Lecture 27

Sorting

Announcements for This Lecture

Prelim/Finals

- Prelims in **handback room**
  - Gates Hall 216
  - Open 12-4pm each day

- **Final: Dec 9th 7:00-9:30pm**
  - Study guide is posted
  - Announce reviews on Thurs.
- **Conflict with Final time?**
  - Submit to conflict to CMS **by this THURSDAY!**

Assignments/Lab

- **A6** is now graded.
  - **Mean**: 89, **Median**: 94
  - **Std Deviation**: 14.2
  - Mean/Median **Time**: 12 hrs
- **A7** is due next **Dec. 11**
  - Will grade if turn in Sun.
- **Lab 13** is *optional*
  - Good study for the final
  - Consultant hours still open

Binary Search

- **Vague:** Look for v in **sorted** sequence segment b[h..k].

Binary Search

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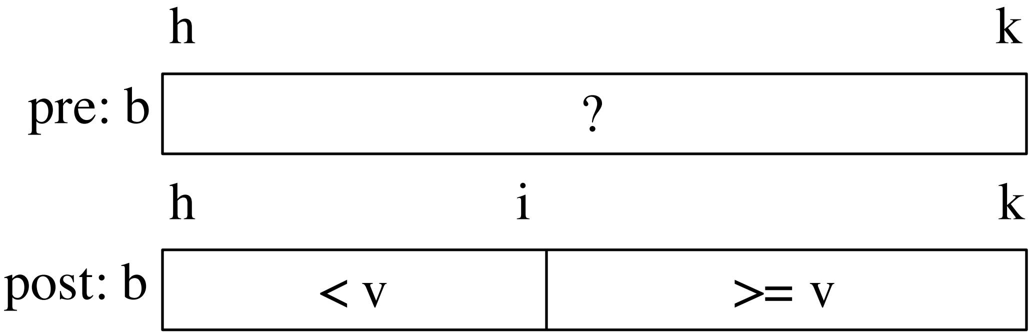
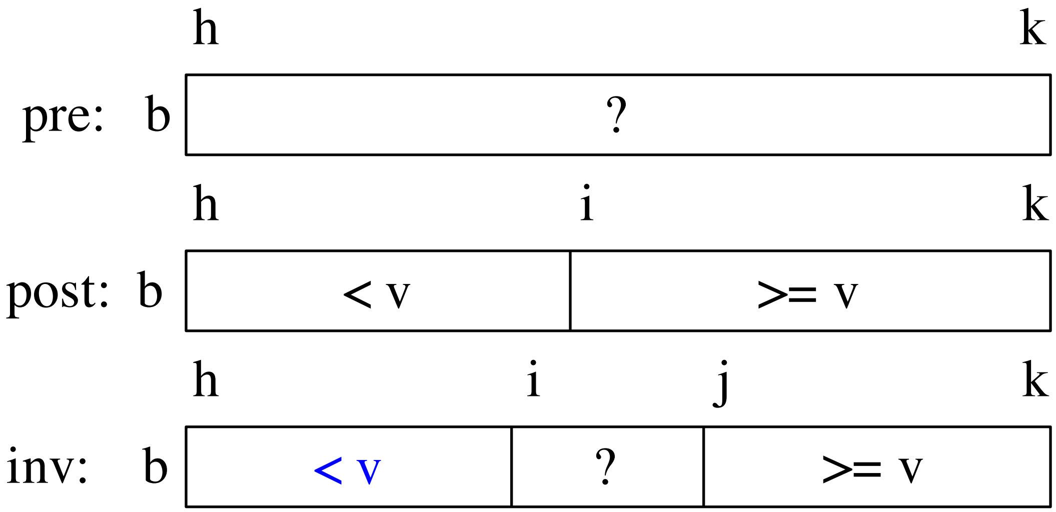
- **Better:**
  - Precondition: b[h..k-1] is sorted (in ascending order).
  - Postcondition: b[h..i] < v and v <= b[i+1..k]
- Below, the array is in non-descending order:



Binary Search

- Look for value v in **sorted** segment b[h..k]



New statement of the invariant guarantees that we get leftmost position of v if found

