

22. Searching a List

Topics:

Linear Search

Binary Search

Measuring Execution Time

The Divide and Conquer Framework

Search

Examples:

Is this song in that playlist?

Is this number in that phone book?

Is this name in that phone book?

Is this fingerprint in that archive of fingerprints?

Is this photo in that yearbook?

More on Using Phone Books

The Manhattan phone book has 1,000,000+ entries.

How is it possible to locate a name by examining just a tiny , tiny fraction of those entries?

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195 Car C

17 566-1282	26 Allen La Esplanade 01938.....	978 356-9960	Carter F 24 Hillcock Ros 02131.....	617 327-1105	Carter Nella E	323 Massachusetts Av Bos 02115.....	617 267-6483	
81 447-4101	18 Jewett Ros 02131.....	617 323-7639	Faye & Ricky	357 Columbus Av Bos 02116.....	617 437-7331	Nicholas S F	115 Randolph Av Mil 02186.....	617 698-5307
00 257-9981	Cartagena Lydia	617 323-7639	Francis S 134 Temple W Rox 02132.....	617 323-6781	Franklin & Anne	Nick & Debbie	106 Herrick Rd Newton 02459.....	617 527-0480
17 566-1282	Cartagena Avith	617 442-9780	Franklin & Anne	221 Mt Auburn Cam 02138.....	617 354-0798	Nicole.....	617 698-0713	
17 364-5188	B Hyd 02136.....	617 361-5253	Fred	42 Haverford Jam 02130.....	617 524-3078	Norman G	38 Chickatawut Dor 02122.....	617 822-1203
361-0380	Jessica 40 Decatur Cha 02139.....	617 241-0152	Fred	96 Hinckley Rd Mil 02186.....	617 698-1343	P A	44 Crestwood Pk Rox 02121.....	617 427-4754
17 566-4548	Melvin 501 Green Cam 02139.....	617 576-1061	G & R	8 Verdun Dor 02124.....	617 436-8906	P E	501 E Sixth St Bos 02127.....	617 268-4213
17 628-8248	Carte Nicholas	617 695-6996	G T	27 Franklin Av Som 02145.....	617 623-7121	P L	44 Hutchings Rox 02121.....	617 427-9170
17 445-5116	Cartegena O 4 Milford Bos 02118.....	617 338-8219	Gayle	25 Frontenac Dor 02124.....	617 825-0322	P R	91 Byrner Jam 02130.....	617 983-8692
17 822-2982	Carten Thos J Sr & Claire	617 698-6163	Geo S	115 Moss Hill Rd Jam 02130.....	617 522-3215	Paul & Constance	134 Anawan Av W Rox 02132.....	617 325-2036
17 427-5712	Thomas & Kathleen	617 696-6919	George	125 Nashua Bos 02114.....	617 367-9548	Paul E	501 E Sixth St Bos 02127.....	617 268-4546
17 569-2698	Carte A Ros 02131.....	617 327-2257	Carte Holliday Associate	107 S Street Bos 02111.....	617 456-1689	Paul M	27 Union Bri 02135.....	617 787-2115
17 667-5190	A Rosbury.....	617 442-5230	Carte Harry F	26 Runging Rk Rd W Rox 02132.....	617 325-5465	Carter Pile Driving Inc	17 Beaver Ct Framingham 01702.....	781 235-8488
17 569-1417	A 31 Bethune Wv Roxbury 02119.....	617 442-1219	Carte Hide Co Inc	146 Summer Bos 02110.....	617 542-7987	Carter Prudence	46 Franklin Watertown 02172.....	617 393-3782
17 338-9110	A M 255 Massachusetts Av Bos 02115.....	617 266-7153	Carte Hilary	61 Harvey Cam 02140.....	617 876-2750	Prudence	46 Franklin Watertown 02172.....	617 926-7063
