24. Sorting a List

Topics:
- Selection Sort
- Merge Sort

Our examples will highlight the interplay between functions and lists

Sorting a List of Numbers

Before:

x --> | 50 | 40 | 10 | 80 | 20 | 60 |

After:

x --> | 10 | 20 | 40 | 50 | 60 | 80 |

We Will First Implement the Method of Selection Sort

At the Start:

x --> | 50 | 40 | 10 | 80 | 20 | 60 |

High-Level:

```
for k in range(len(x)-1)
    Swap x[k] with the smallest
    value in x[k:]
```

Selection Sort: How It Works

Before:

x --> | 50 | 40 | 10 | 80 | 20 | 60 |

Swap x[0] with the smallest value in x[0:]

Selection Sort: How It Works

Before:

x --> | 50 | 40 | 10 | 80 | 20 | 60 |

Swap x[0] with the smallest value in x[0:]

After:

x --> | 10 | 40 | 50 | 80 | 20 | 60 |

Selection Sort: How It Works

Before:

x -->

10	40	50	80	20	60

Swap x[1] with the smallest value in x[1:]

Selection Sort: How It Works

Before:

x -->

10	40	50	80	20	60

Swap x[1] with the smallest value in x[1:]

After:

x -->

10	20	50	80	40	60

Selection Sort: How It Works

Before:

x -->

10	20	50	80	40	60

Swap x[2] with the smallest value in x[2:]

Selection Sort: How It Works

Before:

x -->

10	20	50	80	40	60

Swap x[2] with the smallest value in x[2:]

After:

x -->

10	20	40	80	50	60

Selection Sort: How It Works

Before:

x -->

10	20	40	80	50	60

Swap x[3] with the smallest value in x[3:]

Selection Sort: How It Works

Before:

x -->

10	20	40	80	50	60

Swap x[3] with the smallest value in x[3:]

After:

x -->

10	20	40	50	80	60

Selection Sort: How It Works

Before:

x -->

10	20	40	50	80	60

`Swap x[4] with the smallest value in x[4:]`

Selection Sort: How It Works

Before:

x -->

10	20	40	50	80	60

`Swap x[4] with the smallest value in x[4:]`

After:

x -->

10	20	40	50	60	80

Selection Sort: Recap

50	40	10	80	20	60
10	40	50	80	20	60
10	20	50	80	40	60
10	20	40	80	50	60
10	20	40	50	80	60
10	20	40	50	60	80
10	20	40	50	60	80

The Essential Helper Function: Select(x,i)

```
def Select(x,i):
    """ Swaps the smallest value in
    x[i:] with x[i]

    PreC: x is a list of integers  and
    i is an in that satisfies
    0<=i<len(x)"""
```

Does not return anything and it has a list argument

How Does it Work?

The calling program has a list. E.g.,

```
a -->  0 --->  50
       1 --->  40
       2 --->  10
       3 --->  80
       4 --->  20
       5 --->  60
```

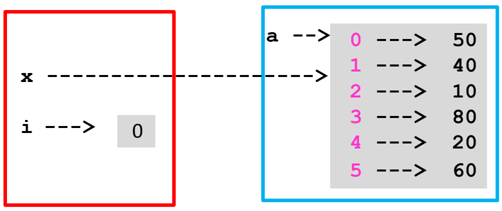
How Does it Work?

The calling program executes `Select(a,0)` and control passes to `Select`

```
a -->  0 --->  50
       1 --->  40
       2 --->  10
       3 --->  80
       4 --->  20
       5 --->  60
```

How Does Select Work?

- Nothing new about the assignment of 0 to i.
- But there is no assignment of the list a to x.
- Instead x now refers to the same list as a.



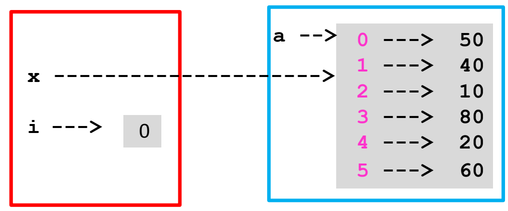
How Does Select Work?

If inside Select we have

```
t = x[0]; x[0] = x[2]; x[2] = t
```

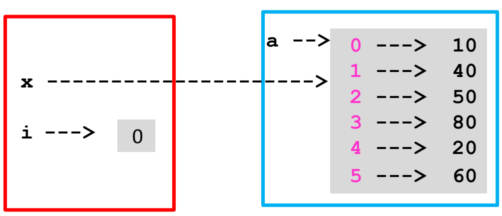
it is as if we said

```
t = a[0]; a[0] = a[2]; a[2] = t
```



How Does Select Work?

