# CS1110

Lecture 12: **Recursion, again**

Announcements

**Prelim preparation**

Study suggestion: be able to re-do labs 2-5 and A1, A2, A3 on paper without much hesitation.

For help on A2: try using the Online Python Tutor.

Fall 2012 prelim 1 and review material will be posted on the exams page.

**Organization suggestion**

Get a three-ring binder and a 3-hole punch. Use these oldie-but-goodie technologies to store your CS1110 handouts 'n stuff.

Slides by D. Gries, L. Lee, S. Marschner, W. White

Announcements

**Many recursion examples are on the lectures page**

These were authored by Prof. Walker White last semester.

- comments in braces are *assertions*: conditions assumed to hold if that line is reached. Example:
  `# {s is empty}`
- We are not currently emphasizing the use of assert statements to enforce preconditions, but they can be quite useful to catch bugs involving accidental precondition violation. Example:
  `assert type(s) == str, `s` + ' is not a string'`
  (backquotes give unambiguous string representation)

Slides by D. Gries, L. Lee, S. Marschner, W. White

## Reminder: our running example

```
def num_es(s):
    """Returns: number of 'e's in <s>. Precond: <s> a string"""
    # Strategy: break off first character, recur on the rest.
1   if s == '':  # base case (no recursion): <s> is empty string
2       return 0

    # recursive case: <s> has at least one char
    # note: s[1:] is '' if len(s) <= 1
3   return ((1 if s[0] == 'e' else 0) + num_es(s[1:]))
```

Let's understand what happens at execution.

## Execution in "typical" recursion case

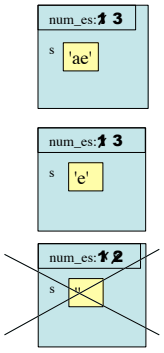
**inside module lec 12**

```
def num_es(s):
1   if s == '':
2       return 0
3   return ((1 if s[0] == 'e' else 0) + num_es(s[1:]))
```

**code with function call**

```
import lec12
print ex.num_es('ae')
```

import creates the function objects that are defined in lec12, like num_es, so we can call them.




## What if we didn't have a base case?

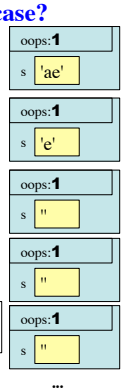
**hypothetical function definition**

```
def oops(s):
1   return ((1 if s[0] == 'e' else 0) + oops (s[1:]))
```

**code with function call**

```
print oops('ae')
```

[lots of copies of the same message]
RuntimeError: maximum recursion depth exceeded




## What if we didn't recur on a "smaller" value?

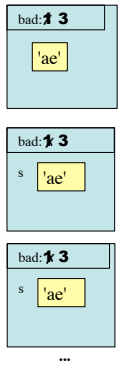
**Hypothetical function definition**

```
def bad(s):
1   if s == '':
2       return 0
3   return ((1 if s[0] == 'e' else 0) + bad(s))
```

**code with function call**

```
print bad('ae')
```

[lots of copies of the same message]
RuntimeError: maximum recursion depth exceeded



## Alternate implementation

```
def num_es2(s):
    """Returns: number of 'e's in <s>. Precond: <s> a string"""
    # Strategy: break into two smaller strings, recur on both.

    # base case: cannot break into two smaller strings
```

| (A) if s == '': | (B) if len(s) == 1: | (C) if len(s) <= 1: | (D) if len(s) <= 2: |
|---|---|---|---|
| ... | ... | ... | ... |

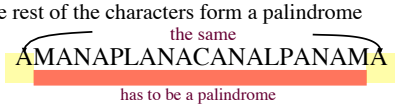
```
    # recursive case: choose a random breakpoint
    i = random integer between 1 and len(s)-1, inclusive
    # return: num of e's from 0 to up to but not including
    # i, plus num of e's from i to the end of the string
```

## How to Think About Recursive Functions

1. **Have a precise function specification.**
   - Test cases generally handy here
2. **Recursive case(s):**
   - Verify recursive cases *with the specification*
3. **Reduction:**
   - Arguments of calls must somehow get "smaller", so each recursive call gets closer to a base case
4. **Base case(s):**
   - When the recursive case doesn't apply
   - When the argument values are as "small" as possible
   - When the answer is determined with little calculation.

## Example: Palindromes

- String with ≥ 2 characters is a *palindrome* if:
  - its first and last characters are the same, and
  - the rest of the characters form a palindrome




- All strings with fewer than 2 characters are palindromes

Practical application: RNA secondary structure: loops form because of "antepalindromes" (G/C and A/U)

## Example: Palindromes

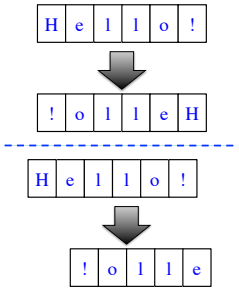
```
def ispalindrome(s):
    """Returns: True if string s is a palindrome, False otherwise"""
    # base case




    # recursive case
```

## Example: Reversing a String

- **Precise Specification**:
  - Returns: reverse of s
- Solving with recursion
  - Suppose we can reverse a smaller string (e.g., one fewer character)
  - Can we use that solution to reverse whole string?




## Example: Reversing a String

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # {s is empty}
    if s == '':
        return s

    # { s at least one char }
    # (reverse of s[1:])+s[0]
    return reverse(s[1:])+s[0]
```

```
def reverse2(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # base case



    # last char + reverse of s up to it
```