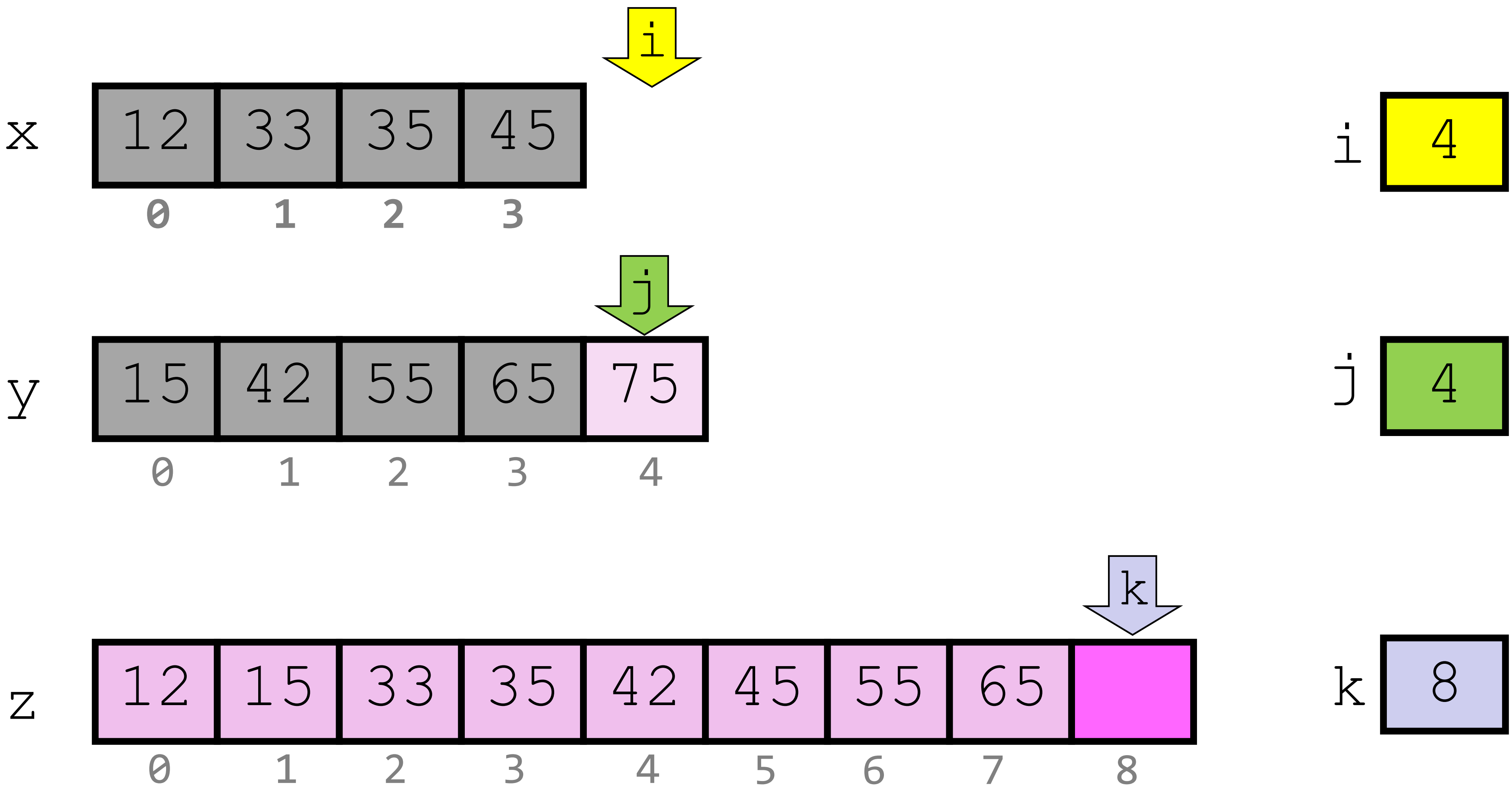