It changes the list a in the calling program.
We say x and a are aliased. They refer to the same list



Let's Assume This Is Implemented

```
def Select(x,i):
    """ Swaps the smallest value in
    x[i:] with x[i]

    PreC: x is a list of integers and
    i is an in that satisfies
    0<=i<len(x)"""
```

After this:	The list a looks like this					
Initialization	50	40	10	80	20	60
Select(a,0)	10	40	50	80	20	60
Select(a,1)	10	20	50	80	40	60
Select(a,2)	10	20	40	80	50	60
Select(a,3)	10	20	40	50	80	60
Select(a,4)	10	20	40	50	60	80
Select(a,5)	10	20	40	50	60	80

In General We Have This

```
def SelectionSort(a):
    n = len(a)
    for k in range(n):
        Select(a,k)
```

Next Problem

Merging Two Sorted Lists
into a
Single Sorted List

Example

x-> | 12 | 33 | 35 | 45 |

y-> | 15 | 42 | 55 | 65 | 75 |

x and **y** are input
They are sorted

z is the output

z-> | 12 | 15 | 33 | 35 | 42 | 45 | 55 | 65 | 75 |

Merging Two Sorted Lists

x-> | 12 | 33 | 35 | 45 |

y-> | 15 | 42 | 55 | 65 | 75 |

ix and iy keep track of where we are in x and y

ix: 0

iy: 0

z-> []

Merging Two Sorted Lists

x-> | 12 | 33 | 35 | 45 |

y-> | 15 | 42 | 55 | 65 | 75 |

ix: 0

iy: 0

z-> []

Do we pick from x? `x[ix] <= y[iy]` ???

Merge

x-> | 12 | 33 | 35 | 45 |

y-> | 15 | 42 | 55 | 65 | 75 |

ix: 0

iy: 0

z-> | 12 |

Yes. So update ix

Merge

x-> | 12 | 33 | 35 | 45 |

y-> | 15 | 42 | 55 | 65 | 75 |

ix: 1

iy: 0

z-> | 12 |

Do we pick from x? `x[ix] <= y[iy]` ???

Merge

x-> 12 | 33 | 35 | 45 ix: 1

y-> 15 | 42 | 55 | 65 | 75 iy: 0

z-> 12 | 15 iz:

No. So update iy

Merge

x-> 12 | 33 | 35 | 45 ix: 1

y-> 15 | 42 | 55 | 65 | 75 iy: 1

z-> 12 | 15

Do we pick from x? `x[ix] <= y[iy]` ???

Merge

x-> 12 | 33 | 35 | 45 ix: 1

y-> 15 | 42 | 55 | 65 | 75 iy: 1

z-> 12 | 15 | 33

Yes. So update ix

Merge

x-> 12 | 33 | 35 | 45 ix: 2

y-> 15 | 42 | 55 | 65 | 75 iy: 1

z-> 12 | 15 | 33

Do we pick from x? `x[ix] <= y[iy]` ???

