Lecture 25

Sorting

Announcements for This Lecture

Prelim/Finals

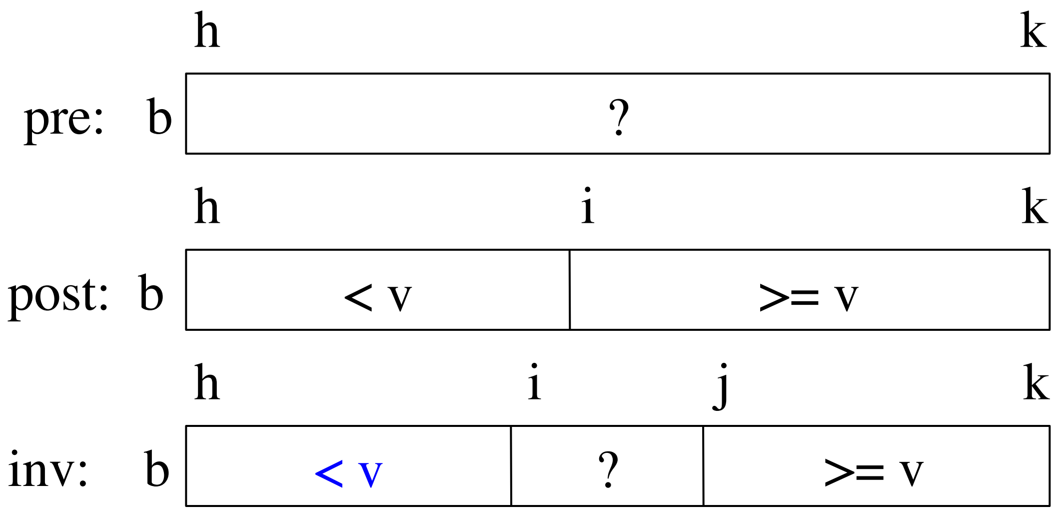
- Prelims in **handback room**
  - Upson Hall 305
  - Open "business hours"
  - Get them any day this week
- **Final: Dec 16th 9:00-11:30a**
  - Study guide by end of week
- **Conflict with Final time?**
  - Submit to Final Conflict assignment on CMS
  - **Must be in by December 9th**

Assignments/Lab

- **A5** will be graded by Thurs.
  - Will give grade breakdown
  - Will review survey too
- **A6** is due next **Monday**
  - One week left
  - Keep up with deadlines
- **Lab 12** is the last lab
  - Due before final exam
  - Consultant hours still open

Binary Search

- Look for value v in **sorted** segment b[h..k]

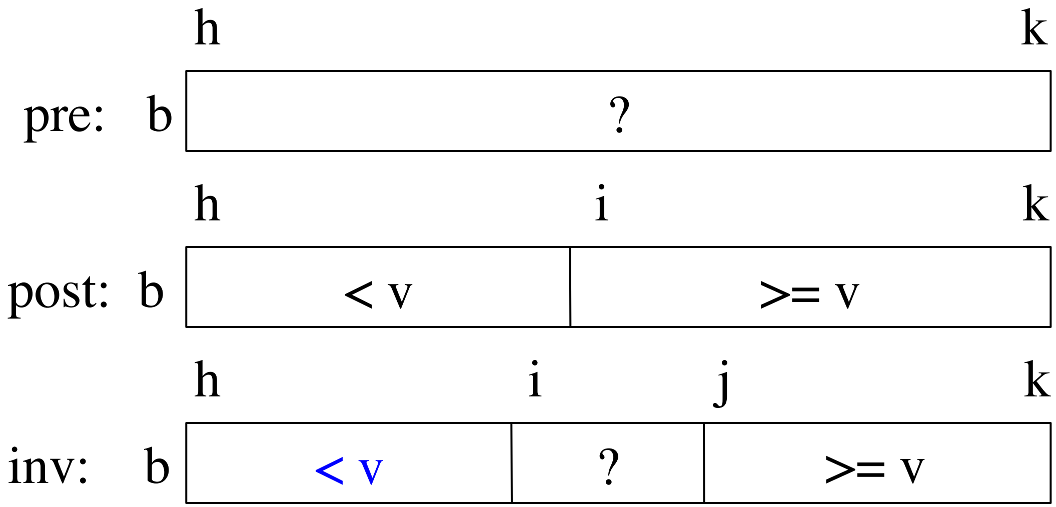


New statement of the invariant guarantees that we get leftmost position of v if found

