CS1110

Lecture 20: Sequence algorithms

Announcements

Upcoming schedule

Today (April 4) A6 out: A4 due **tomorrow**. Fix to memotable printing posted; see Piazza @303.

Tu Apr 9: lecture on searching &sorting – last material on the exam

Probably a new lab exercise, for prelim exercise

Th Apr 11: lecture = review session

Sat Apr 13: A6 due (yes, we *cancelled* A5!).

Tu Apr 16: lecture = office hours, in Thurston 102

Exam that evening, same location as before

Probably no new lab exercise that week

Slides by D. Gries, L. Lee, S. Marschner, W. White

Sorting: A Key Algorithmic Family

Q: Given a list of items, how can we arrange for them to be sorted in increasing order, in a time- and space-efficient manner?

Applications: making items easier to find.[1]

```
def sort(b, h, k):
    ""Sort  b[h..k] in place.  Pre: b: list of ints; k>=h-1"""
    # Start with b[h], and organize the rest according to it??
    # Note: we have h & k explicit to simplify recursive
    # structure.
```

[1] Also, computing poker-hand scores.

Motivation: A Famous Sorting Function

```
def qsort(b, h, k):
    """Make b[h..k] sorted.
    Pre: b: list of ints; k>=h-1"""
```

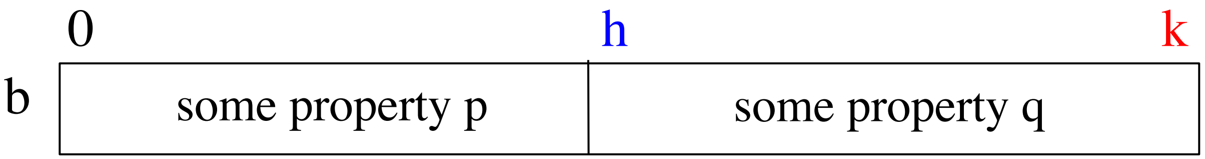
Clicker Q2: base case

```
    i = partition(b, h, k)
```

Clicker Q1: recursive case

```
def partition(b, h, k):
    """Let x = b[h] be the pivot
value.  Rearrange b[h..k] so
that there is an i where
b[h..i-1] <= x, b[i]=x; b[i+1..k]
>=x.  Return i.
Pre: k>=h"""
# Can you do this without
# creating extra lists?
```

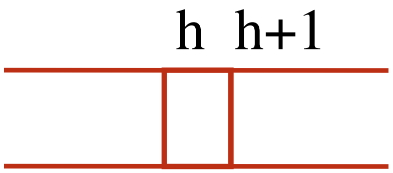
Pictorial Notation for Sequence Assertions



Equivalent to:

Property p holds on all items in b[0..h-1], and property q holds on all items in b[h..k].

(The precise location of the "vertical bars" matters.)

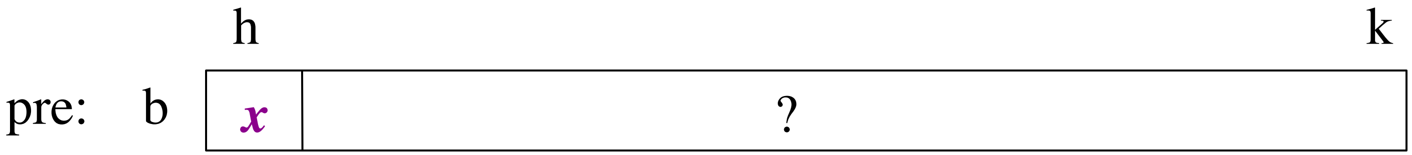


Can also indicate single items.

((h +1) – h = 1; it's all consistent, hurrah.)

Partition Algorithm

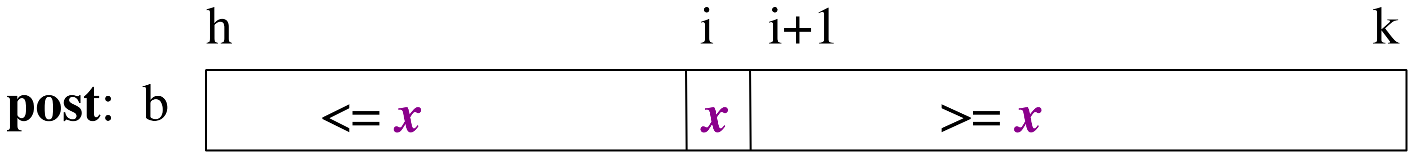
- Given a sequence b[h..k] with some *value* x in b[h]:



h k

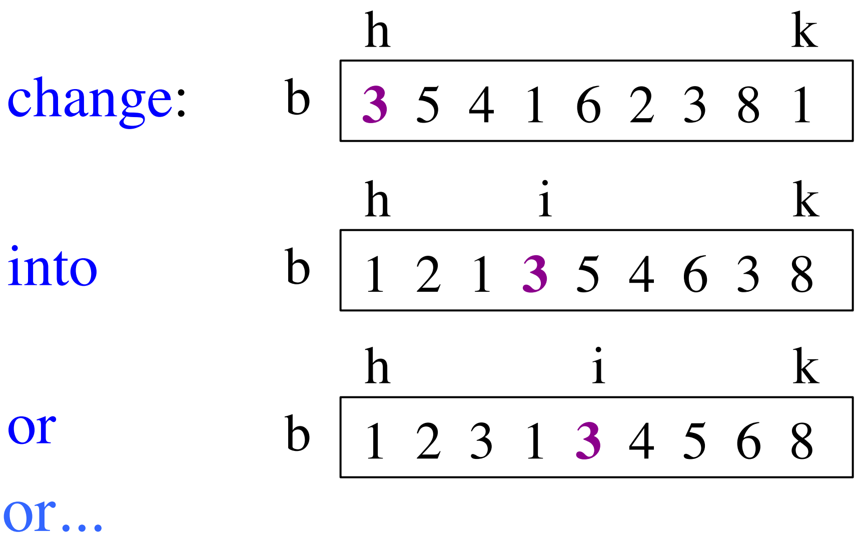
pre: b | *x* | ?

- Swap elements of b[h..k] and store in i to truthify postcondition:



h i i+1 k

post: b | <= *x* | *x* | >= *x*



change: b

h k

3 5 4 1 6 2 3 8 1

into b

h i k

1 2 1 3 5 4 6 3 8

or b

h i k

1 2 3 1 3 4 5 6 8

or…

- *x* is called the pivot value
  - *x* is not a program variable, but a standin for a number: value initially in b[h]

Motivation: A Famous Sorting Function

```
def qsort(b, h, k):
    """Make b[h..k] sorted.
    Pre: b: list of ints; k>=h-1"""
```

Clicker Q2: base case

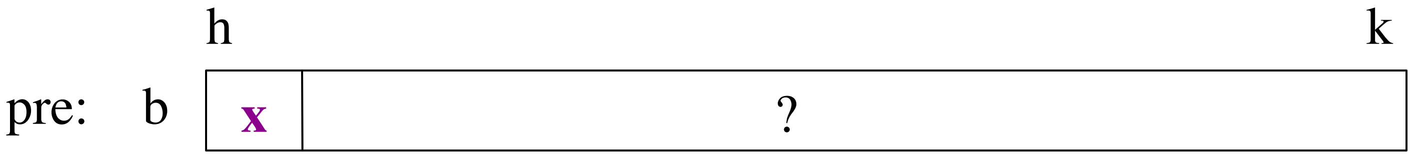
```
    if k < h:  # empty is sorted
        return
    i = partition(b, h, k)
```

Clicker Q1: recursive case

```
def partition(b, h, k):
    """Let x = b[h] be the pivot
value.  Rearrange b[h..k] so
that there is an i where
b[h..i-1] <= x, b[i]=x; b[i+1..k]
>=x.  Return i.
Pre: k>=h"""
# Can you do this in place,
# i.e., w/out
# creating extra lists?
```

An Invariant to Guide Our Thinking

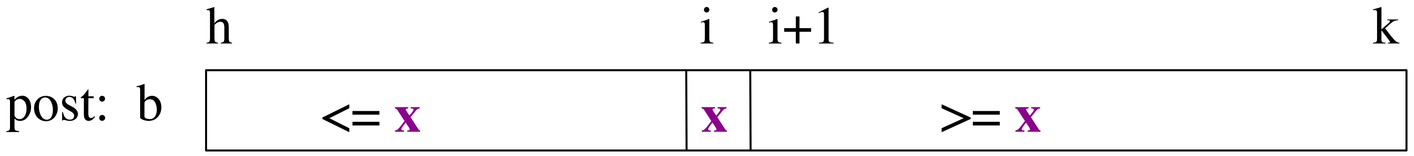
- Given a sequence b[h..k] with some value x in b[h]:



h k

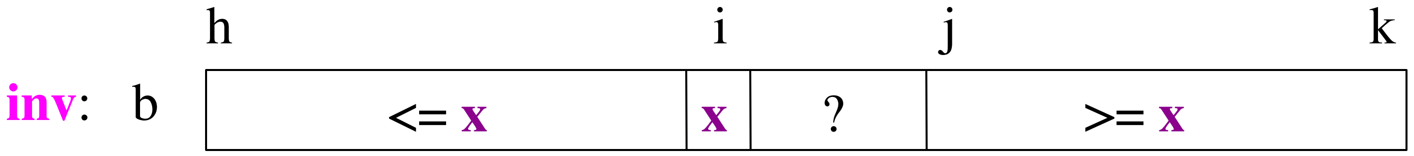
pre: b | x | ?