Merge

x-> 12 | 33 | 35 | 45 ix: 2

y-> 15 | 42 | 55 | 65 | 75 iy: 1

z-> 12 | 15 | 33 | 35

Yes. So update ix

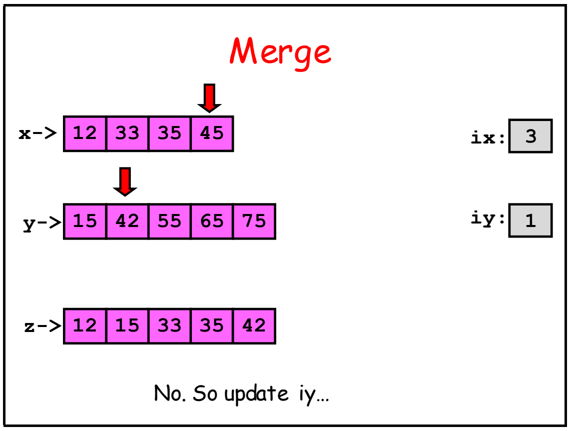
Merge

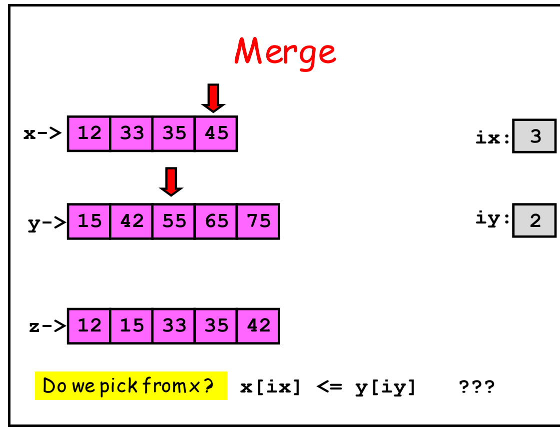
x-> 12 | 33 | 35 | 45 ix: 3

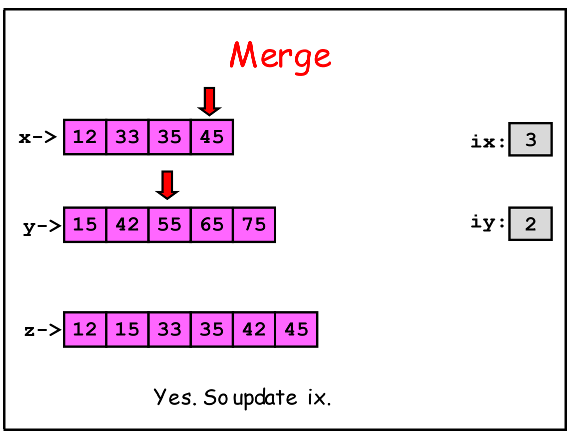
y-> 15 | 42 | 55 | 65 | 75 iy: 1

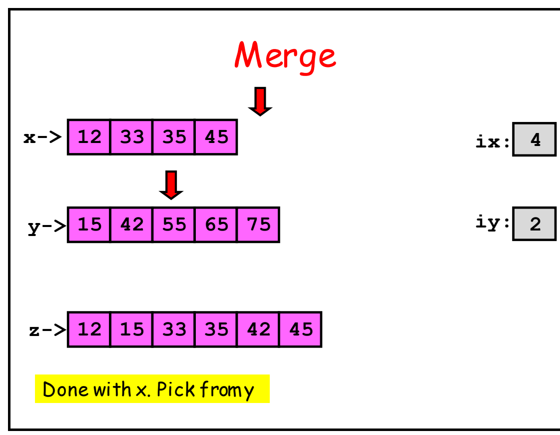
z-> 12 | 15 | 33 | 35

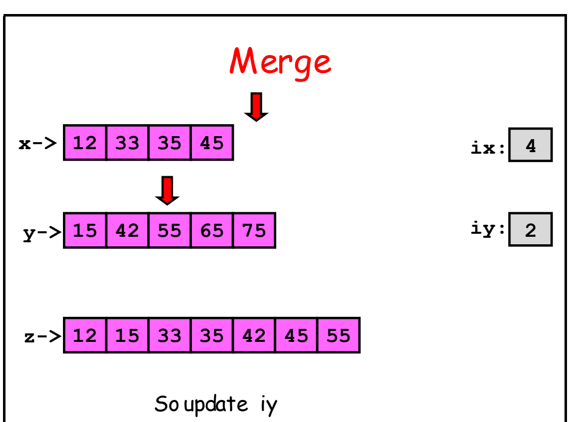
Do we pick from x? `x[ix] <= y[iy]` ???

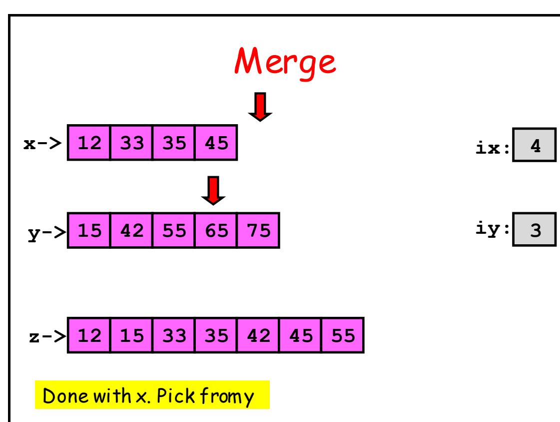

Merge
x-> 12 33 35 45
ix: 3
y-> 15 42 55 65 75
iy: 1
z-> 12 15 33 35 42
No. So update iy...

