Lecture 26

Sorting

Announcements for This Lecture

Prelim/Finals

- Prelims in **handback room**
  - Gates Hall 216
  - Open “business hours”
  - Get them any day this week
- **Final: Dec 17th 2:00-4:30pm**
  - Study guide by end of week
- **Conflict with Final time?**
  - Submit to Final Conflict assignment on CMS
  - **Must be in by December 10th**

Assignments/Lab

- **A6** will be graded by Thurs.
  - Will give grade breakdown
  - Will review survey too
- **A7** is due next **Wednesday**
  - One week left
  - Keep up with deadlines
- **Lab 13** is *optional*
  - Good study for the final
  - Consultant hours still open

Linear Search

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Linear Search

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- **Better**: Store an integer in i to truthify result condition post:

  post: 1. v is not in b[h..i-1]

  2. i = k OR v = b[i]

Linear Search

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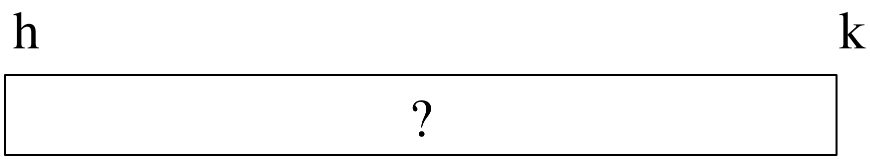
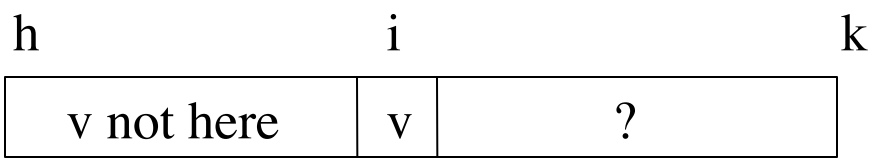
- **Better**: Store an integer in i to truthify result condition post:

post: 1. v is not in b[h..i-1]

2. i = k OR v = b[i]

pre: b


post: b


Linear Search

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- **Better**: Store an integer in i to truthify result condition post:

post: 1. v is not in b[h..i-1]

2. i = k OR v = b[i]

	h			k
pre: b	?			

	h	i		k
post: b	v not here	v	?	

OR

	h		k, i
b	v not here		

Linear Search

h k

pre: b | ? |

h i k

post: b | v not here | v | ? |

OR

i

h k

b | v not here |

h i k

inv: b | v not here | ? |

Linear Search

```
def linear_search(b,c,h):
    """Returns: first occurrence of c in b[h..]"""
    # Store in i the index of the first c in b[h..]
    i = h

    # invariant: c is not in b[0..i-1]
    while i < len(b) and b[i] != c:
        i = i + 1

    # post: c is not in b[h..i-1]
    #       i >= len(b) or b[i] == c
    return i if i < len(b) else -1
```

Analyzing the Loop

1. Does the initialization make **inv** true?
2. Is **post** true when **inv** is true and **condition** is false?
3. Does the repetend make progress?
4. Does the repetend keep the invariant **inv** true?

Binary Search

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Binary Search

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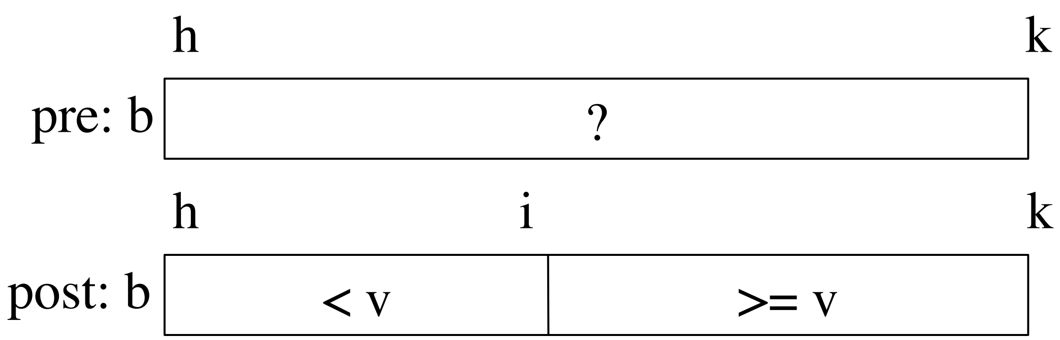
- **Better:**
  - Precondition: b[h..k-1] is sorted (in ascending order).
  - Postcondition: b[h..i] < v and v <= b[i+1..k-1]
- Below, the array is in non-descending order:



Binary Search

- Look for value v in **sorted** segment b[h..k]

	h	k
pre: b	?	

	h	i	k
post: b	< v	>= v	

	h	i	j	k
inv: b	< v	?	>= v	

New statement of the invariant guarantees that we get leftmost position of v if found

	h									k
	0	1	2	3	4	5	6	7	8	9
Example b	3	3	3	3	3	4	4	6	7	7

- if v is 3, set i to 0
- if v is 4, set i to 5
- if v is 5, set i to 7
- if v is 8, set i to 10

Binary Search

- **Vague:** Look for v in **sorted** sequence segment b[h..k].
- **Better:**
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  - Postcondition: b[h..i] <= v and v < b[i+1..k-1]
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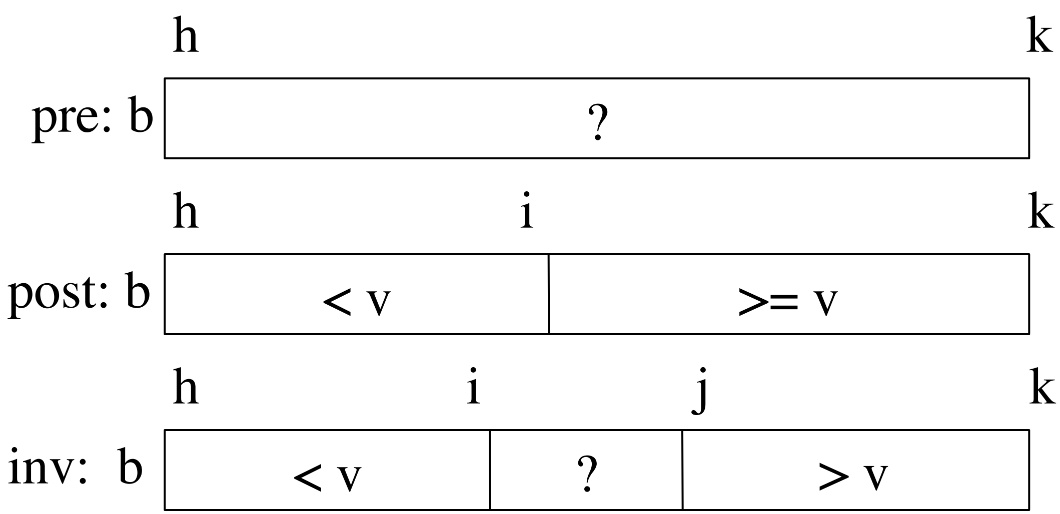


Called binary search because each iteration of the loop cuts the array segment still to be processed in half

Binary Search

		h			k
pre:	b	?			

		h	i		k
post:	b	< v	>= v		

		h	i	j	k
inv:	b	< v	?	>= v	

New statement of the invariant guarantees that we get leftmost position of v if found

```
i = h; j = k+1;
while i != j:
```

Looking at b[i] gives linear search from left.

Looking at b[j-1] gives linear search from right.

Looking at middle: b[(i+j)/2] gives binary search.

Sorting: Arranging in Ascending Order

	0	n
pre: b	?	

	0	n
post: b	sorted	

Insertion Sort:

	0	i	n
inv: b	sorted	?	

```
i = 0
while i < n:
    # Push b[i] down into its
    # sorted position in b[0..i]
    i = i+1
```