	h									k
	0	1	2	3	4	5	6	7	8	9
Example b	3	3	3	3	3	4	4	6	7	7

- if v is 3, set i to 0
- if v is 4, set i to 5
- if v is 5, set i to 7
- if v is 8, set i to 10

Binary Search



New statement of the invariant guarantees that we get leftmost position of v if found

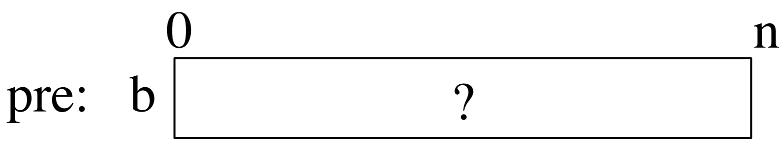
```
i = h; j = k+1;
while i != j:
```

Looking at b[i] gives linear search from left.

Looking at b[j-1] gives linear search from right.

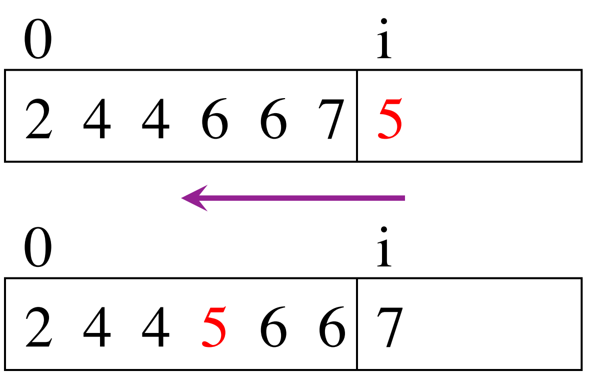
Looking at middle: b[(i+j)/2] gives binary search.

Sorting: Arranging in Ascending Order



Insertion Sort:


```
i = 0
while i < n:
    # Push b[i] down into its
    # sorted position in b[0..i]
    i = i+1
```

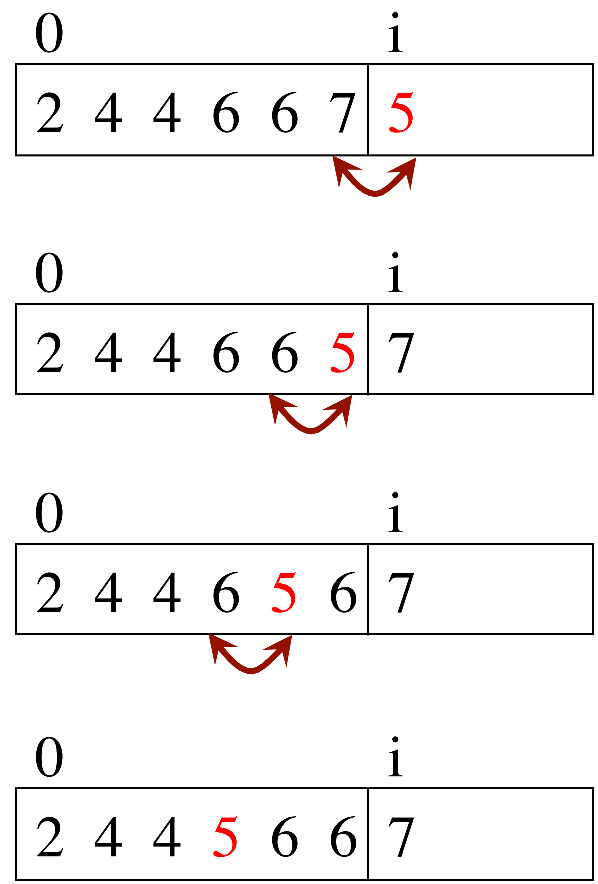


Insertion Sort: Moving into Position

```
i = 0
while i < n:
    push_down(b,i)
    i = i+1

def push_down(b, i):
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

swap shown in the lecture about lists



The Importance of Helper Functions