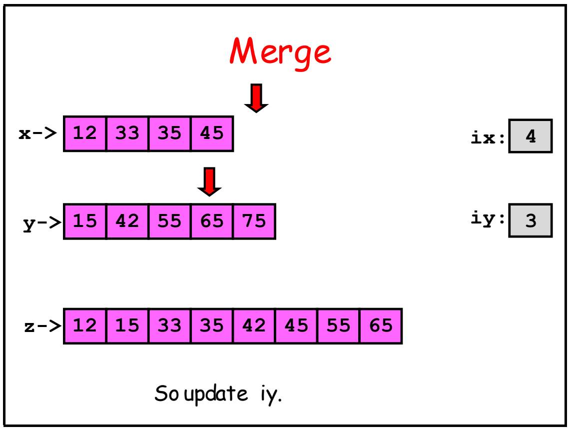

Merge
x-> 12 33 35 45
ix: 3
y-> 15 42 55 65 75
iy: 2
z-> 12 15 33 35 42
Do we pick from x?
x[ix] <= y[iy] ???

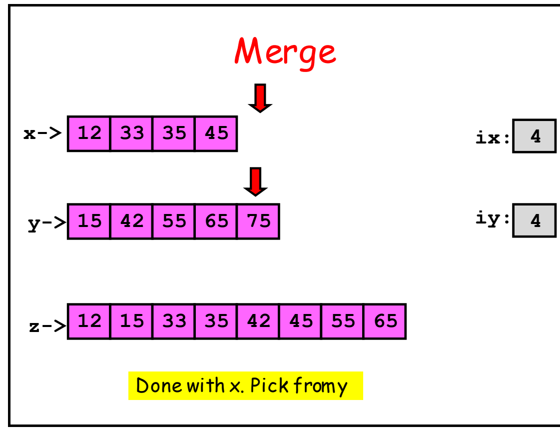

Merge
x-> 12 33 35 45
ix: 3
y-> 15 42 55 65 75
iy: 2
z-> 12 15 33 35 42 45
Yes. So update ix.

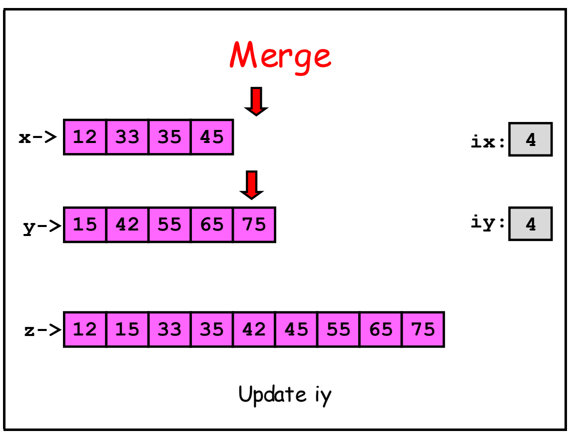

Merge
x-> 12 33 35 45
ix: 4
y-> 15 42 55 65 75
iy: 2
z-> 12 15 33 35 42 45
Done with x. Pick from y

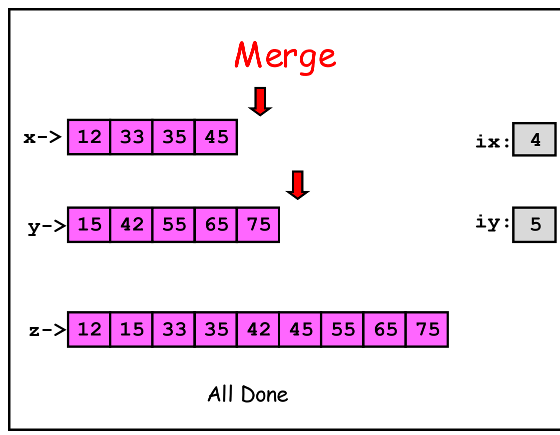

Merge
x-> 12 33 35 45
ix: 4
y-> 15 42 55 65 75
iy: 2
z-> 12 15 33 35 42 45 55
So update iy


Merge
x-> 12 33 35 45
ix: 4
y-> 15 42 55 65 75
iy: 3
z-> 12 15 33 35 42 45 55
Done with x. Pick from y









The Python Implementation…

```
def Merge(x,y):
    n = len(x); m = len(y);
    ix = 0; iy = 0; z = []
    for iz in range(n+m):
        if   ix>=n:
            z.append(y[iy]);  iy+=1
        elif iy>=m:
            z.append(x[ix]);  ix+=1
        elif x[ix] <= y[iy]:
            z.append(x[ix]);  ix+=1
        elif x[ix] > y[iy]:
            z.append(y[iy]);  iy+=1
    return z
```