0						i
2	4	4	6	6	7	5

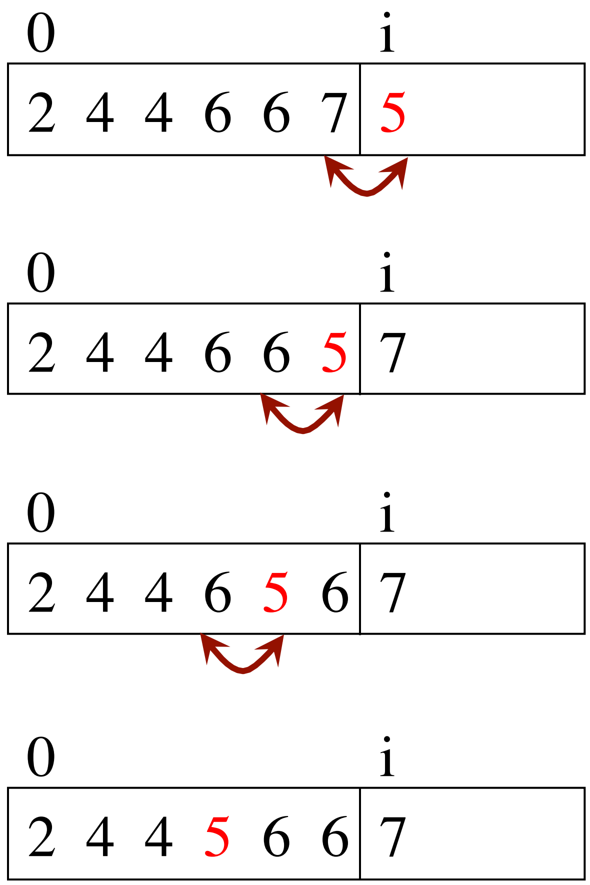
0						i
2	4	4	5	6	6	7

Insertion Sort: Moving into Position

```
i = 0
while i < n:
    push_down(b,i)
    i = i+1

def push_down(b, i):
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

swap shown in the lecture about lists



The Importance of Helper Functions

```
i = 0
while i < n:
    push_down(b,i)
    i = i+1

def push_down(b, i):
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

VS

Can you understand all this code below?

```
i = 0
while i < n:
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            temp = b[j]
            b[j] = b[j-1]
            b[j-1] = temp
        j = j -1
    i = i +1
```

Insertion Sort: Performance

```
def push_down(b, i):
    """Push value at position i into
    sorted position in b[0..i-1]"""
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

- b[0..i-1]: i elements
- Worst case:
  - i = 0: 0 swaps
  - i = 1: 1 swap
  - i = 2: 2 swaps
- Pushdown is in a loop
  - Called for i in 0..n
  - i swaps each time

Insertion sort is an n^2 algorithm

Total Swaps: 0 + 1 + 2 + 3 + … (n-1) = (n-1)*n/2

Algorithm "Complexity"

- **Given**: a list of length n and a problem to solve
- **Complexity**: *rough* number of steps to solve worst case
- Suppose we can compute 1000 operations a second:

Complexity	n=10	n=100	n=1000
n	0.01 s	0.1 s	1 s
n log n	0.016 s	0.32 s	4.79 s
n^2	0.1 s	10 s	16.7 m
n^3	1 s	16.7 m	11.6 d
2^n	1 s	4×10^{19} y	3×10^{290} y

Major Topic in 2110: Beyond scope of this course

Sorting: Changing the Invariant

	0	n
pre: b	?	

	0	n
post: b	sorted	

Selection Sort:

	0	i	n
inv: b	sorted, ≤ b[i..]	≥ b[0..i-1]	

First segment always contains smaller values

```
i = 0
while i < n:
    # Find minimum in b[i..]
    # Move it to position i
    i = i+1
```

					i						n
2	4	4	6	6	8	9	9	7	8	9	

					i						n
2	4	4	6	6	7	9	9	8	8	9	

						i					n
2	4	4	6	6	7	9	9	8	8	9	

Sorting: Changing the Invariant

	0	n
pre: b	?	

	0	n
post: b	sorted	

Selection Sort:

	0	i	n
inv: b	sorted, ≤ b[i..]	≥ b[0..i-1]	

First segment always contains smaller values