17 825-9195	Adams 361 Centre St Mil 02186.....	617 698-9074	Horace	241 Walnut Av Rosbury 02119.....	617 442-5307	Reginald	106 Brunswick Dorchester 02121.....	617 541-2843
17 296-1593	Alice 108 Kilmarnock Bos 02215.....	617 425-0193	Howard Jr	26 Notre Dame Rox 02119.....	617 445-5552	Renee & Andrew	10 Walnut Bos 02108.....	617 720-3765
17 670-2078	Alice 45 Market Cambridge 02139.....	617 945-2711	J	15 Chatham Bro 02446.....	617 232-7990	Carter Rice Dowd	Bulky Danton Publishing 163 Main Wilmington 01887	800 638-1671
17 623-9001	Andrew F 62 Vinyl Av Som 02143.....	617 625-7623	J	518 Harvard Bro 02446.....	617 730-9483	Call Free-Dial '1' & Then.....	800 619-7447	
17 296-4725	Carter Anne MD	617 739-1022	J	75 Viny Flwy West Roxbury 02132.....	617 323-5574	Call Free-Dial '1' & Then.....	800 648-7447	
17 542-1521	Carter Athens	617 536-6329	Carte J Jacques MD	1 Brookline Pl Bro 02446.....	617 735-8787	Headquarters	613 Main Wilmington 01887	978 988-7447
17 364-5232	B E 68 Gloucester Av Mat 02126.....	617 296-6911	Carte J M	1410 Columbia Rd S Bos 02127.....	617 464-1040	Call.....	800 638-1673	
17 541-5649	Carter Barbara L MD	617 523-4368	Carte J H Ornamental Ironworks	Call.....	617 436-5353	Call.....	800 638-1673	
17 739-2662	Tuffs-New England Medical Center Bos 02111	617 636-0051	Carte J Veal Co	48 Newmarket Sq Rox 02118.....	617 442-1775	Call.....	800 638-1673	
17 879-0030	Carter Becky Bos 02114.....	617 523-4368	Carte James	1573 Cambridge St Cam 02138.....	617 492-1214	Call.....	800 638-1673	
17 541-3948	Bernard J	617 567-3430	James	182 Fisher Av Rosbury 02120.....	617 739-2193	Carter Richard	1079 Commwth Av Brighton 02215.....	617 987-0836
17 436-1513	Bithiah 25 Medway Dor 02124.....	617 298-8713	James L	37 Gold Star Rd Cambridge 02140.....	617 876-8841	Richard A	97 Mt Vernon Bos 02108.....	617 566-7293
17 569-4119	Blake 26 Mt Vernon Bos 02108.....	617 367-9931	Jane	14 Roseberry Rd Mat 02126.....	617 361-0773	Carte Richard A MD	170 Commwth Av Bos 02116.....	617 267-0710
300 569-8782	Carter Broadcasting Co	617 423-0210	Jeffrey	41 Warren Av Bos 02116.....	617 426-5994	Carter Richard K	15 Mercer S Bos 02127.....	617 268-0448
	Carter & Burgess Consultants Inc	617 225-0200	John	11 Mansfield Bri 02134.....	617 987-2163	Robert L	175 Richdale Av Cam 02140.....	617 864-1535
	C 2000 Commwth Av Bri 02135.....	617 782-2118	John	327 Summer Bos 02210.....	617 423-4334	Roger	150 St Botolph Bos 02115.....	617 424-6148
	C 228 Fywood Av East Boston 02128.....	617 569-1545	John	40 Westwind Rd Dor 02125.....	617 282-1235	Royce	44 Concord Av Cam 02138.....	617 491-6115
	C 359 Harvard Cam 02138.....	617 491-8822	June O	329 A Summit Av Bri 02136.....	617 734-6109			
	C 610 Walk Hill Mat 02126.....	617 296-6392	K	38 Browning Av Dorchester 02124.....	617 265-8456			
	Chon 02128	617 524-9558	K	17 Esmond Dorchester 02121.....	617 282-1593			

There must be a great search algorithm behind the scenes.