Build z up via repeated appending

x-list exhausted y-list exhausted x-value smaller y-value smaller

```
def Merge(x,y):
    n = len(x); m = len(y);
    ix = 0; iy = 0; z = []
    for iz in range(n+m):
        if   ix>=n:
             z.append(y[iy]);  iy+=1
        elif iy>=m:
             z.append(x[ix]);  ix+=1
        elif x[ix] <= y[iy]:
             z.append(x[ix]);  ix+=1
        elif x[ix] > y[iy]:
             z.append(y[iy]);  iy+=1
    return z
```

len(x)+len(y) is the total length of the merged list

x-list exhausted y-list exhausted x-value smaller y-value smaller

Implementation Using Pop

```
def Merge(x,y):
    u = list(x)
    v = list(y)
    z = []
    while len(u)>0 and len(v)>0 :
        if u[0]<= v[0]:
            g = u.pop(0)
        else:
            g = v.pop(0)
        z.append(g)
    z.extend(u)
    z.extend(v)
    return z
```

Make copies of the Incoming lists

Implementation Using Pop

```
def Merge(x,y):
    u = list(x)
    v = list(y)
    z = []
    while len(u)>0 and len(v)>0 :
        if u[0]<= v[0]:
            g = u.pop(0)
        else:
            g = v.pop(0)
        z.append(g)
    z.extend(u)
    z.extend(v)
    return z
```

Build z up via repeated appending

Implementation Using Pop

```
def Merge(x,y):
    u = list(x)
    v = list(y)
    z = []
    while len(u)>0 and len(v)>0 :
        if u[0]<= v[0]:
            g = u.pop(0)
        else:
            g = v.pop(0)
        z.append(g)
    z.extend(u)
    z.extend(v)
    return z
```

Every “pop” reduces the length by 1. The loop shuts down when one of u or v is exhausted

Implementation Using Pop

```
def Merge(x,y):
    u = list(x)
    v = list(y)
    z = []
    while len(u)>0 and len(v)>0 :
        if u[0]<= v[0]:
            g = u.pop(0)
        else:
            g = v.pop(0)
        z.append(g)
    z.extend(u)
    z.extend(v)
    return z
```

g gets the popped value and it is appended to z

Implementation Using Pop

```
def Merge(x,y):
    u = list(x)
    v = list(y)
    z = []
    while len(u)>0 and len(v)>0 :
        if u[0]<= v[0]:
            g = u.pop(0)
        else:
            g = v.pop(0)
        z.append(g)
    z.extend(u)
    z.extend(v)
    return z
```

Add what is left in u. OK if u is the empty list

Implementation Using Pop

```
def Merge(x,y):
    u = list(x)
    v = list(y)
    z = []
    while len(u)>0 and len(v)>0 :
        if u[0]<= v[0]:
            g = u.pop(0)
        else:
            g = v.pop(0)
        z.append(g)
    z.extend(u)
    z.extend(v)
    return z
```

Add what is left in v.
OK if v is the empty list

MergeSort

Binary Search is an example of a "divide and conquer" approach top probkem solving.

A method for sorting a list that features this strategy is MergeSort

Motivation

You are asked to sort a list but you have two "helpers": H1 and H2.

Idea:

1. Split the list in half and have each helper sort one of the halves.

2. Then merge the two sorted lists into a single larger list.

This idea can be repeated if H1 has two helpers and H2 has two helpers.