- Swap elements of b[h..k] and store in i to truthify post:



h i i+1 k

post: b | <= x | x | >= x



h i j k

inv: b | <= x | x | ? | >= x

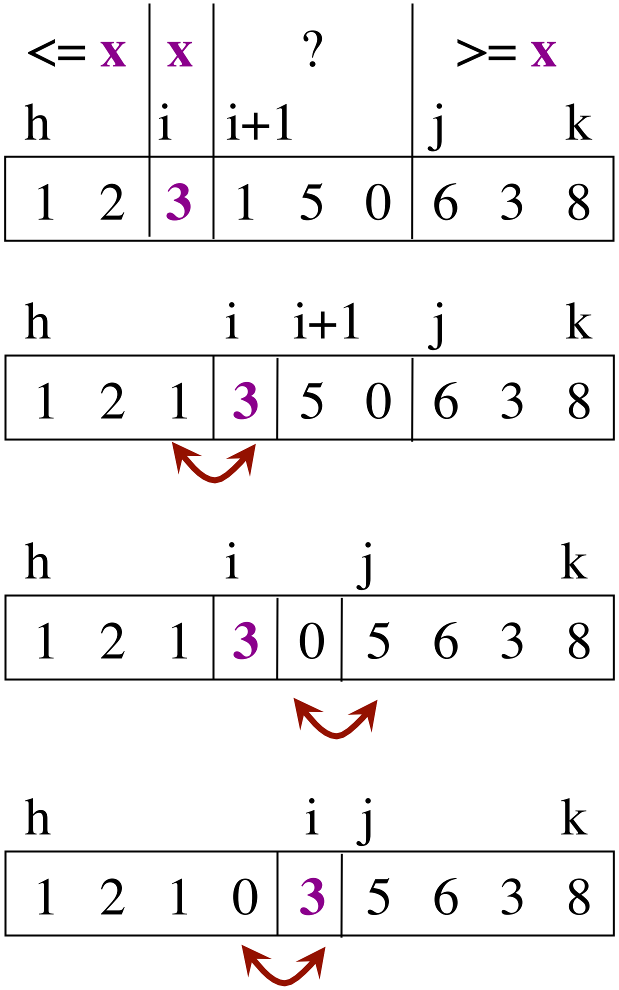
- Agrees with precondition when i = h, j = k+1
- Agrees with postcondition when j = i+1

Partition Algorithm Implementation

```
def partition(b, h, k):
    """Partition list b[h..k] around a pivot x = b[h];
    Return index of pivot point. ~~Assume a swap function _swap(b,ind1, ind2).~~
   Pre: k>=h"""
   CLICKER Q5
    # invariant: b[h..i-1] < x, b[i] = x, b[j..k] >= x, b[i+1..j-1] unknown
    while CLICKER Q4
          if b[i+1] >= x:
           # Move to end of block.
           b[i+1], b[j-1] = b[j-1], b[i+1]
           j = j - 1
       else:   # b[i+1] < x
          CLICKER Q3
    # post: b[h..i-1] < x, b[i] is x, and b[i+1..k] >= x
    return i
```

Partition Algorithm Implementation

```
def partition(b, h, k):
    """Partition list b[h..k] around a pivot x = b[h]"""
    i = h; j = k+1; x = b[h]
    # invariant: b[h..i-1] < x, b[i] = x, b[j..k] >= x
    while i < j-1:
        if b[i+1] >= x:
            # Move to end of block.
            b[i+1], b[j-1] = b[j-1], b[i+1]
            j = j - 1
        else:   # b[i+1] < x
            b[i], b[i+1] = b[i+1], b[i]
            i = i + 1
    # post: b[h..i-1] < x, b[i] is x, and b[i+1..k] >= x
    return i
```