Linear Search

LinSearch: The Spec

```
def LinSearch(x, a) :  
    """ Returns an int k with the  
    property that a[k]==x is True.  
    If no such k exists, then  
    k==-1.  
  
    PreC: a is a nonempty list of  
    ints and x is an int.  
    """
```

Could also apply the same ideas for searching a list of strings.

Linear Search

0 1 2 3 4 5 6 7 8 9 10 11

a ->

86	73	43	35	23	45	42	62	15	25	51	35
----	----	----	----	----	----	----	----	----	----	----	----

k ->

0

x ->

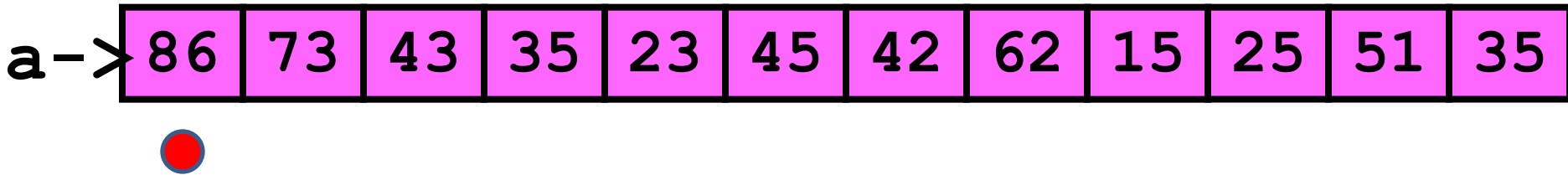
23

```
def LinSearch(x, a):  
    for k in range(len(a)):  
        if x == a[k]:  
            return k  
    return -1
```

Walk down the list looking for a match

Linear Search

0 1 2 3 4 5 6 7 8 9 10 11



k- >

0

x- >

23

```
def LinSearch(x, a):  
    for k in range(len(a)):  
        if x == a[k]:  
            return k  
    return -1
```

Walk down the list looking for a match

Linear Search

0 1 2 3 4 5 6 7 8 9 10 11

a ->

86	73	43	35	23	45	42	62	15	25	51	35
----	----	----	----	----	----	----	----	----	----	----	----



k ->

1

x ->

23

```
def LinSearch(x, a):  
    for k in range(len(a)):  
        if x == a[k]:  
            return k  
    return -1
```

Walk down the list looking for a match

Linear Search

0 1 2 3 4 5 6 7 8 9 10 11

a ->

86	73	43	35	23	45	42	62	15	25	51	35
----	----	----	----	----	----	----	----	----	----	----	----



k ->

2

x ->

23

```
def LinSearch(x, a):  
    for k in range(len(a)):  
        if x == a[k]:  
            return k  
    return -1
```

Walk down the list looking for a match

Linear Search

0 1 2 3 4 5 6 7 8 9 10 11

a ->

86	73	43	35	23	45	42	62	15	25	51	35
----	----	----	----	----	----	----	----	----	----	----	----



k ->

3

x ->

23

```
def LinSearch(x, a):  
    for k in range(len(a)):  
        if x == a[k]:  
            return k  
    return -1
```

Walk down the list looking for a match

Linear Search

0 1 2 3 4 5 6 7 8 9 10 11

a ->

86	73	43	35	23	45	42	62	15	25	51	35
----	----	----	----	----	----	----	----	----	----	----	----



k ->

4

x ->

23

```
def LinSearch(x, a):  
    for k in range(len(a)):  
        if x == a[k]: Yup  
            return k  
    return -1
```

Walk down the list looking for a match

Linear Search

0 1 2 3 4 5 6 7 8 9 10 11

a ->

86	73	43	35	23	45	42	62	15	25	51	35
----	----	----	----	----	----	----	----	----	----	----	----



k ->

4

x ->

23

```
def LinSearch(x, a):  
    for k in range(len(a)):  
        if x == a[k]:  
            return k All done  
    return -1
```