```
i = 0
while i < n:
    push_down(b,i)
    i = i+1

def push_down(b, i):
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

VS

Can you understand all this code below?

```
i = 0
while i < n:
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            temp = b[j]
            b[j] = b[j-1]
            b[j-1] = temp
        j = j -1
    i = i +1
```

Insertion Sort: Performance

```
def push_down(b, i):
    """Push value at position i into
    sorted position in b[0..i-1]"""
    j = i
    while j > 0:
        if b[j-1] > b[j]:
            swap(b,j-1,j)
        j = j-1
```

- b[0..i-1]: i elements
- Worst case:
  - i = 0: 0 swaps
  - i = 1: 1 swap
  - i = 2: 2 swaps
- Pushdown is in a loop
  - Called for i in 0..n
  - i swaps each time

Insertion sort is an n^2 algorithm

Total Swaps: 0 + 1 + 2 + 3 + … (n-1) = (n-1)*n/2

Algorithm "Complexity"

- **Given**: a list of length n and a problem to solve
- **Complexity**: *rough* number of steps to solve worst case
- Suppose we can compute 1000 operations a second:

Complexity	n=10	n=100	n=1000
n	0.01 s	0.1 s	1 s
n log n	0.016 s	0.32 s	4.79 s
n^2	0.1 s	10 s	16.7 m
n^3	1 s	16.7 m	11.6 d
2^n	1 s	4×10^{19} y	3×10^{290} y

Major Topic in 2110: Beyond scope of this course

Sorting: Changing the Invariant

pre: b [0 .. n) : ?

post: b [0 .. n) : sorted

Selection Sort:

inv: b [0 .. i) : sorted, ≤ b[i..] ; [i .. n) : ≥ b[0..i-1]

First segment always contains smaller values

```
i = 0
while i < n:
    # Find minimum in b[i..]
    # Move it to position i
    i = i+1
```

b[0..i-1]	b[i..n-1]
2 4 4 6 6	8 9 9 7 8 9

→

b[0..i-1]	b[i..n-1]
2 4 4 6 6	7 9 9 8 8 9

b[0..i-1]	b[i..n-1]
2 4 4 6 6 7	9 9 8 8 9

Sorting: Changing the Invariant

	0	n
pre: b	?	

	0	n
post: b	sorted	

Selection Sort:

	0	i	n
inv: b	sorted, ≤ b[i..]	≥ b[0..i-1]	

First segment always contains smaller values

```
i = 0
while i < n:
    j = index of min of b[i..n-1]
    swap(b,i,j)
    i = i+1
```

					i					n
2	4	4	6	6	8	9	9	7	8	9

					i					n
2	4	4	6	6	7	9	9	8	8	9

Selection sort also is an n^2 algorithm

Partition Algorithm

- Given a list segment b[h..k] with some value x in b[h]:

pre: b

h		k
x	?	

- Swap elements of b[h..k] and store in j to truthify post:

post: b

h	i	i+1	k
<= **x**	**x**	>= **x**	

change: b

h								k
3	5	4	1	6	2	3	8	1

into b

h			i					k
1	2	1	**3**	5	4	6	3	8

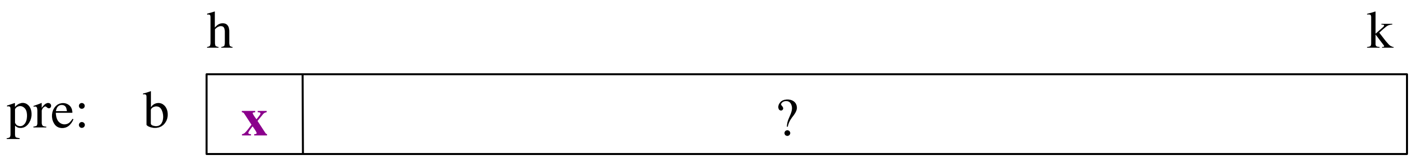
or b

h				i				k
1	2	3	1	**3**	4	5	6	8

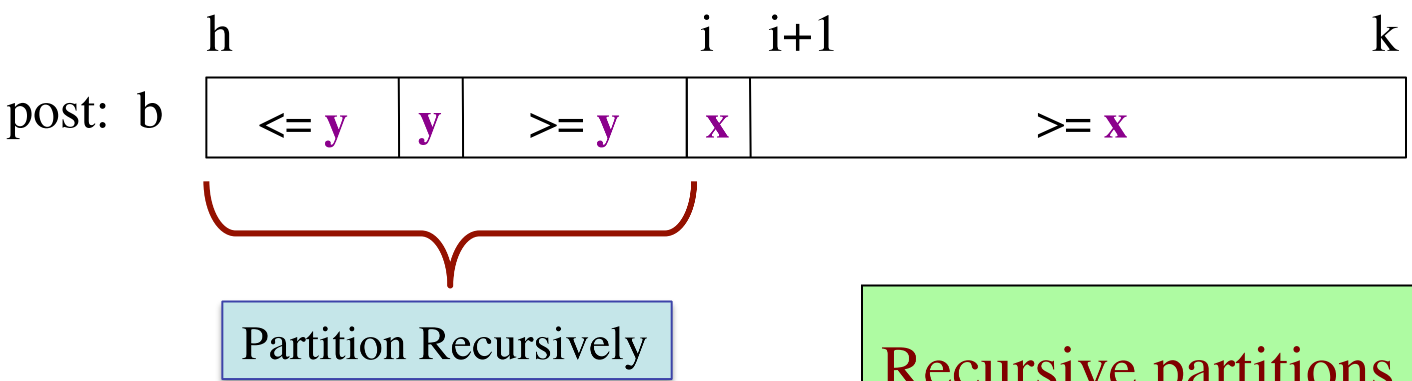
- x is called the pivot value
  - x is not a program variable
  - denotes value initially in b[h]

Sorting with Partitions

- Given a list segment b[h..k] with some value x in b[h]:



- Swap elements of b[h..k] and store in j to truthify post:



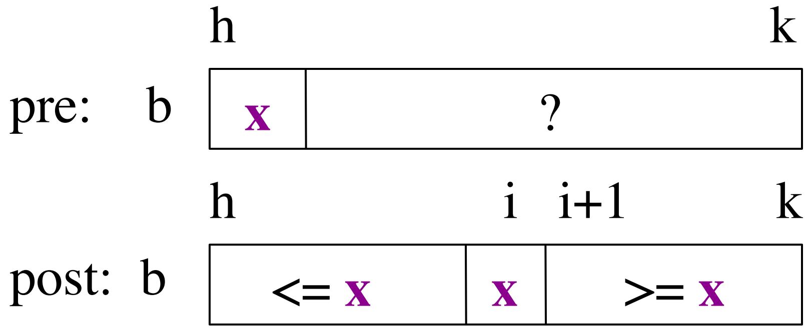
Recursive partitions = sorting

- Called **QuickSort** (why???)
- Popular, fast sorting technique

QuickSort

```
def quick_sort(b, h, k):
    """Sort the array fragment b[h..k]"""
    if  b[h..k] has fewer than 2 elements:
        return
    j = partition(b, h, k)
    # b[h..j–1] <= b[j] <= b[j+1..k]
    # Sort b[h..j–1]  and  b[j+1..k]
    quick_sort (b, h, j–1)
    quick_sort (b, j+1, k)
```

- **Worst Case:** array already sorted
  - Or almost sorted
  - n^2 in that case
- **Average Case:** array is scrambled
  - n log n in that case
  - Best sorting time!



Final Word About Algorithms

- **Algorithm:**
  - Step-by-step way to do something
  - Not tied to specific language
- **Implementation:**
  - An algorithm in a specific language
  - Many times, not the "hard part"
- Higher Level Computer Science courses:
  - We teach advanced algorithms (pictures)
  - Implementation you learn on your own

List Diagrams

Demo Code