Subdivide the Sorting Task

H E M G B K A Q F L P D R C J N

H E M G B K A Q   F L P D R C J N

Subdivide Again

H E M G B K A Q   F L P D R C J N

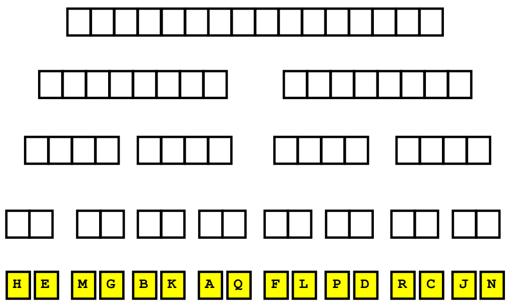
H E M G   B K A Q   F L P D   R C J N

And Again

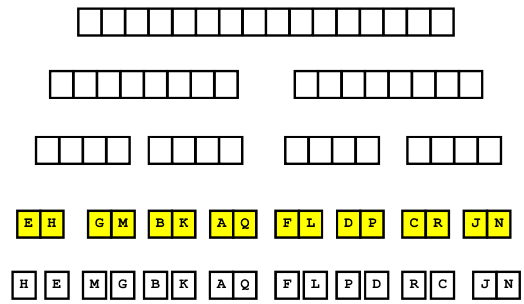
H E M G   B K A Q   F L P D   R C J N

H E   M G   B K   A Q   F L   P D   R C   J N

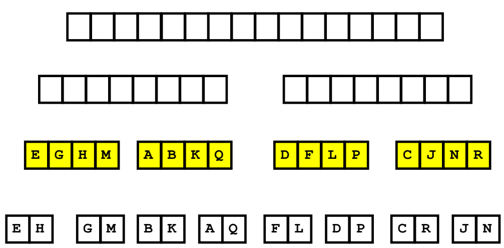
And One Last Time



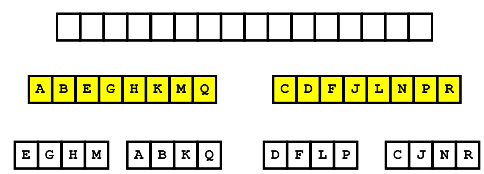
Now Merge



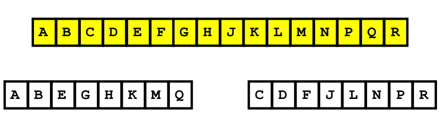
And Merge Again



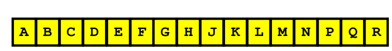
And Again



And One Last Time



Done!



Done!

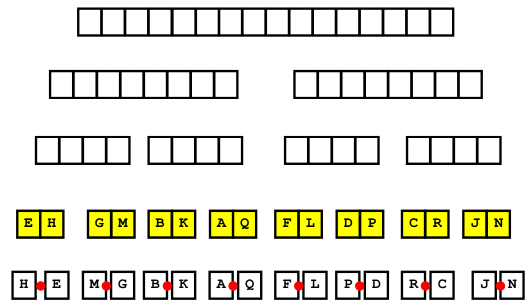
A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q	R

Let's write a function to do this making use of

```
def Merge(x,y):
     """ Returns a float list that is the
     merge of sorted lists x and y.

     PreC: x and y are lists of floats
     that are sorted from small to big.
     """
```

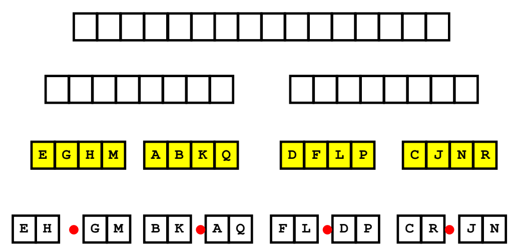
8 Merges Producing length-2 lists



Handcoding the n =16 case

```
A0 = Merge(a[0],a[1])
A1 = Merge(a[2],a[3])
A2 = Merge(a[4],a[5])
A3 = Merge(a[6],a[7])
A4 = Merge(a[8],a[9])
A5 = Merge(a[10],a[11])
A6 = Merge(a[12],a[13])
A7 = Merge(a[14],a[15])
```

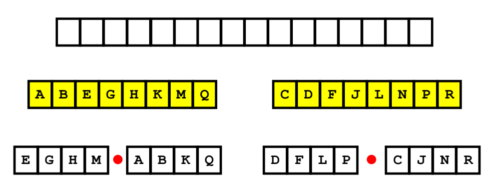
4 Merges Producing Length-4 lists



Handcoding the n =16 case

```
B0 = Merge(A0,A1)
B1 = Merge(A2,A3)
B2 = Merge(A4,A5)
B3 = Merge(A6,A7)
```

2 Merges Producing Length-8 Lists



Handcoding the n =16 case

```
C0 = Merge(B0,B1)
C1 = Merge(B2,B3)
```

1 Merge Producing a Length-16 List

A	B	C	D	E	F	G	H	J	K	L	M	N	P	Q	R

A	B	E	G	H	K	M	Q

•

C	D	F	J	L	N	P	R

All Done!

```
D0 = Merge(C0,C1)
```

For general n, it can be handled using recursion.

Recursive Merge Sort

```
def MergeSort(a):
     n = length(a)
     if n==1:
         return a
     else:
         m  = n/2
         u0 = list(a[:m])
         u1 = list(a[m:])
         y0 = MergeSort(u0)
         y1 = MergeSort(u1)
         return Merge(y0,y1)
```

A function can call Itself!