Walk down the list looking for a match

Linear Search: No Match Case

0 1 2 3 4 5 6 7 8 9 10 11

a ->

86	73	43	35	23	45	42	62	15	25	51	35
----	----	----	----	----	----	----	----	----	----	----	----



k ->

11

x ->

7

```
def LinSearch(x, a):  
    for k in range(len(a)):  
        if x == a[k]: Nope  
            return k  
    return -1
```

Walk down the list looking for a match

Linear Search: No Match Case

0 1 2 3 4 5 6 7 8 9 10 11

a →

86	73	43	35	23	45	42	62	15	25	51	35
----	----	----	----	----	----	----	----	----	----	----	----



x →

7

```
def LinSearch(x, a):  
    for k in range(len(a)):  
        if x == a[k]:  
            return k  
    return -1
```

Yup

Return -1 if no match

Linear Search: While Implementation

```
def LinSearchW(x, a):  
    k=0  
    while k<len(a) and a[k]!=x:  
        k+=1  
    if k==len(a):  
        return -1  
    else:  
        return k
```

Binary Search

Now we assume that the list
to be searched is sorted
from little to big.

```
a = [10, 20, 40, 60, 90]
```

```
a = ['brown', 'dog', 'fox', 'lazy', 'quick', 'the']
```


Back to Using Phone Books

The Ithaca phone book has 10,000+ entries.

The Manhattan phone book has 1,000,000+ entries. But it does not take 100 x longer to look something up. Why?

