Binary Search

- Look for value v in **sorted** segment b[h..k]

pre: b[h..k] = ?

post: b[h..i-1] < v, b[i..k] >= v

inv: b[h..i-1] < v, b[i..j-1] ?, b[j..k] >= v

New statement of the invariant guarantees that we get leftmost position of v if found

Example b:

h									k
0	1	2	3	4	5	6	7	8	9
3	3	3	3	3	4	4	6	7	7

- if v is 3, set i to 0
- if v is 4, set i to 5
- if v is 5, set i to 7
- if v is 8, set i to 10

Binary Search

pre: b[h..k] = ?

post: b[h..i-1] < v, b[i..k] >= v

inv: b[h..i-1] < v, b[i..j-1] ?, b[j..k] >= v

New statement of the invariant guarantees that we get leftmost position of v if found

```
i = h; j = k+1;
while i != j:
    Looking at b[i] gives linear search from left.
    Looking at b[j-1] gives linear search from right.
    Looking at middle: b[(i+j)/2] gives binary search.
```

Flag of Mauritius

< 0, o			< 0, e			?			≥ 0, e	
h			r=s			i			t	k
-1	-3	-7	-4	-2	-6	-5	1	0	2	4
-1	-3	-7	-4	-2	-6	-5	1	0	2	4

Need two swaps for two spaces

BUT NOT ALWAYS!

Have to check if second swap is okay

Sorting: Arranging in Ascending Order

pre: b[0..n] = ? post: b[0..n] = sorted

Insertion Sort:

inv: b[0..i-1] sorted, b[i..n] ?

```
i = 0
while i < n:
    # Push b[i] down into its
    # sorted position in b[0..i]
    i = i+1
```

0					i
2	4	4	6	6	7	5

0					i
2	4	4	5	6	6	7

Insertion Sort: Moving into Position

```
i = 0
while i < n:
    push_down(b,i)
    i = i+1

def push_down(b, i):
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

swap shown in the lecture about lists

2 4 4 6 6 7 5 (i)

2 4 4 6 6 5 7

2 4 4 6 5 6 7

2 4 4 5 6 6 7

Insertion Sort: Performance

```
def push_down(b, i):
    """Push value at position i into
    sorted position in b[0..i-1]"""
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

- b[0..i-1]: i elements
- Worst case:
  - i = 0: 0 swaps
  - i = 1: 1 swap
  - i = 2: 2 swaps
- Pushdown is in a loop
  - Called for i in 0..n
  - i swaps each time

Insertion sort is an n^2 algorithm

Total Swaps: 0 + 1 + 2 + 3 + … (n-1) = (n-1)*n/2

Algorithm "Complexity"

- **Given**: a list of length n and a problem to solve
- **Complexity**: *rough* number of steps to solve worst case
- Suppose we can compute 1000 operations a second:

Complexity	n=10	n=100	n=1000
n	0.01 s	0.1 s	1 s
n log n	0.016 s	0.32 s	4.79 s
n^2	0.1 s	10 s	16.7 m
n^3	1 s	16.7 m	11.6 d
2^n	1 s	$4x10^{19}$ y	$3x10^{290}$ y

Major Topic in 2110: Beyond scope of this course

Sorting: Changing the Invariant

pre: b [0 … n: ?] post: b [0 … n: sorted]

Selection Sort:

inv: b [0 … i: sorted, ≤ b[i..] | i … n: ≥ b[0..i-1]]

First segment always contains smaller values

```
i = 0
while i < n:
    j = index of min of b[i..n-1]
    swap(b,i,j)
    i = i+1
```

b: 2 4 4 6 6 | 8 9 9 7 8 9 (i, n)

b: 2 4 4 6 6 | 7 9 9 8 8 9 (i, n)

Selection sort also is an n^2 algorithm

Partition Algorithm

- Given a list segment b[h..k] with some value x in b[h]:

pre: b [h: x | ? :k]

- Swap elements of b[h..k] and store in j to truthify post:

post: b [h: <= x | i: x | i+1: >= x :k]

change: b 3 5 4 1 6 2 3 8 1 (h … k)

into b 1 2 1 3 5 4 6 3 8 (h, i, k)

or b 1 2 3 1 3 4 5 6 8 (h, i, k)

- x is called the pivot value
  - x is not a program variable
  - denotes value initially in b[h]

Sorting with Partitions

- Given a list segment b[h..k] with some value x in b[h]:

pre: b [h: x | ? :k]

- Swap elements of b[h..k] and store in j to truthify post:

post: b [h: <= y | y | >= y | i: x | i+1: >= x :k]

Partition Recursively

Recursive partitions = sorting
- Called **QuickSort** (why???)
- Popular, fast sorting technique

QuickSort

```
def quick_sort(b, h, k):
    """Sort the array fragment b[h..k]"""
    if b[h..k] has fewer than 2 elements:
        return
    j = partition(b, h, k)
    # b[h..j–1] <= b[j] <= b[j+1..k]
    # Sort b[h..j–1]  and  b[j+1..k]
    quick_sort (b, h, j–1)
    quick_sort (b, j+1, k)
```

- **Worst Case:** array already sorted
  - Or almost sorted
  - n^2 in that case
- **Average Case:** array is scrambled
  - n log n in that case
  - Best sorting time!

pre: b [h: x | ? :k]

post: b [h: <= x | i: x | i+1: >= x :k]

Final Word About Algorithms

- **Algorithm:**
  - Step-by-step way to do something
  - Not tied to specific language
- **Implementation:**
  - An algorithm in a specific language
  - Many times, not the "hard part"
- Higher Level Computer Science courses:
  - We teach advanced algorithms (pictures)
  - Implementation you learn on your own

List Diagrams

Demo Code