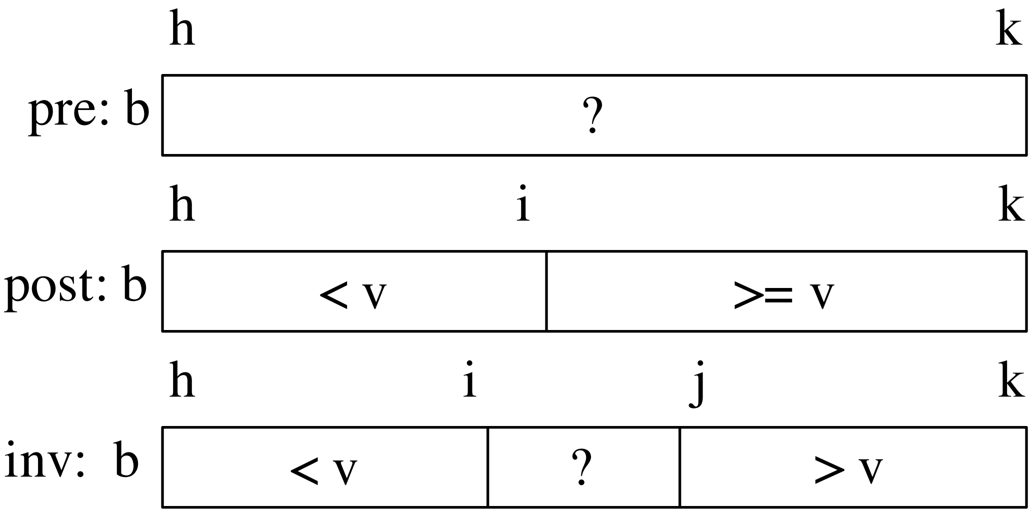
Example b

	h									k
	0	1	2	3	4	5	6	7	8	9
b	3	3	3	3	3	4	4	6	7	7

- if v is 3, set i to 0
- if v is 4, set i to 5
- if v is 5, set i to 7
- if v is 8, set i to 10

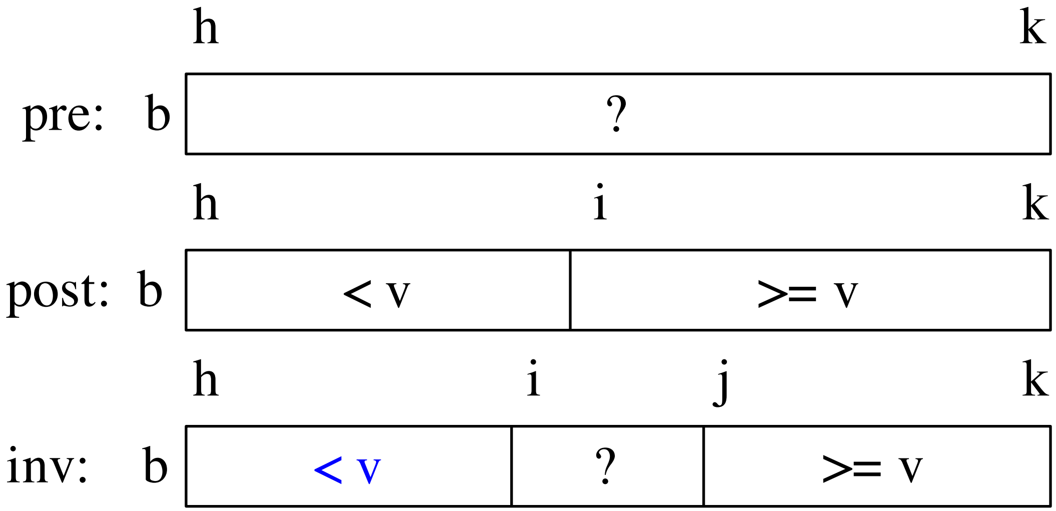
Binary Search

- **Vague:** Look for v in **sorted** sequence segment b[h..k].
- **Better:**
  - Precondition: b[h..k-1] is sorted (in ascending order).
  - Postcondition: b[h..i] <= v and v < b[i+1..k]
- Below, the array is in non-descending order:



Called binary search because each iteration of the loop cuts the array segment still to be processed in half

Binary Search



New statement of the invariant guarantees that we get leftmost position of v if found

```
i = h; j = k+1;
while i != j:
```

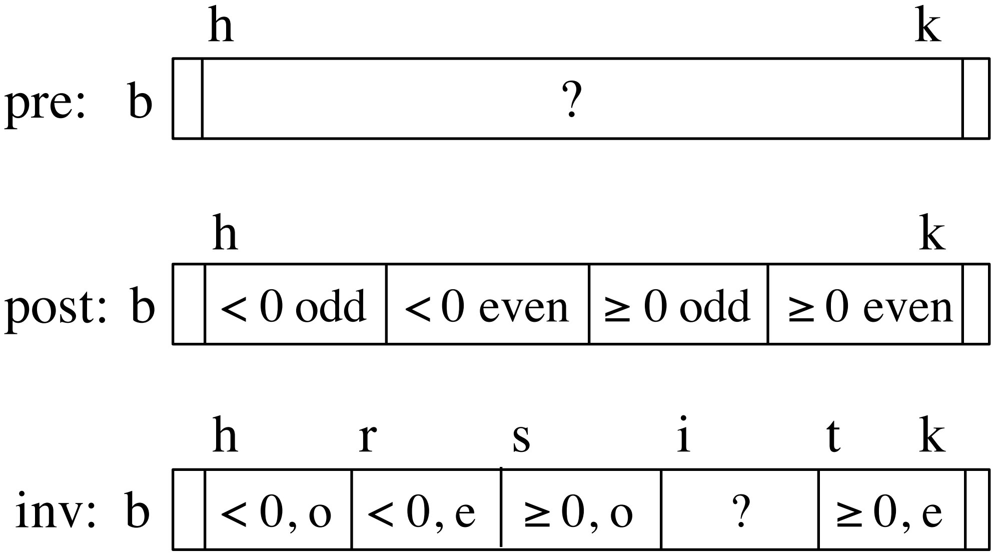
Looking at b[i] gives linear search from left.

