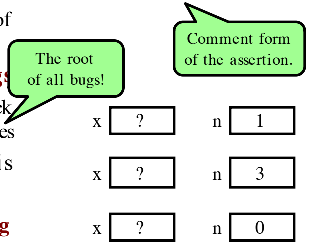
Recall: Important Terminology

- **assertion**: true-false statement placed in a program to *assert* that it is true at that point
  - Can either be a **comment**, or an **assert** command
- **invariant**: assertion supposed to "always" be true
  - If temporarily invalidated, must make it true again
  - **Example**: class invariants and class methods
- **loop invariant**: assertion supposed to be true before and after each iteration of the loop
- **iteration of a loop**: one execution of its body

Assertions versus Asserts

- Assertions **prevent bugs**
  - Help you keep track of what you are doing
- Also **track down bugs**
  - Make it easier to check belief/code mismatches
- The assert statement is a (type of) assertion
  - One you are **enforcing**
  - Cannot always convert a comment to an assert

 # x is the sum of 1..n

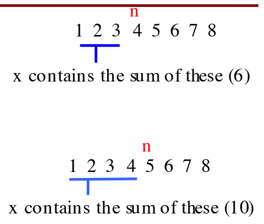


Preconditions & Postconditions

precondition

```
# x = sum of 1..n-1
x = x + n
n = n + 1
# x = sum of 1..n-1
```

postcondition



x contains the sum of these (6)

x contains the sum of these (10)

- **Precondition:** assertion placed before a segment
- **Postcondition:** assertion placed after a segment

Relationship Between Two

If precondition is true, then postcondition will be true

Solving a Problem

precondition

```
# x = sum of 1..n

n = n + 1
# x = sum of 1..n
```

postcondition

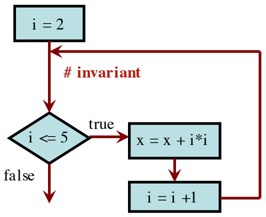
What statement do you put here to make the postcondition true?

A: x = x + 1
B: x = x + n
C: x = x + n+1
D: None of the above
E: I don't know

Invariants: Assertions That Do Not Change

- **Loop Invariant**: an assertion that is true before and after each iteration (execution of repetend)

```
x = 0; i = 2
while i <= 5:
    x = x + i*i
    i = i +1
# x = sum of squares of 2..5
```



Invariant:

x = sum of squares of 2..i-1

in terms of the range of integers that have been processed so far

The loop processes the range 2..5

Invariants: Assertions That Do Not Change

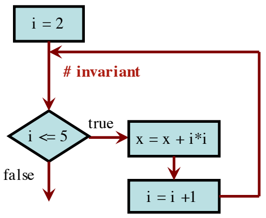
```
x = 0; i = 2
# Inv: x = sum of squares of 2..i-1
while i <= 5:
    x = x + i*i
    i = i +1
# Post: x = sum of squares of 2..5
```

x ~~0~~ ~~4~~ ~~13~~ ~~29~~ 54

i ~~[illegible]~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ 6

Integers that have been processed: 2, 3, 4, 5

Range 2..i-1: 2..5



Invariant was always true just before test of loop condition. So it's true when loop terminates

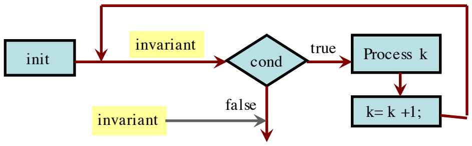
The loop processes the range 2..5

Designing Integer while-loops

```
# Process integers in a..b
# inv: integers in a..k-1 have been processed
k = a
while  k <= b:
    process integer k
    k = k + 1
# post: integers in a..b have been processed
```

Command to do something

Equivalent postcondition



Designing Integer while-loops

1. Recognize that a range of integers b..c has to be processed
2. Write the command and equivalent postcondition
3. Write the basic part of the while-loop
4. Write loop invariant
5. Figure out any initialization
6. Implement the repetend (process k)

```
# Process  b..c
Initialize  variables  (if necessary)   to make invariant  true
# Invariant:  range b..k-1  has been  processed
while  k <= c:
    # Process k
    k = k + 1
# Postcondition:  range b..c  has been  processed
```

Finding an Invariant

Command to do something

```
# Make b True if n is prime, False otherwise
b = True
k = 2
# invariant: b is True if no int in 2..k-1 divides n, False otherwise
while  k < n:
    # Process k;
    if n % k == 0:
        b = False
    k = k +1
# b is True if no int in 2..n-1 divides n, False otherwise
```

Equivalent postcondition

What is the invariant?   1 2 3 … k-1 k k+1 … n

Finding an Invariant

```
# set x to # adjacent equal pairs in s


while k < len(s):
    # Process k

    k = k + 1
# x = # adjacent equal pairs in s[0..len(s)-1]
```

Command to do something

for s = 'ebeee',   x = 2

Equivalent postcondition

k: next integer to process.
Which have been processed?

A: 0..k
B: 1..k
C: 0..k–1
D: 1..k–1
E: I don't know

Finding an Invariant

```
# set x to # adjacent equal pairs in s
x = 0

# inv: x = # adjacent equal pairs in s[0..k-1]
while k < len(s):
    # Process k

    k = k + 1
# x = # adjacent equal pairs in s[0..len(s)-1]
```

Command to do something

for s = 'ebeee',   x = 2

Equivalent postcondition

k: next integer to process.
What is initialization for k?

A: k = 0
B: k = 1
C: k = –1
D: I don't know

Reason carefully about initialization

```
# s is a string;  len(s)  >= 1
# Set c to largest  element   in s
c = ??
k = ??
# inv: c is largest element in s[0..k–1]
while k < len(s):
    # Process k
    k = k+1
#   c = largest char in s[0..len(s)–1]
```

Command to do something

Equivalent postcondition

1. What is the invariant?
2. How do we initialize c and k?

A: k = 0; c = s[0]
B: k = 1; c = s[0]
C: k = 1; c = s[1]
D: k = 0; c = s[1]
E: None of the above