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195 Car C

17 566-1282	Cartage New England Inc 26 Allen Ln Norwich 01938.....	978 356-9960	Cartar F 24 Hillcock Ros 02131.....	617 327-1105	Carter Nella E 323 Massachusetts Av Bos 02115.....	617 267-6483
81 447-4101	Cartagema Lydia 18 Jewett Ros 02131.....	617 323-7639	Faye & Ricky 357 Columbus Av Bos 02116.....	617 437-7331	Nicholas S F 115 Randolph Av Mil 02186.....	617 698-5307
00 257-9981	Cartagema Avith 9 Bancroft Row 02119.....	617 442-9780	Francis S 134 Temple W Ros 02132.....	617 323-6781	Franklin & Anne 271 Mt Auburn Cam 02138.....	617 354-0798
17 566-1282	B Hyd 02136.....	617 361-5253	Fred 42 Haverford Jam 02138.....	617 524-3078	Nick & Debbie 106 Herrick Rd Newton 02459.....	617 527-0480
17 364-5188	Lucilla 174 Harvard Cam 02139.....	617 491-5621	Fred 96 Hinckley Rd Mil 02186.....	617 698-1343	Nicole.....	617 698-0713
361-0380	M 95 Rowe Ros 02131.....	617 323-9713	G & R 8 Verdun Dor 02124.....	617 436-8906	Norman G 38 Chickatawout Dor 02122.....	617 822-1203
17 566-4548	Melvin 501 Green Cam 02139.....	617 576-1061	G T 27 Franklin Av Som 02145.....	617 623-7121	P 44 Crestwood Pk Row 02121.....	617 427-4754
17 628-8248	Carte Nicholas 18 Appleton Boston 02116.....	617 695-6996	Gayle 25 Frontenac Dor 02124.....	617 825-0322	P E 501 E Sixth St Bos 02127.....	617 268-4213
17 445-5116	Cartena O 4 Milford Bos 02118.....	617 338-8219	Geo S 115 Moss Hill Rd Jam 02130.....	617 522-3215	P L 44 Hutchings Row 02121.....	617 427-9170
17 822-2982	Carten Thos J Sr & Claire 1 Paradise Rd Mil 02186.....	617 698-6163	George 125 Nashua Bos 02114.....	617 367-9548	P R 91 Byrner Jam 02130.....	617 983-8692
17 427-5712	Thomas & Kathleen 50 Thompson St Mil 02186.....	617 696-6919	Cartar Halliday Associate 107 S Street Bos 02111.....	617 456-1689	Paul & Constance 114 Anawan Av W Ros 02132.....	617 325-2036
17 569-2698	Cartar A Ros 02131.....	617 327-2257	Cartar Harry F 26 Ruess Rte Rd W Ros 02132.....	617 325-5465	Paul E 501 E Sixth St Bos 02127.....	617 268-4546
17 667-5190	A Rosbury A 31 Bethune Wy Rosbury 02119.....	617 442-5230	Cartar Hide Co Inc 146 Summer Bos 02110.....	617 542-7987	Paul M 27 Union Bri 02135.....	617 787-2115
17 569-1417	A M 255 Maschick Av Bos 02115.....	617 492-4174	Cartar Hilary 61 Harvey Cam 02140.....	617 876-2750	Carter Pile Driving Inc 17 Beaver Ct Framingham 01702.....	617 235-8488
17 338-9110	Adams 361 Centre St Mil 02186.....	617 698-9074	Horace 241 Walnut Av Rosbury 02119.....	617 442-5307	Carter Prudence 46 Franklin Watertown 02127.....	617 393-3782
17 825-9195	Alice 108 Kilmarnock Bos 02215.....	617 425-0193	Howard Jr 26 Notre Dame Row 02119.....	617 445-5552	Prudence 46 Franklin Watertown 02127.....	617 926-7063
17 296-1593	Alice 45 Market Cambridge 02139.....	617 945-2711	J Cam.....	617 354-2688	Reginald 106 Brunswick Dorchester 02121.....	617 541-2843
17 670-2078	Andrew F 62 Vinal Av Som 02143.....	617 625-7623	J 15 Chatham Bro 02446.....	617 232-7990	Renee & Andrew 10 Walnut Bos 02188.....	617 720-3765
17 623-9001	Cartar Anne MD 1101 Beacon Bro 02446.....	617 739-1022	J 518 Harvard Bro 02446.....	617 730-9483	Carter Rice Doud Bulkyer Danton Publishing 163 Main Wilmington 01887	800 638-1671
17 296-4725	Cartar Athens 272 Newbury Boston 02116.....	617 536-6329	J 775 Vna Pkwy West Rosbury 02132.....	617 323-5574	Toll Free-Dial '1' & Then.....	800 619-7447
17 364-5232	B E 58 Gladeside Av Mat 02126.....	617 296-6911	Cartar J M 1410 Columbia Rd S Bos 02127.....	617 464-1040	Cut Svc-Printing 613 Main Wilmington Toll Free-Dial '1' & Then.....	800 648-7447
17 541-5649	Cartar Barbara L MD Tufts-New England Medical Center Bos 02111	617 636-0051	Cartar J M Ornamental Ironworks Call.....	617 436-5353	Headquarters 613 Main Wilmington 01887	978 988-7447
17 739-2662	Cartar Becky Bos 02114.....	617 523-4368	Cartar J Veal Co 48 Newmarket Sq Rox 02118.....	617 442-1775	Call.....	800 638-1673
17 879-0030	Bernard J 112 Gladstone E Bos 02128.....	617 567-3430	Cartar James 1573 Cambridge St Cam 02138.....	617 492-1214	Carter Richard 1079 Commwith Av Brighton 02215.....	617 987-0836
17 541-3948	Bithiah 25 Medway Dor 02124.....	617 298-8713	James 182 Fisher Av Rosbury 02120.....	617 739-2193	Richard A 97 Mt Vernon Bos 02108.....	617 566-7293
17 436-1513	Blake 26 Mt Vernon Bos 02108.....	617 367-9931	James 37 Gold Star Rd Cambridge 02140.....	617 876-8841	Carter Richard A MD 170 Commwith Av Bos 02116.....	617 267-0710
17 569-4119	Cartar Broadcasting Co 20 Park Pl Bos 02116.....	617 423-0210	Jas L 14 Roseberry Rd Mat 02126.....	617 361-0773	Carter Richard K 15 Mercer S Bos 02127.....	617 268-0448
300 569-8782	Cartar & Burgess Consultants Inc 23 East St Cam 02141.....	617 225-0200	Jane 114 Adams Rd Newton 02465.....	617 964-0435	Robert L 175 Richards Av Cam 02140.....	617 864-1535
	C 2000 Commwith Av Bri 02135.....	617 782-2118	Jeffrey 41 Warren Av Bos 02116.....	617 426-5994	Roger 150 St Botolph Bos 02115.....	617 424-6148
	C 228 Fywood Av East Boston 02128.....	617 569-1545	John 11 Mansfield Bri 02134.....	617 987-2163	Roy 44 Concord Av Cam 02138.....	617 491-6115
	C 359 Harvard Cam 02138.....	617 491-8822	John 327 Summer Bos 02110.....	617 423-4334	Royce 18 Seminary Cha 02129.....	617 241-0418
	C 610 Walk Hill Mat 02126.....	617 296-6392	John 40 Westwind Rd Dor 02125.....	617 282-1235		
	C & M 43 Burroughs Jam 02130.....	617 524-9558	June O 329 A Summit Av Bri 02135.....	617 734-6109		
			K 38 Browning Av Dorchester 02124.....	617 265-8456		
			K 17 Esmond Dorchester 02121.....	617 282-1593		