Looking at b[j-1] gives linear search from right.

Looking at middle: b[(i+j)/2] gives binary search.

Flag of Mauritius

- Now we have four colors!
  - Negatives: ‘red’ = odd, ‘purple’ = even
  - Positives: ‘yellow’ = odd, ‘green’ = even



h k

pre: b | ? |

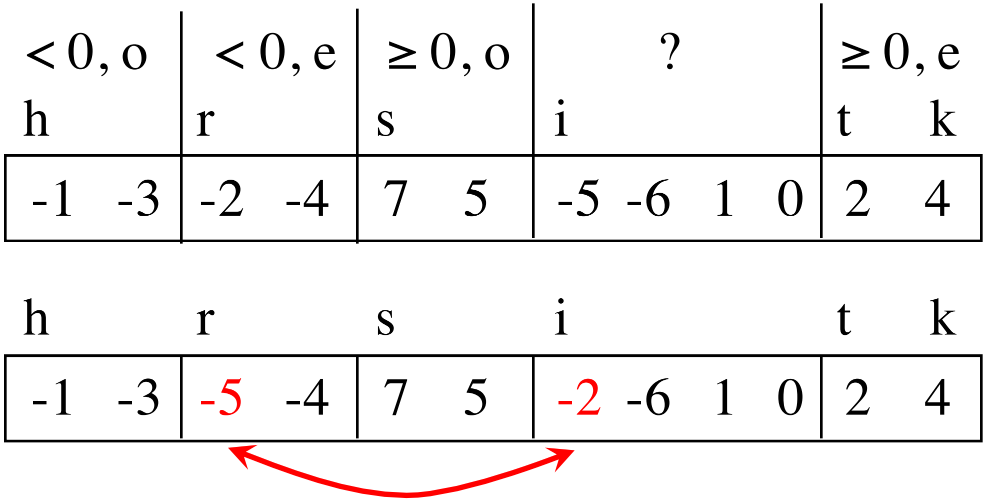
h k

post: b | < 0 odd | < 0 even | ≥ 0 odd | ≥ 0 even |

h r s i t k

inv: b | < 0, o | < 0, e | ≥ 0, o | ? | ≥ 0, e |

Flag of Mauritius



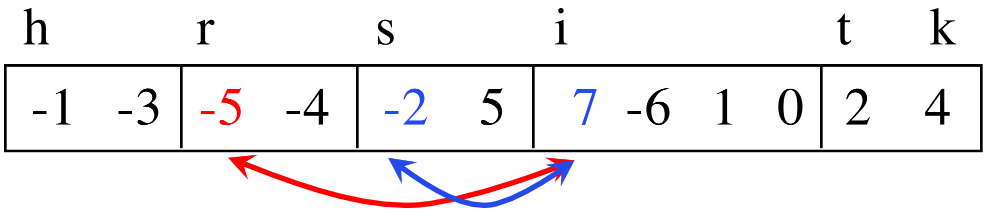
< 0, o		< 0, e		≥ 0, o		?				≥ 0, e	
h		r		s		i				t	k
-1	-3	-2	-4	7	5	-5	-6	1	0	2	4

h		r		s		i				t	k
-1	-3	-5	-4	7	5	-2	-6	1	0	2	4

One swap is not good enough

Flag of Mauritius

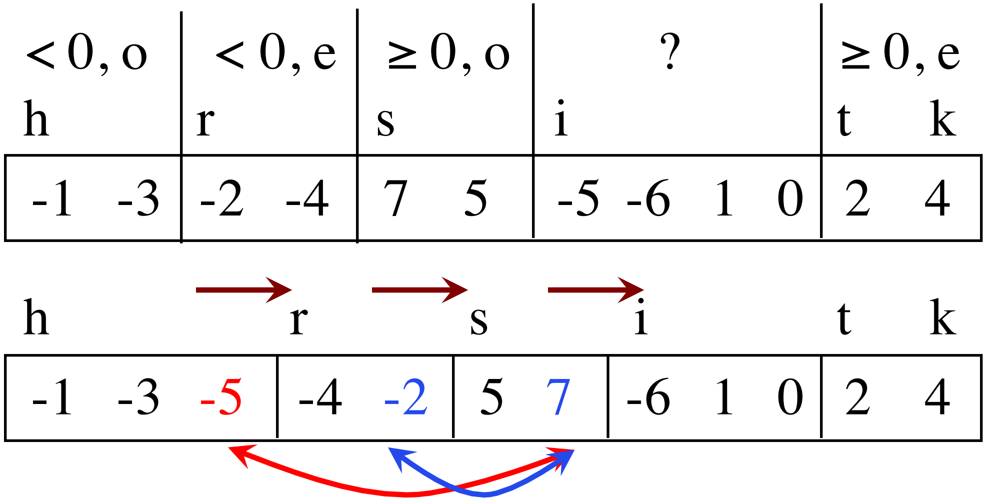
< 0, o		< 0, e		≥ 0, o		?				≥ 0, e	
h		r		s		i				t	k
-1	-3	-2	-4	7	5	-5	-6	1	0	2	4