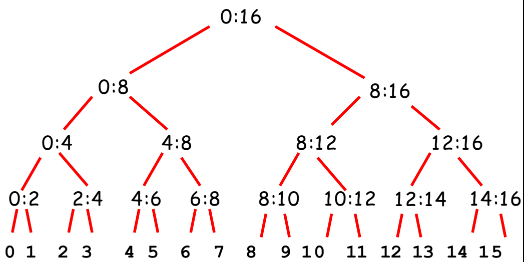
Back To Merge Sort

Recursive Merge Sort

```
def MergeSort(a):
     n = length(a)
     if n==1:
         return a
     else:
         m  = n/2
         u0 = list(a[:m])
         u1 = list(a[m:])
         y0 = MergeSort(u0)
         y1 = MergeSort(u1)
         return Merge(y0,y1)
```

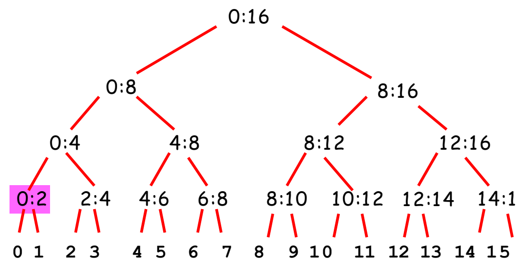
A function can call Itself!

A Schematic



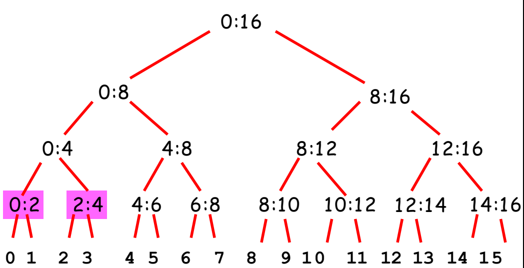
A Sorted List is produced at each ":" Let's look at the order in which

A Schematic



A Sorted List is produced at each ":" Let's look at the order in which lists are sorted.

A Schematic



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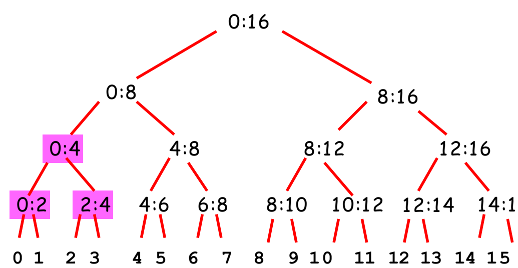

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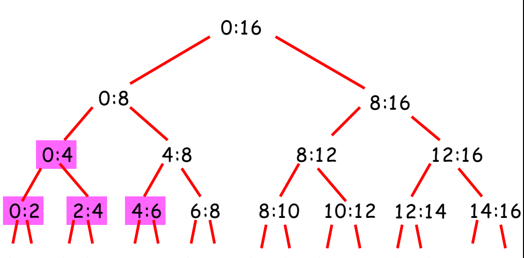

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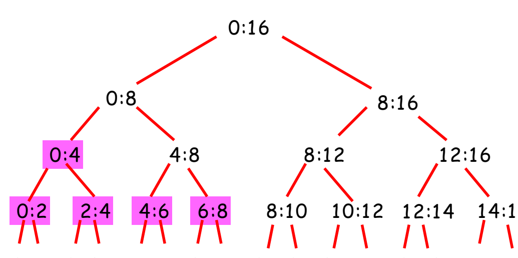

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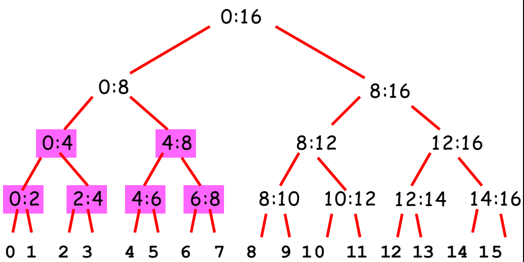
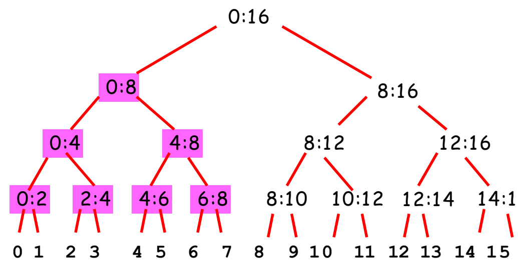