Key Idea: Repeated Halving

To find Derek Jeter's number...

B = phone book

while (B is longer than 1 page):

1. P = middle page of B

2. Let Q be the first name on P

3. **if** 'Jeter' comes before Q:

 Rip away the 2nd half of B

else:

 Rip away the 1st half of B.

Scan remaining page P line-by-line for 'Jeter'

What Happens to Phone Book Length?

Original: 3000 pages

After 1 rip: 1500 pages

After 2 rips: 750 pages

After 3 rips: 375 pages

After 4 rips: 188 pages

After 5 rips: 94 pages

After 12 rips: 1 page

Binary Search

The idea of repeatedly halving the size of the "search space" is the main idea behind the method of binary search.

An item in a sorted array of length n can be located with approximately $\log_2 n$ comparisons.

$$\log_2 8 = 3 \quad \log_2 64 = 7 \quad \log_2 2^{**k} = k$$

What is $\log_2(n)$?

n	$\text{ceil}(\log_2(n))$
10	4
100	7
1000	10
10000	14
100000	17
1000000	20

BinSearch: The Spec

```
def BinSearch(x, a) :  
    """ Returns an int k with the  
    property that a[k]==x is True.  
    If no such k exists, then  
    k==-1.  
  
    PreC: a is a nonempty list of  
    ints that is sorted from smallest  
    to largest. x is an int.  
    """
```

Example:
Does this List have an Element
With Value Equal to 70?

0 1 2 3 4 5 6 7 8 9 10 11

12	15	33	35	42	45	51	62	73	75	86	98
----	----	----	----	----	----	----	----	----	----	----	----

Let's Look For x in a

0 1 2 3 4 5 6 7 8 9 10 11



L: 0

Mid: 5

R: 11

$a[\text{Mid}] \leq x$????

x: 70

$\text{Mid} = (\text{L} + \text{R}) / 2$

The Midpoint Computations

L	R	$(L+R) / 2$
0	11	5
2	6	4
1	100	50

Let's Look For x in a

0 1 2 3 4 5 6 7 8 9 10 11

$a \rightarrow$

12	15	33	35	42	45	51	62	73	75	86	98
----	----	----	----	----	----	----	----	----	----	----	----



L:

0

$a[\text{Mid}] \leq x$????

Mid:

5

R:

11

x:

70

Let's Look For x in a

0 1 2 3 4 5 6 7 8 9 10 11



L: 0

Mid: 5

R: 11

$a[\text{Mid}] \leq x$

Yes!

x : 70

So throw away
The "left half"

Let's Look For x in a

0 1 2 3 4 5 6 7 8 9 10 11



L: 0

Mid: 5

R: 11

x: 70

$a[\text{Mid}] \leq x$

Yes!