h		r		s		i				t	k
-1	-3	-5	-4	-2	5	7	-6	1	0	2	4

Need two swaps for two spaces

Flag of Mauritius

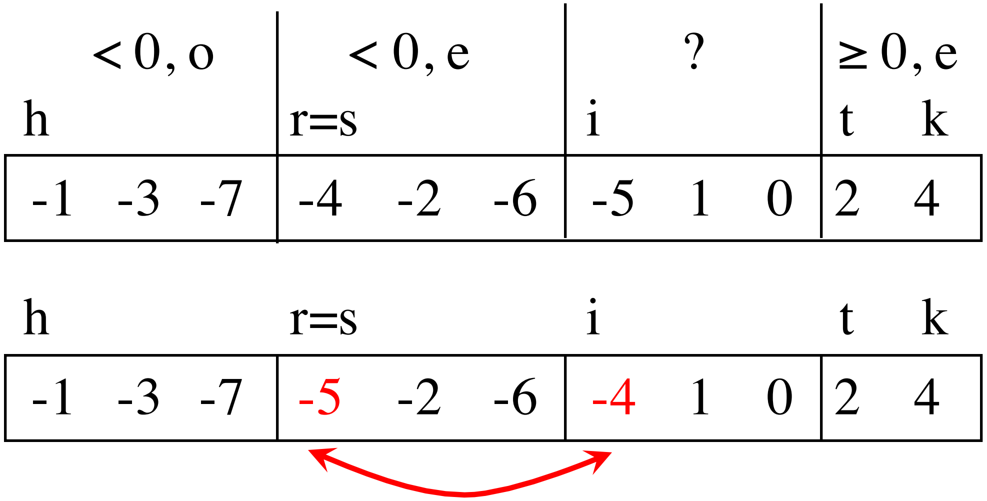


< 0, o	< 0, e	≥ 0, o	?	≥ 0, e
h	r	s	i	t k
-1 -3	-2 -4	7 5	-5 -6 1 0	2 4

h	r	s	i	t k
-1 -3 -5	-4 -2	5 7	-6 1 0	2 4

And adjust the loop variables

Flag of Mauritius

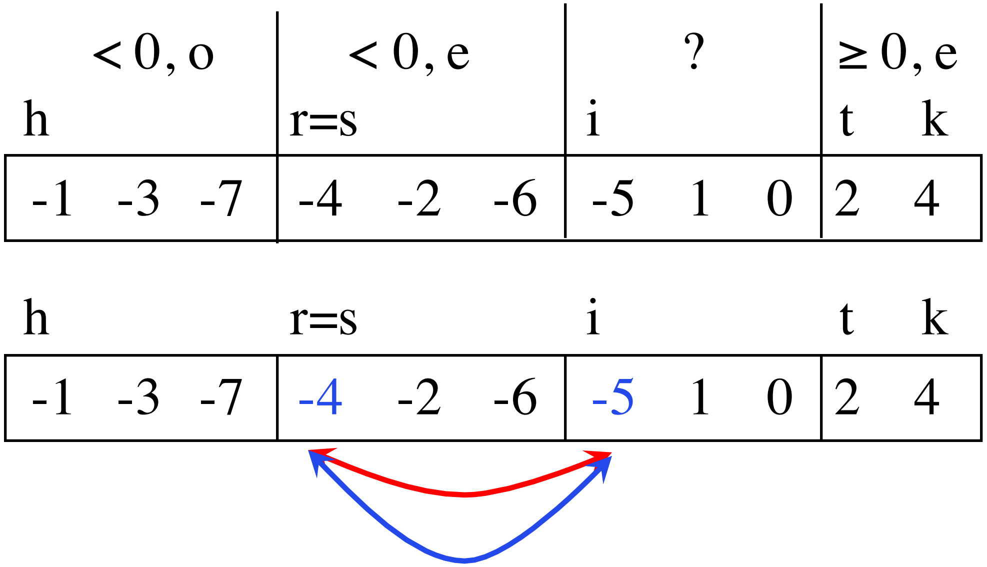


< 0, o	< 0, e	?	≥ 0, e
h	r=s	i	t k
-1 -3 -7	-4 -2 -6	-5 1 0	2 4

h	r=s	i	t k
-1 -3 -7	-5 -2 -6	-4 1 0	2 4

BUT NOT ALWAYS!