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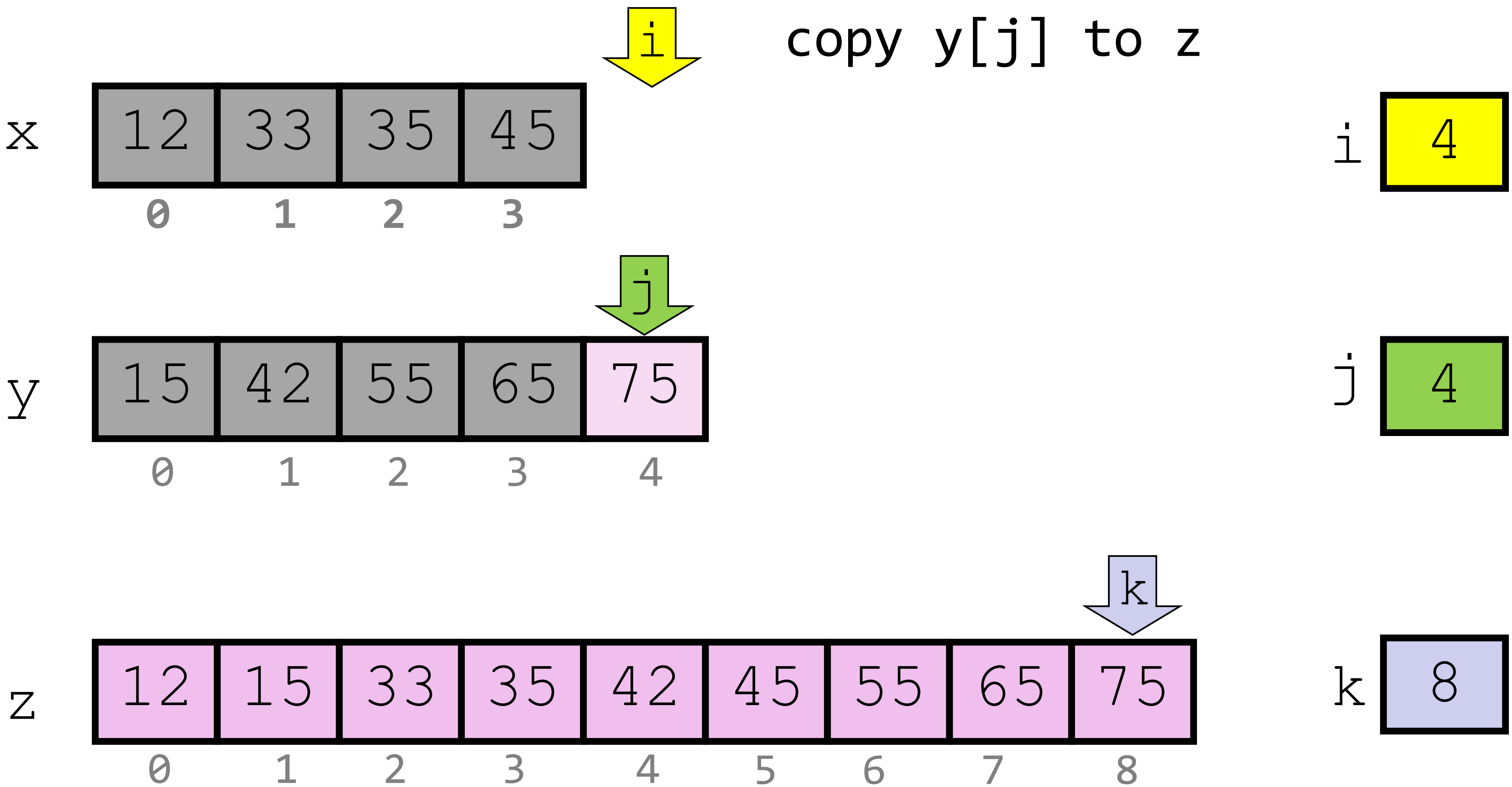