```
i = 0
while i < n:
    j = index of min of b[i..n-1]
    swap(b,i,j)
    i = i+1
```

					i						n
2	4	4	6	6	8	9	9	7	8	9	

					i						n
2	4	4	6	6	7	9	9	8	8	9	

Selection sort also is an n^2 algorithm

Partition Algorithm

- Given a list segment b[h..k] with some value x in b[h]:

	h		k
pre: b	**x**	?	

- Swap elements of b[h..k] and store in j to truthify post:

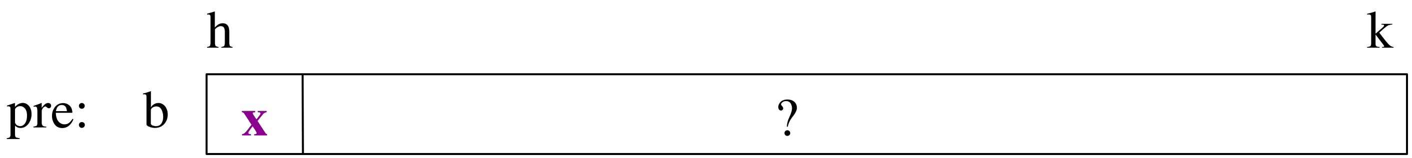
	h	i	i+1	k
post: b	<= **x**	**x**	>= **x**	

		h								k
change:	b	**3**	5	4	1	6	2	3	8	1
		h			i					k
into	b	1	2	1	**3**	5	4	6	3	8
		h				i				k
or	b	1	2	3	1	**3**	4	5	6	8

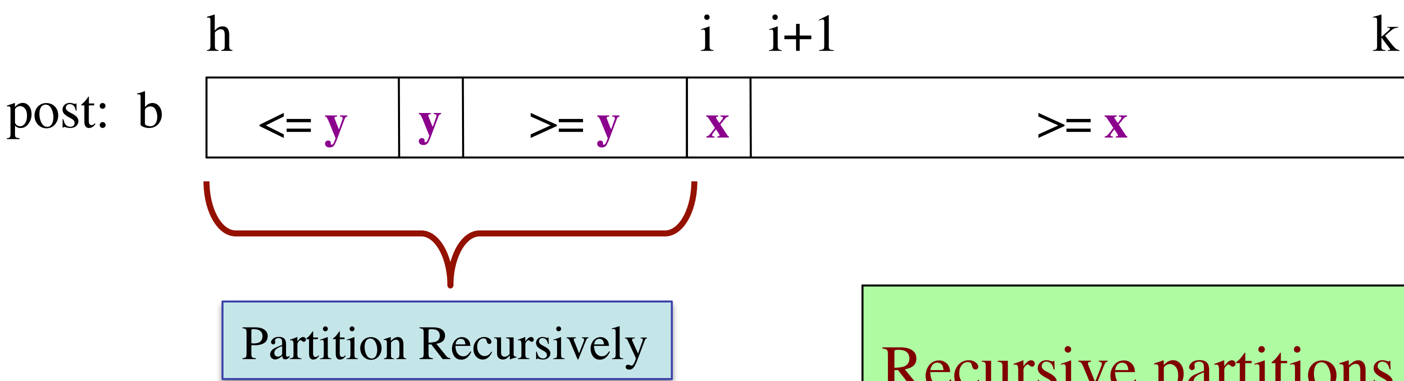
- x is called the pivot value
  - x is not a program variable
  - denotes value initially in b[h]

Sorting with Partitions

- Given a list segment b[h..k] with some value x in b[h]:



- Swap elements of b[h..k] and store in j to truthify post:



Recursive partitions = sorting

- Called **QuickSort** (why???)
- Popular, fast sorting technique

QuickSort

```
def quick_sort(b, h, k):
    """Sort the array fragment b[h..k]"""
    if b[h..k] has fewer than 2 elements:
        return
    j = partition(b, h, k)
    # b[h..j–1] <= b[j] <= b[j+1..k]
    # Sort b[h..j–1] and b[j+1..k]
    quick_sort (b, h, j–1)
    quick_sort (b, j+1, k)
```

- **Worst Case:** array already sorted
  - Or almost sorted
  - n^2 in that case
- **Average Case:** array is scrambled
  - n log n in that case
  - Best sorting time!

pre: b

h		k
x	?	

post: b

h	i	i+1 k
<= x	x	>= x

Final Word About Algorithms

- **Algorithm:**
  - Step-by-step way to do something
  - Not tied to specific language
- **Implementation:**
  - An algorithm in a specific language
  - Many times, not the "hard part"
- Higher Level Computer Science courses:
  - We teach advanced algorithms (pictures)
  - Implementation you learn on your own

List Diagrams

Demo Code