Flag of Mauritius



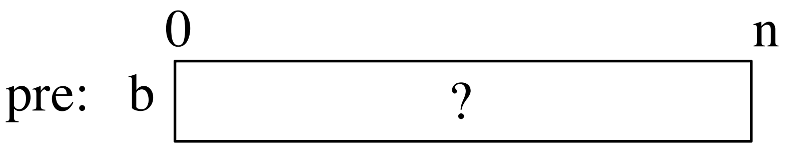
< 0, o	< 0, e	?	≥ 0, e
h	r=s	i	t k
-1 -3 -7	-4 -2 -6	-5 1 0	2 4

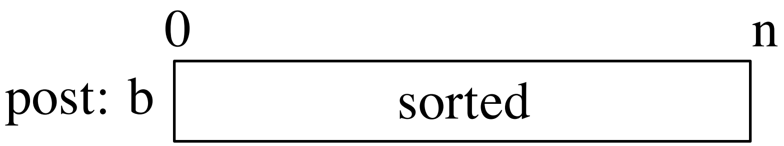
h	r=s	i	t k
-1 -3 -7	-4 -2 -6	-5 1 0	2 4

BUT NOT ALWAYS!

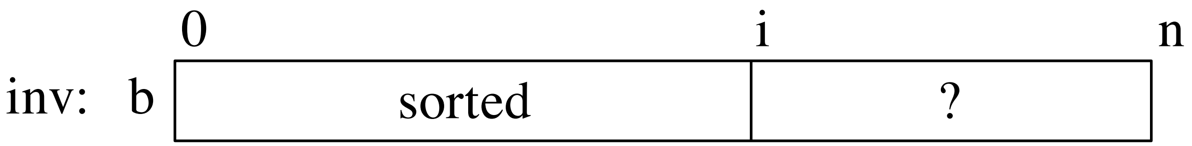
Have to check if second swap is okay

Sorting: Arranging in Ascending Order

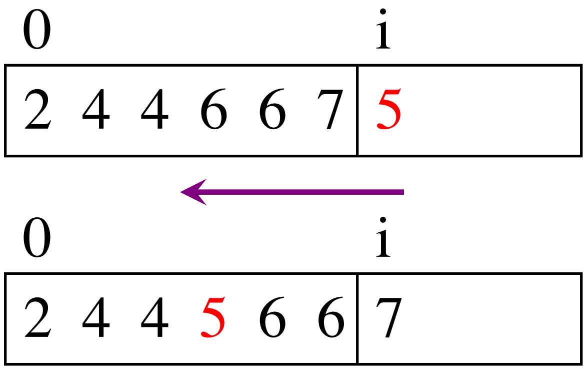




Insertion Sort:



```
i = 0
while i < n:
    # Push b[i] down into its
    # sorted position in b[0..i]
    i = i+1
```

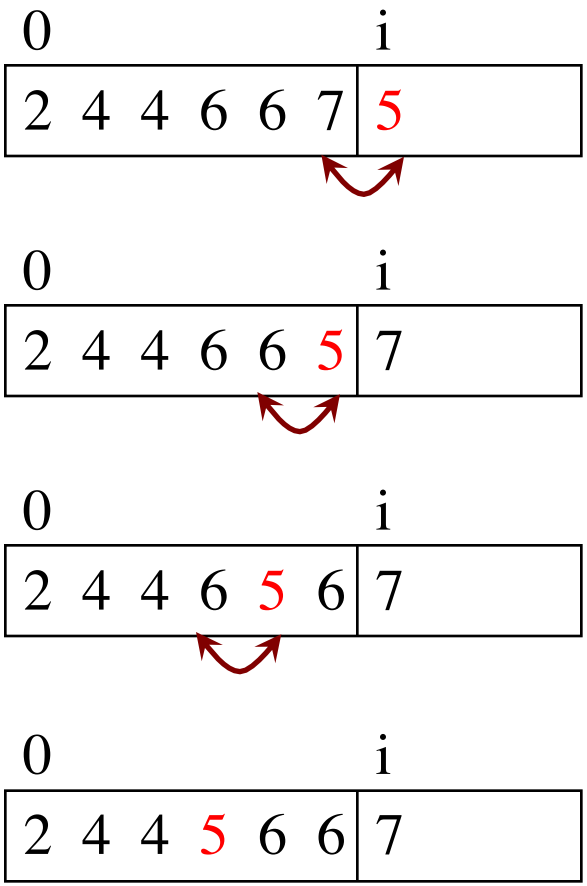


Insertion Sort: Moving into Position

```
i = 0
while i < n:
    push_down(b,i)
    i = i+1

def push_down(b, i):
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

swap shown in the lecture about lists



The Importance of Helper Functions

```
i = 0
while i < n:
    push_down(b,i)
    i = i+1

def push_down(b, i):
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

VS

Can you understand all this code below?

```
i = 0
while i < n:
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            temp = b[j]
            b[j] = b[j-1]
            b[j-1] = temp
        j = j -1
    i = i +1
```