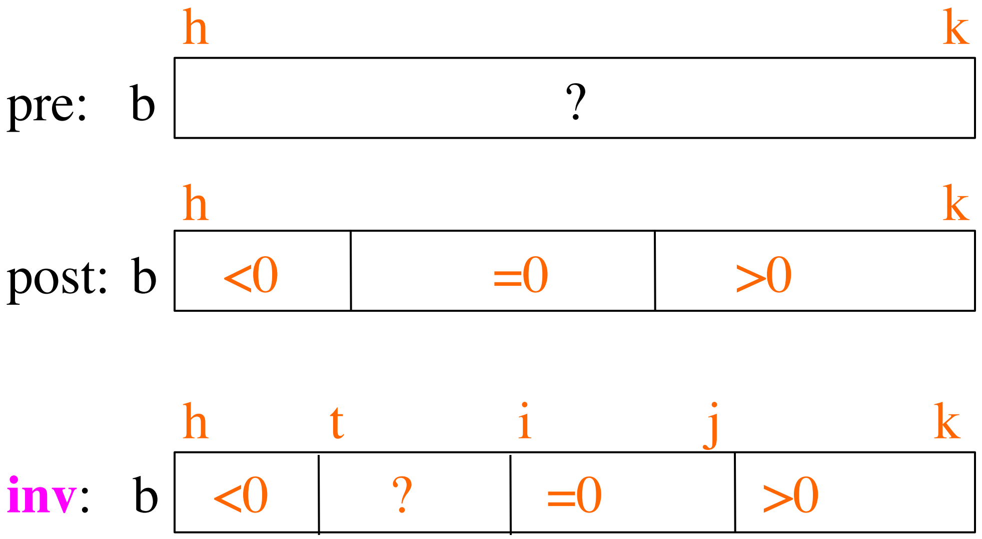
```
<= x | x | ?       | >= x
h      i   i+1       j      k
1  2 | 3 | 1  5  0 | 6  3  8

h            i   i+1   j      k
1  2  1 | 3 | 5  0 | 6  3  8

h            i      j             k
1  2  1 | 3 | 0 | 5  6  3  8

h                i   j             k
1  2  1  0 | 3 | 5  6  3  8
```

Developing Algorithms on Sequences

- Specify the algorithm by giving its precondition and postcondition as pictures.
- Draw the invariant by drawing another picture that "generalizes" the precondition and postcondition
  - The invariant is true at the beginning and at the end
- The four loop design questions (memorize them)
  1. How does loop start (how to make the invariant true)?
  2. How does it stop (is the postcondition true)?
  3. How does repetend make progress toward termination?
  4. How does repetend keep the invariant true?

Famous "Sort-Like" Example

- Dutch national flag: tri-color
  - Sequence of h..k of red (<0), white (=0), blue (>0) "pixels"
  - Arrange to put <0 first, then =0 , then >0, return "split pts"



h k

pre: b ?

(values in h..k are unknown)