So throw away
The "left half"

Let's Look For x in a

0 1 2 3 4 5 6 7 8 9 10 11

a →

12	15	33	35	42	45	51	62	73	75	86	98
----	----	----	----	----	----	----	----	----	----	----	----



L: 0

Mid: 5

R: 11

$a[\text{Mid}] \leq x$

Revise L and Mid

x: 70

Let's Look For x in a

0 1 2 3 4 5 6 7 8 9 10 11

a →

12	15	33	35	42	45	51	62	73	75	86	98
----	----	----	----	----	----	----	----	----	----	----	----



L: 5

a[Mid] <= x ???

Mid: 8

R: 11

x: 70

Let's Look For x in a

0 1 2 3 4 5 6 7 8 9 10 11



L: 5

Mid: 8

R: 11

$a[\text{Mid}] \leq x$

No

x : 70

So throw away the
"right half"

Let's Look For $x = 70$

0 1 2 3 4 5 6 7 8 9 10 11

a →

12	15	33	35	42	45	51	62	73	75	86	98
----	----	----	----	----	----	----	----	----	----	----	----



L: 5

Mid: 8

R: 11

$a[\text{Mid}] \leq x$

Revise R and Mid

x: 70

Let's Look For $x = 70$

0 1 2 3 4 5 6 7 8 9 10 11

a →

12	15	33	35	42	45	51	62	73	75	86	98
----	----	----	----	----	----	----	----	----	----	----	----



L: 5

$a[\text{Mid}] \leq x$

Mid: 6

Revise R and Mid

R: 8

x: 70

Let's Look For x in a

0 1 2 3 4 5 6 7 8 9 10 11



L:



a[Mid] <= x ????

Mid:



R:



x:



Let's Look For x in a

0 1 2 3 4 5 6 7 8 9 10 11

a →

12	15	33	35	42	45	51	62	73	75	86	98
----	----	----	----	----	----	----	----	----	----	----	----



L:

5

$a[\text{Mid}] \leq x$

Mid:

6

Yes

R:

8

x:

70

Throw away the
Left half

Let's Look For x in a

0 1 2 3 4 5 6 7 8 9 10 11

a ->

12	15	33	35	42	45	51	62	73	75	86	98
----	----	----	----	----	----	----	----	----	----	----	----



L: 6

Mid: 7

R: 8

$a[\text{Mid}] \leq x$

Yes

x: 70

Let's Look For x in a

0 1 2 3 4 5 6 7 8 9 10 11

a →

12	15	33	35	42	45	51	62	73	75	86	98
----	----	----	----	----	----	----	----	----	----	----	----



L: 6

Mid: 7

R: 8

x: 70

$a[\text{Mid}] \leq x$

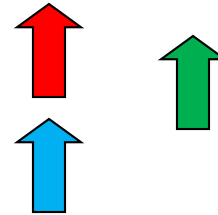
Throw away the
left half

Let's Look For x in a

0 1 2 3 4 5 6 7 8 9 10 11

$a \rightarrow$

12	15	33	35	42	45	51	62	73	75	86	98
----	----	----	----	----	----	----	----	----	----	----	----



L:

6

Mid:

7

R:

8

x :

70

Done! At this point we just compare x with $a[L]$ and $a[L+1]$.

What We Just Did

```
L = 0
R = len(a) - 1
while R - L > 1:
    # a[L] <= x <= a[R]
    Mid = (L + R) / 2
    if x <= a[mid]:
        R = Mid
    else:
        L = Mid
```

A Loop
Invariant

Note that $a[L] \leq x \leq a[R]$ remains True throughout the loop

What We Just Did

```
L = 0
R = len(a) - 1
while R - L > 1:
    # a[L] <= x <= a[R]
    Mid = (L + R) / 2
    if x <= a[mid]:
        R = Mid
    else:
        L = Mid
```

What is the situation when the loop terminates?