Insertion Sort: Performance

```
def push_down(b, i):
    """Push value at position i into
    sorted position in b[0..i-1]"""
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

- b[0..i-1]: i elements
- Worst case:
  - i = 0: 0 swaps
  - i = 1: 1 swap
  - i = 2: 2 swaps
- Pushdown is in a loop
  - Called for i in 0..n
  - i swaps each time

Insertion sort is an n^2 algorithm

Total Swaps: 0 + 1 + 2 + 3 + … (n-1) = (n-1)*n/2

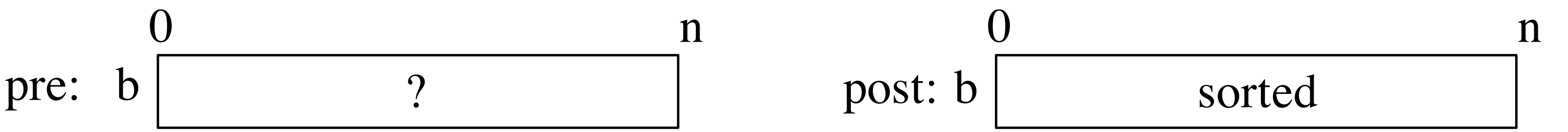
Algorithm “Complexity”

- **Given**: a list of length n and a problem to solve
- **Complexity**: *rough* number of steps to solve worst case
- Suppose we can compute 1000 operations a second:

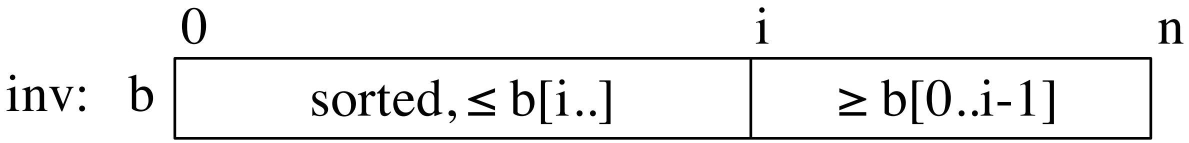
Complexity	n=10	n=100	n=1000
n	0.01 s	0.1 s	1 s
n log n	0.016 s	0.32 s	4.79 s
n^2	0.1 s	10 s	16.7 m
n^3	1 s	16.7 m	11.6 d
2^n	1 s	$4\text{x}10^{19}$ y	$3\text{x}10^{290}$ y

Major Topic in 2110: Beyond scope of this course

Sorting: Changing the Invariant

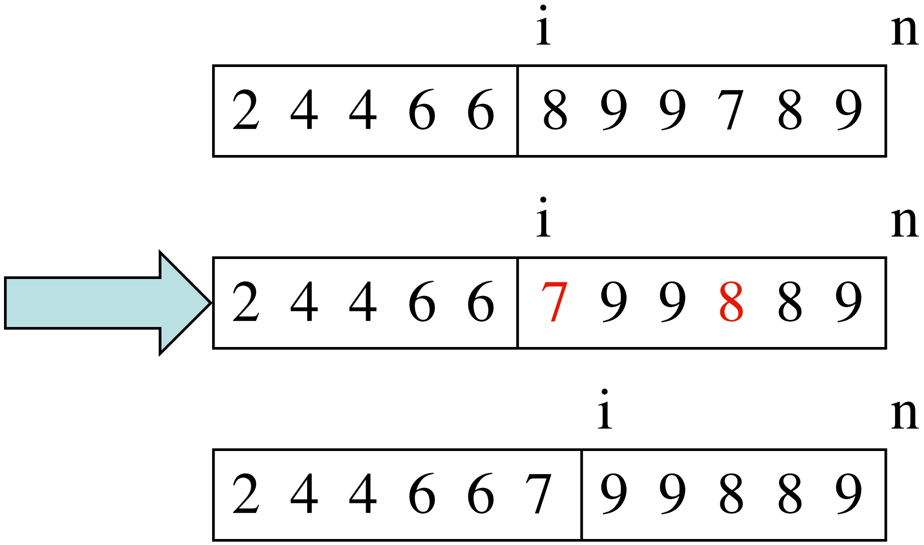


Selection Sort:

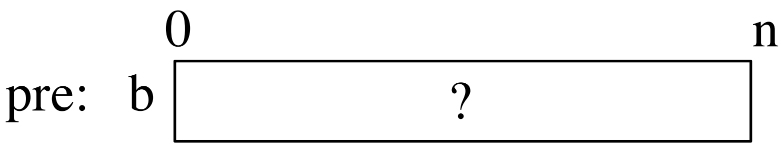


First segment always contains smaller values