How to Merge

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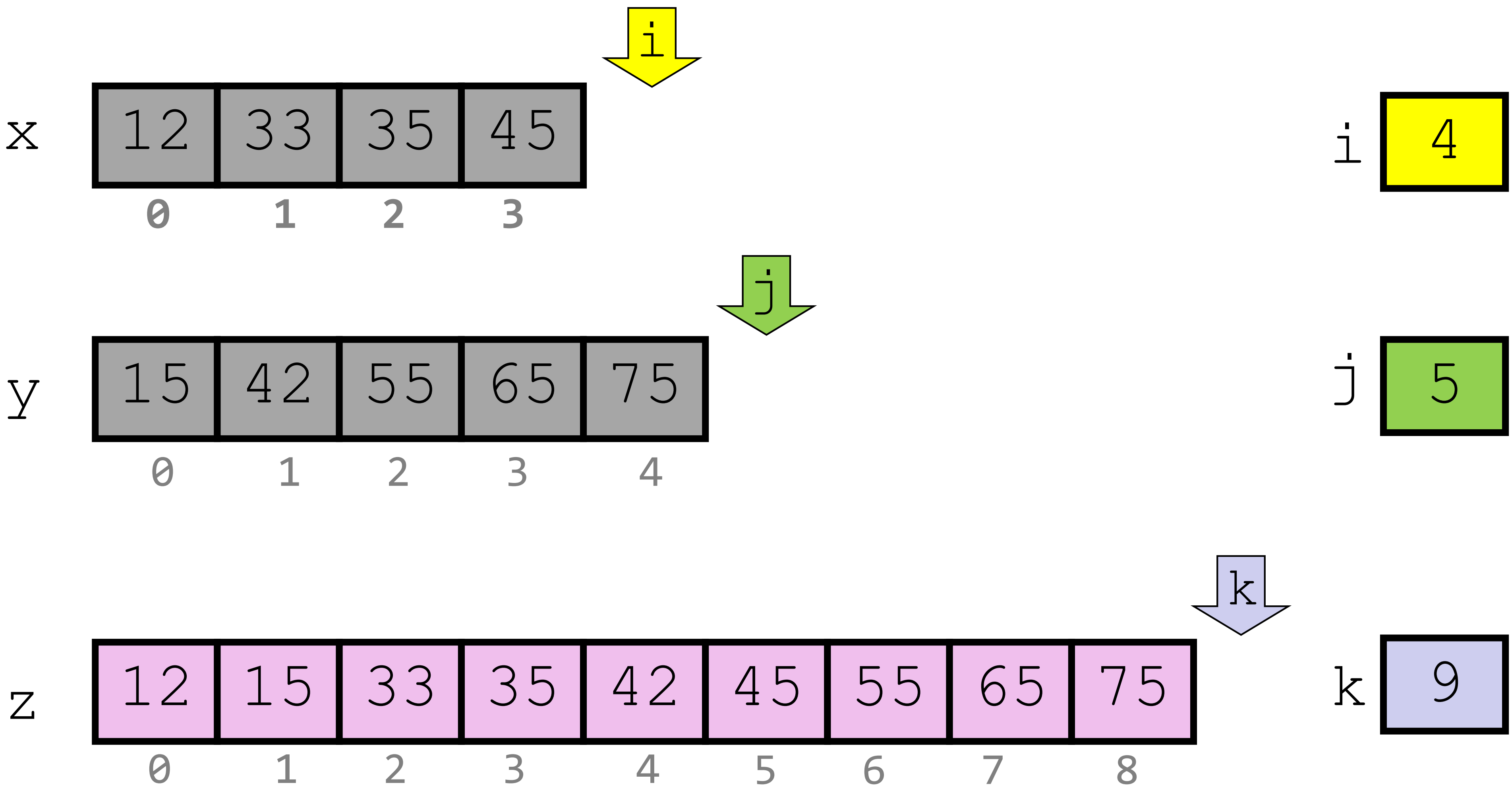

```
def merge(x, y, z):
    # Given: sorted lists x and y
    # list z, has the combined length of x and y...
    nx = len(x); ny = len(y)

    i = 0; j = 0; k = 0;
    while i<nx and j<ny:




    # Deal with remaining values in x or y
```

```
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    i = 0; j = 0; k = 0;
    while i<nx and j<ny:
        if  x[i] <= y[j]:
            z[k]= x[i];  i=i+1
        else:
            z[k]= y[j];  j=j+1
        k=k+1
    # Deal with remaining values in x or y
```

```python
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    while i<nx and j<ny:
        if  x[i] <= y[j]:
            z[k]= x[i];  i=i+1
        else:
            z[k]= y[j];  j=j+1
        k=k+1
    # Deal with remaining values in x or y
    while i<nx:  # copy any remaining x-values
        z[k]= x[i];  i=i+1;  k=k+1

    while j<ny:  # copy any remaining y-values
        z[k]= y[j];  j=j+1;  k=k+1
```

```python
def mergeSort(li):
    """Sort list li using Merge Sort"""
    if len(li) > 1:
        # Divide into two parts
        mid= len(li)/2
        left= li[:mid]
        right= li[mid:]

        # Recursive calls
        mergeSort(left)
        mergeSort(right)

        # Merge left & right back to li
        merge(left, right, li)
```

Sorting Algorithms

- Sorting data is a common task
  - Insertion sort: on the order of n^2
    - input doubles? → work **quadruples**! (yikes)

