Announcements for This Lecture

Recursion

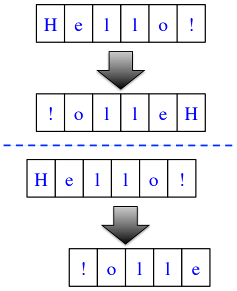
- Read: 15.1, p. 415
- PLive, activity 15-2.1
- Work on many exercises
  - Today's (& Wed) lab
- Remember you need
  - Good function specification
  - Base case(s) are correct
  - Progress toward termination
  - Recursive case(s) are correct

Prelim 1

- Thursday 7:30-9pm
  - Abel–Price (Upson B17)
  - Rabbit–Teo (Upson 111)
  - Ting–Zytariuk (Upson 109)
- Graded late Thursday
  - Will have grade Fri morn
  - In time for drop day
- Make-up, Friday 4:30
  - For preapproved students

Example: Reversing a String

- **Precise Specification**:
  - Yield: reverse of String s
- Solving with recursion
  - Suppose we could reverse a smaller string (e.g. less one character)
  - Can we use that solution to reverse whole string?
- Often easy to understand first without Java
  - Then sit down and code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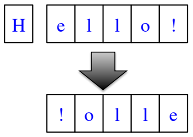
Example: Reversing a String

```
/** Yields: reverse of string s */
public static String reverse(String s) {
  if (s.length() == 0) {
    return s;
  }

  // {s is not empty}
  // (reverse of s[1..])+s[0]
  return reverse(s.substring(1)) +
           s.charAt(0);
}
```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✔ 1. Precise specification?
✔ 2. Base case: correct?
✔ 3. Recursive case: progress to termination?
✔ 4. Recursive case: correct?

Example: Palindromes

- String with ≥ 2 characters is a palindrome if:
  - its first and last characters are equal, and
  - the rest of the characters form a palindrome
- **Example:**

have to be the same

AMANAPLANACANALPANAMA

has to be a palindrome

- **Precise Specification:**

```
/** Yields: "s is a palindrome" */
public static boolean isPalindrome(String s)
```

Example: Palindromes

- String with ≥ 2 characters is a palindrome if:
  - its first and last characters are equal, and
  - the rest of the characters form a palindrome
- **Recursive Method:**

Recursive Definition

```
/** Yields: "s is a palindrome" */
public static boolean isPalindrome(String s) {
  if (s.length() <= 1) { return true; }   Base case
  // { s has at least two characters }
  return s.charAt(0) == s.charAt(s.length()–1) &&   Recursive case
         isPalindrome(s.substring(1, s.length()–1));
}
```

Example: More Palindromes

```
/** Yields: "s is a palindrome".
 *  Case of characters and punctuation is ignored. */
public static boolean isPalindrome3(String s) {
  return isPalindrome2(depunct(s));
}

/** Yields: s with the punctuation removed
public static String depunct(String s) {
  if (s.length() == 0)  { return s; }
  // {s is not empty}
  if (!Character.isLetter(s.charAt(0))) { return depunct(s.substring(1)); }
  // {s is not empty and s[0] is not punctuation}
  return s.charAt(0) + depunct (s.substring(1));
}
```

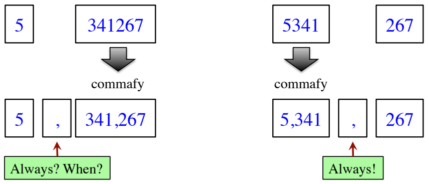
Use helper methods!
- Often easy to break a problem into two
- Can use recursion more than once to solve

How to Break Up a Recursive Method?

```
/** Yields: String with commas every 3 digits
 *   Precondition: s represents a non-negative int
 *   e.g. commafy("5341267") = "5,341,267" */
public static String commafy(String s)
```

Approach 1

Approach 2



How to Break Up a Recursive Solution?

```
/** Yields: String with commas every 3 digits
 *   Precondition: s represents a non-negative int
 *   e.g. commafy("5341267") = "5,341,267" */
public static String commafy(String s) {
   // No commas if too few digits.
   if (s.length() <= 3) { return s; }                Base case

   // Add the comma before last 3 digits
   return commafy(s.substring(0,s.length()-3)) + "," +
            s.substring(s.length()-3);             Recursive case
}
```

How to Break Up a Recursive Method?

```
/** Yields: bc
 *   Precondition: c ≥ 0 */
public static double exp(double b, int c)
```

Approach 1

$$12^{256} = 12 \times (\boxed{12^{255}})$$

Recursive

$$b^{c} = b \times (b^{c})$$

Approach 2

$$12^{256} = (\boxed{12^{128}}) \times (\boxed{12^{128}})$$

Recursive Recursive

$$b^{c} = (b^{c/2}) \times (b^{c-c/2})$$

Raising a Number to an Exponent

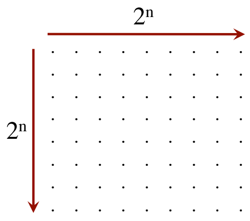
```
/** Yields: bc
 *   Precondition: c ≥ 0 */
public static double exp(double b, int c) {
   // b⁰ is 1
   if (c == 0) {
      return 1;
   }

   // bᶜ = (bᶜ/²)*(bᶜ⁻ᶜ/²)
   int mid = c/2;
   return exp(b,mid)*exp(b,c-mid);
}
```

c	# of calls
0	0
1	1
2	2
4	3
8	4
16	5
32	6
2^{n}	n + 1

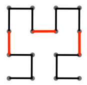
32768 is 215
b^{32768} needs only 16 calls!

Hilbert's Space Filling Curve

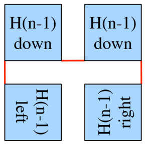


Hilbert(1):

Hilbert(2):



Hilbert(n):



Hilbert's Space Filling Curve

Basic Idea

- Given a box
- Draw $2^{n}\times2^{n}$ grid in box
- Trace the curve
- As n -> ∞, curve fills box