```
i = 0
while i < n:
    # Find minimum in b[i..]
    # Move it to position i
    i = i+1
```



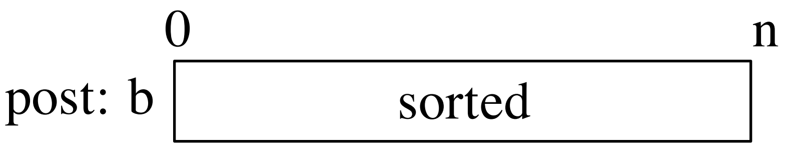
Sorting: Changing the Invariant



pre: b

0 n

?

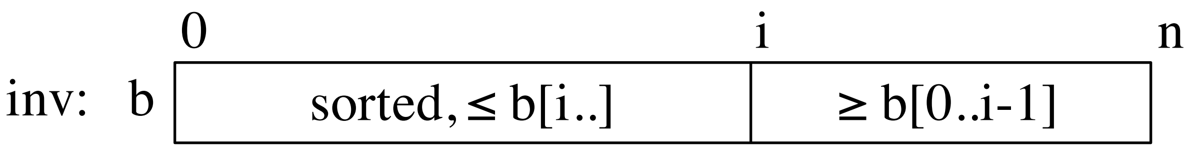


post: b

0 n

sorted

Selection Sort:



inv: b

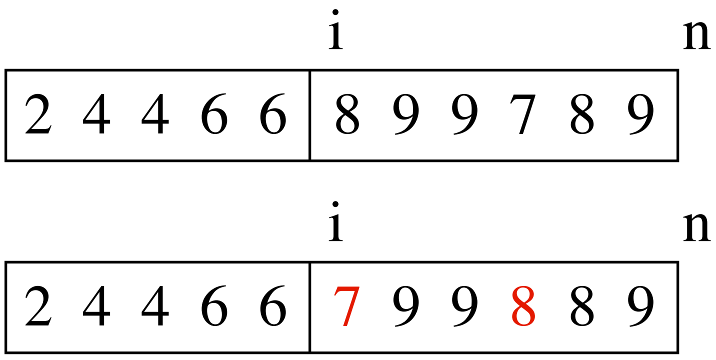
0 i n

sorted, ≤ b[i..]

≥ b[0..i-1]

First segment always contains smaller values