What We Just Did

```
L = 0
R = len(a) - 1
while R - L > 1:
    # a[L] <= x <= a[R]
    Mid = (L + R) / 2
    if x <= a[mid]:
        R = Mid
    else:
        L = Mid
```

$R - L \leq 1$ implies $R = L + 1$

After the Loop Ends



This is True: $a[L] \leq x \leq a[L+1]$

After the Loop Ends



```
if x==a[L]:  
    return L  
elif x==a[L+1]:  
    return L+1  
else:  
    return -1
```

Measuring Execution Time

We now have two ways to search a list:

`LinSearch(x, a)`

`BinSearch(x, a)`

Intuition: `BinSearch` much faster.

Can we quantify this with a "stop watch"?

The `timeit` Module

This module can be used to time how long it takes to execute a chunk of code.

Typical chunk = some function of interest.

This is called benchmarking.

Benchmarking

Let's benchmark `LinSearch(x, a)` and `BinSearch(x, a)`.

Compare how long it takes when `len(a)` equals 1000, 10000, 100000, and 1000000.

Our intuition tells us that as `len(a)` increases, `BinSearch` will be dramatically faster.

BinSearch vs LinSearch

n	tBin	tLin	tLinW
1000	0.0007	0.0064	0.0119
10000	0.0009	0.0668	0.1203
100000	0.0011	0.8296	1.2082
1000000	0.0015	17.7388	13.9341

tBin = time for BinSearch

tLin = time for LinSearch (for loop version)

tLinW = time for LinSearch (while-loop version)

BinSearch vs LinSearch

n	tLin/tBin
1000	9
10000	74
100000	754
1000000	7095

Reporting ratios is more illuminating since we do not really care about the time units in this informal comparison

Using the `timeit` Module

We show how this module was use to get the results on the previous slides.

Our `LinSearch` vs `BinSearch` example is very typical: is one function faster than another?

A Benchmarking Framework

```
from timeit import *
```

```
S = """
```

Set-up code

```
"""
```

```
B = """
```

Code to Benchmark

```
"""
```

```
p = 10; m = 100
```

```
t = min(Timer(B, setup=S) . repeat(p, m) )
```

Yes, these are doc strings.

The Set-Up and Bench Codes

```
from random import randint as randi
from ShowSearch import BinSearch
n = 10000
s = [randi(0,10*n) for i in range(n)]
s.sort()
x = s[n/2]
```

```
k=BinSearch(x,s)
```

The set-up code is run once.

It is not timed.

It just sets up the code to be timed.

A Benchmarking Framework

```
from timeit import *
```

```
S = """
```

```
    Set-up code
```

```
    """
```

```
B = """
```

```
    Code to Benchmark
```

```
    """
```

```
p = 10; m = 100
```

```
t = min(Timer(B, setup=S).repeat(p, m))
```

An “experiment” consists of running the blue code m times.

The stopwatch will time how long it takes to do one experiment

Larger values necessary if the blue code executes very quickly

A Benchmarking Framework

```
from timeit import *
```

```
S = """
```

Set-up code

```
"""
```

```
B = """
```

Code to Benchmark

```
"""
```

```
p = 10; m = 100
```

```
t = min(Timer(B, setup=S).repeat(p, m))
```

Timer returns a length-p list. Each element is the stopwatch time for 1 experiment

This helps control for other stuff that may be running on your computer.

A Benchmarking Framework

```
from timeit import *
```

```
S = """
```

Set-up code

```
"""
```

```
B = """
```

Code to Benchmark

```
"""
```

```
p = 10; m = 100
```

```
t = min(Timer(B, setup=S).repeat(p, m))
```

In general, it is best to take the minimum as the most reliable. The benchmark time is assigned to t

This helps control for other stuff that may be running on your computer.

Why Benchmarking is Important

Confirms/refutes what our intuition might say about efficiency.

Makes us sensitive to the various issues that affect efficiency.

Steers us away from simplistic comparisons of different methods that can be used on the same problem.