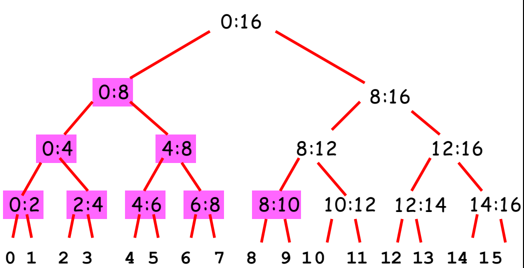
A Sorted List is produced at each ":" Let's look at the order in which

A Schematic



A Sorted List is produced at each ":" Let's look at the order in which lists are sorted.

A Schematic



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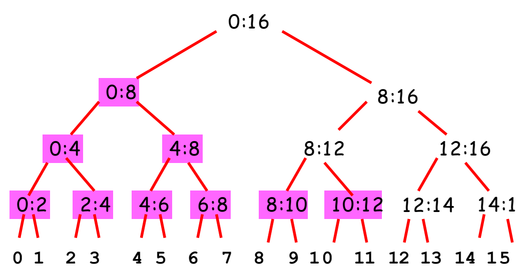

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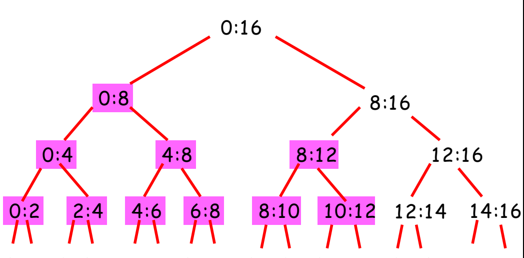

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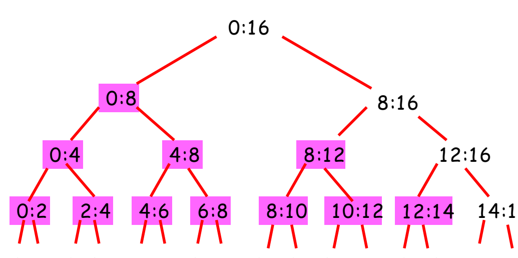

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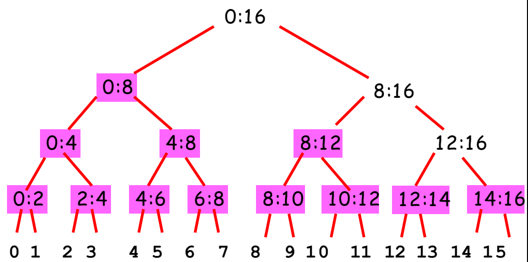

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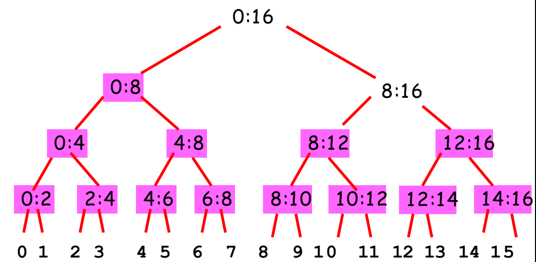

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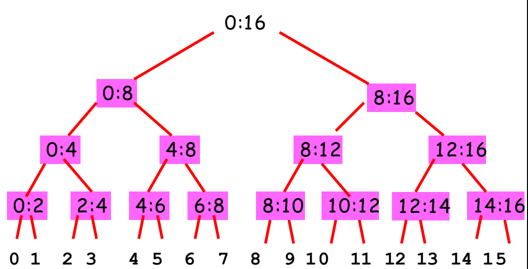

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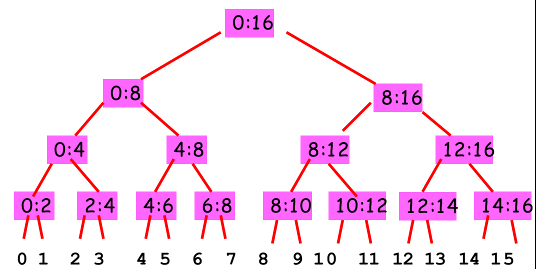

All Done!

Some Conclusions

Infinite recursion (like infinite loops) can happen so careful reasoning is required.

Will we reach the "base case"?

In **MergeSort**, a recursive call always involves a list that is shorter than the input list. So eventually we reach the len(a)==1 base case.