```
i = 0
while i < n:
    j = index of min of b[i..n-1]
    swap(b,i,j)
    i = i+1
```



i n

2 4 4 6 6 | 8 9 9 7 8 9

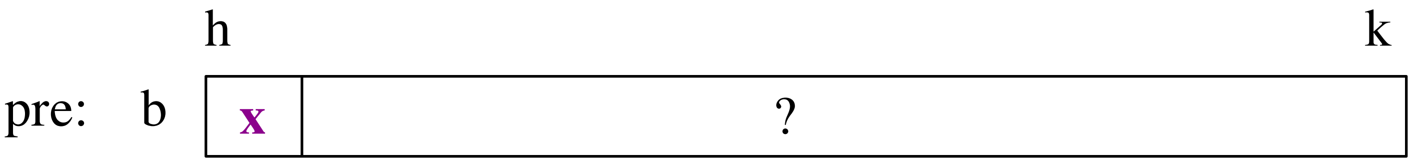
i n

2 4 4 6 6 | 7 9 9 8 8 9

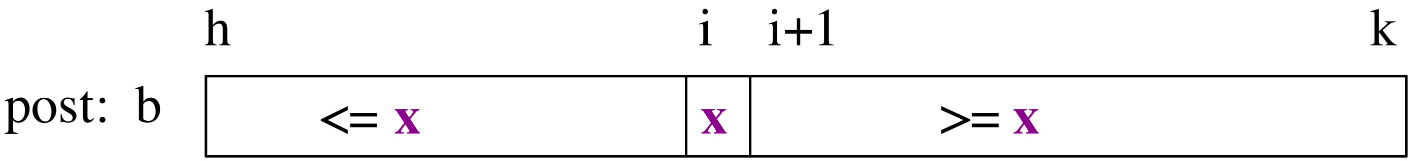
Selection sort also is an n^2 algorithm

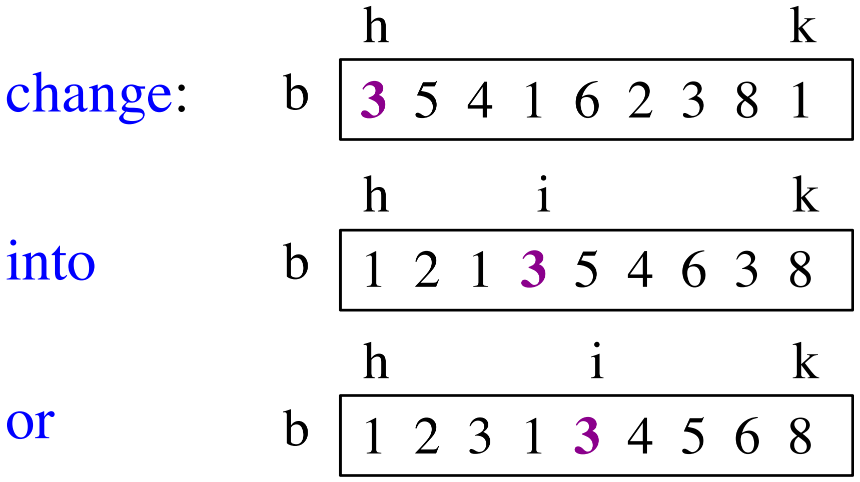
Partition Algorithm

- Given a list segment b[h..k] with some value x in b[h]:



- Swap elements of b[h..k] and store in j to truthify post:





h k

change: b 3 5 4 1 6 2 3 8 1

h i k

into b 1 2 1 3 5 4 6 3 8

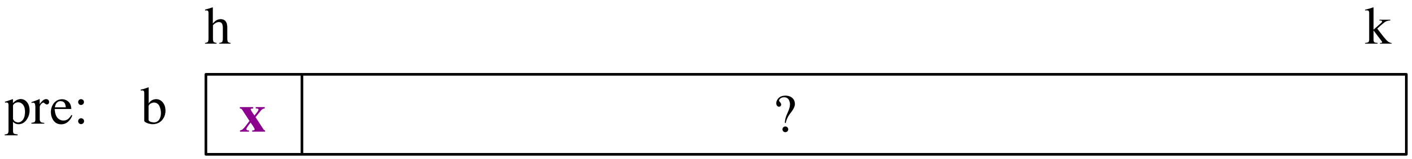
h i k

or b 1 2 3 1 3 4 5 6 8

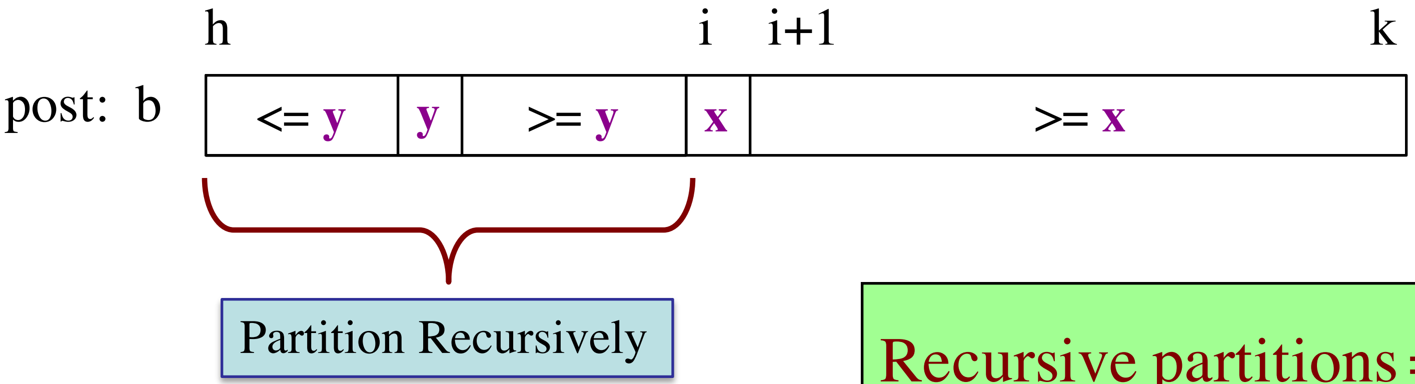
- x is called the pivot value
    - x is not a program variable
    - denotes value initially in b[h]

Sorting with Partitions

- Given a list segment b[h..k] with some value x in b[h]:



- Swap elements of b[h..k] and store in j to truthify post:



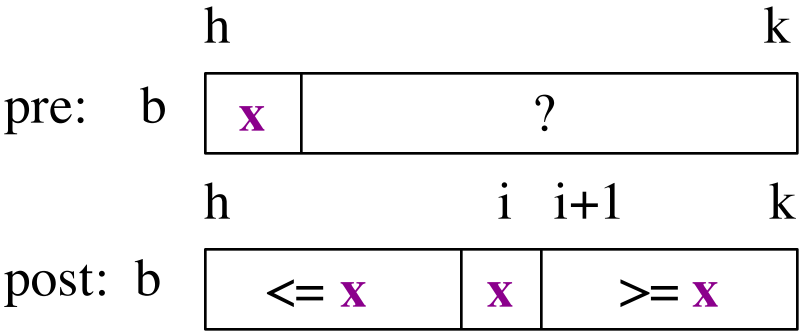
Recursive partitions = sorting

- Called **QuickSort** (why???)
- Popular, fast sorting technique

QuickSort

```
def quick_sort(b, h, k):
    """Sort the array fragment b[h..k]"""
    if  b[h..k] has fewer than 2 elements:
        return
    j = partition(b, h, k)
    # b[h..j–1] <= b[j] <= b[j+1..k]
    # Sort b[h..j–1]  and  b[j+1..k]
    quick_sort (b, h, j–1)
    quick_sort (b, j+1, k)
```

- **Worst Case:** array already sorted
    - Or almost sorted
    - n^2 in that case
- **Average Case:** array is scrambled
    - n log n in that case
    - Best sorting time!



h k

pre: b | x | ?

h i i+1 k

post: b | <= x | x | >= x

Final Word About Algorithms

- **Algorithm:**
  - Step-by-step way to do something
  - Not tied to specific language

List Diagrams

- **Implementation:**
  - An algorithm in a specific language
  - Many times, not the “hard part”

Demo Code

- Higher Level Computer Science courses:
  - We teach advanced algorithms (pictures)
  - Implementation you learn on your own