h k

post: b <0 =0 >0

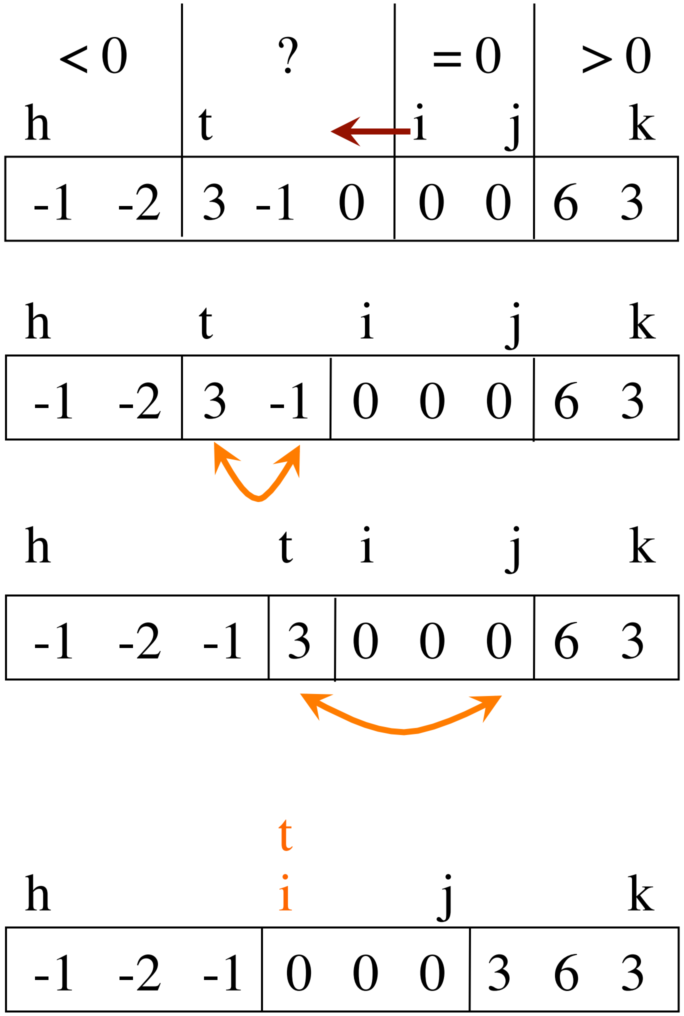
h t i j k

inv: b <0 ? =0 >0

b[h..t-1] <0, b[t..i-1] unknown, b[i..j] =0, b[j+1..k] >0

Dutch National Flag Algorithm

```
def dnf(b, h, k):
    """(DNF explanation omitted for space.)
    Returns: split-points as a tuple (i,j)"""
   # init?
    # inv: b[h..t-1] < 0, b[t..i-1] ?, b[i..j] = 0, b[j+1..k] > 0
    while t < i:
        if b[i-1] < 0:
           # what?

        elif b[i-1] == 0:
           # what?
        else:
           # what?

    # post: b[h..i-1] < 0, b[i..j] = 0, b[j+1..k] > 0
    return (i, j)
```



< 0 ? = 0 > 0

h t i j k

-1 -2 | 3 -1 0 | 0 0 | 6 3

h t i j k

-1 -2 | 3 -1 | 0 0 0 | 6 3

h t i j k

-1 -2 -1 | 3 | 0 0 0 | 6 3

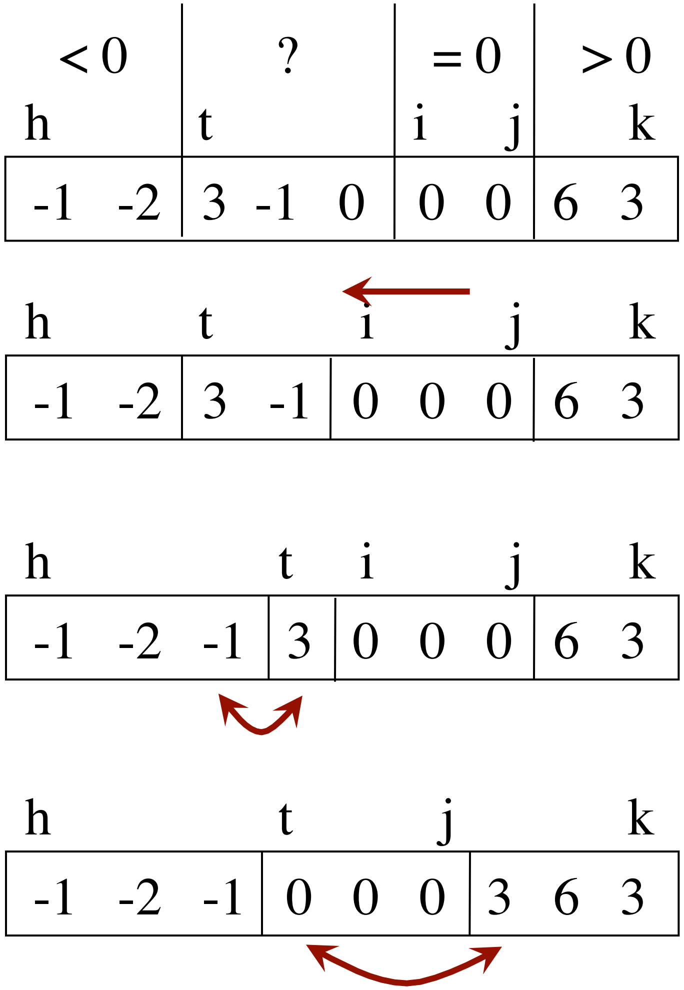
t

h i j k

-1 -2 -1 | 0 0 0 | 3 6 3

Dutch National Flag Algorithm

```
def dnf(b, h, k):
    """Returns: partition points as a tuple (i,j)"""
    t = h; i = k+1, j = k;
    # inv: b[h..t-1] < 0, b[t..i-1] ?, b[i..j] = 0, b[j+1..k] > 0
    while t < i:
        if b[i-1] < 0:
            b[i-1], b[t] = b[t], b[i-1]
            t = t+1
        elif b[i-1] == 0:
            i = i-1
        else:
            b[-1], b[j] = b[j], b[i-1
            i = i-1; j = j-1
    # post: b[h..i-1] < 0, b[i..j] = 0, b[j+1..k] > 0
    return (i, j)
```



< 0 | ? | = 0 | > 0

h t i j k

-1 -2 | 3 -1 0 | 0 0 | 6 3

h t i j k

-1 -2 | 3 -1 | 0 0 0 | 6 3

h t i j k

-1 -2 -1 | 3 | 0 0 0 | 6 3

h t j k

-1 -2 -1 | 0 0 0 | 3 6 3