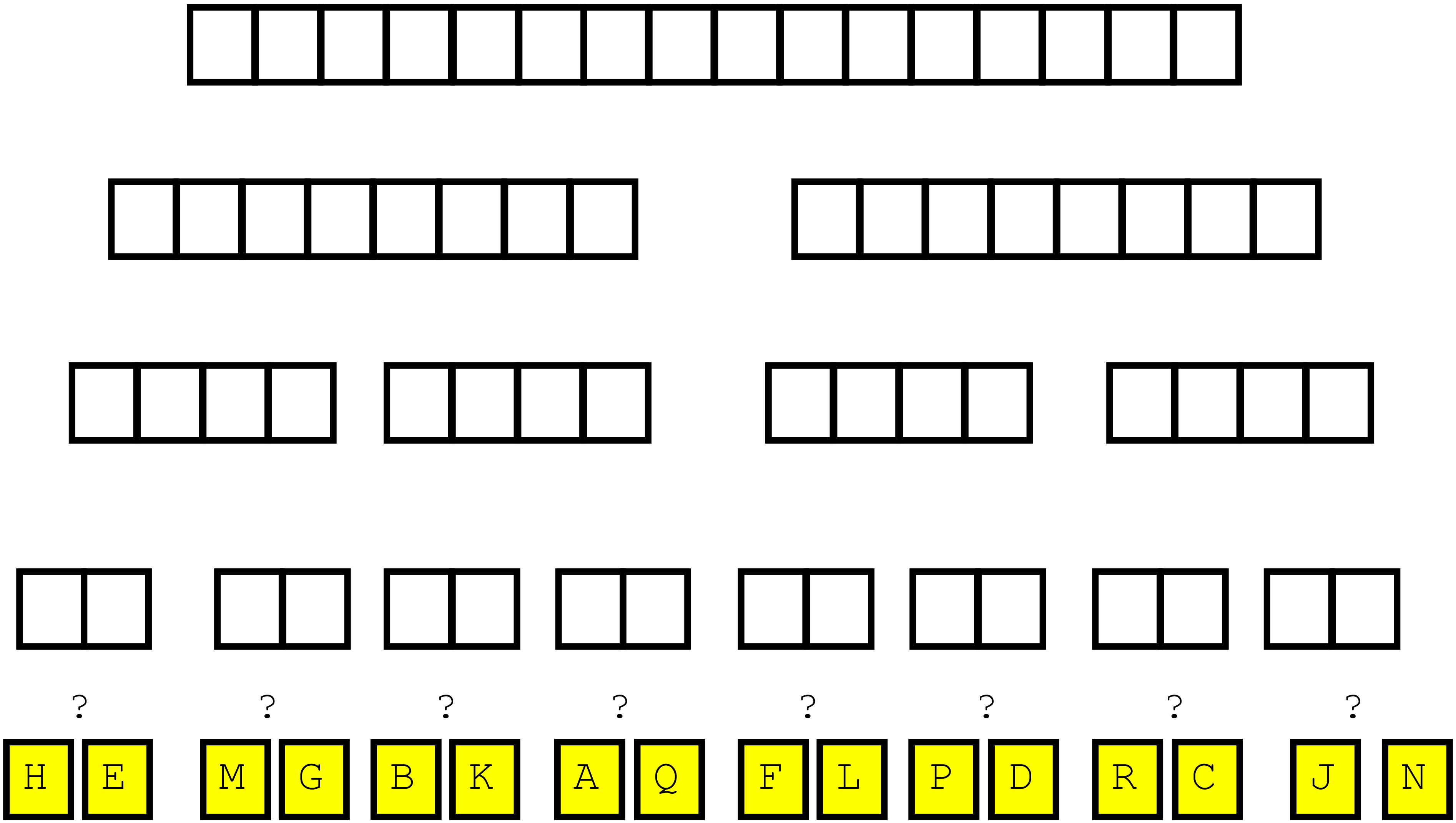
- Today's topic:
  - Merge sort: *did we do better than Insertion Sort?*

work = one comparison

How many comparisons do we make?

Merge sort:

~ $\log_2(n)$ "levels" X ~ n comparisons each level



Sorting Algorithms

- Sorting data is a common task
  - Insertion sort: on the order of n^2
    - input doubles? → work **quadruples**! (yikes)
  - Merge sort: on the order of $n \cdot \log_2(n)$

Order of magnitude difference

Should we always use merge sort then?

Python's **sort** *actually combines merge and insertion sort!*

For fun, check out the visualizations:
https://www.youtube.com/watch?v=xxcpvCGrCBc
https://www.youtube.com/watch?v=ZRPoEKHXTJg