Lecture 21

Loop Invariants

Announcements for This Lecture

Assignments

- A5 graded by weekend
  - Graders needed a break
  - Also working on exam
- Should be working on A6
  - Due week from Thursday
  - Start tomorrow at latest
  - Work on a method a day
  - Should start stenography no later than Sunday

Prelim 2

- **Today** 7:30-9pm
  - A–Q (Kennedy 1116)
  - R–T (Warren 131)
  - U–Z (Warren 231)
- Make-ups on Wednesday
  - Should have been contacted
  - If not, see me now!
- Graded **Wednesday** night
  - Too much to do Today

While-Loops and Flow

```
print 'Before while'
count = 0
i = 0
while i < 3:
    print 'Start loop '+`i`
    count = count + i
    i = i + 1
    print 'End loop '
print 'After while'
```

Output:

```
Before while
Start loop 0
End loop
Start loop 1
End loop
Start loop 2
End loop
After while
```

Some Important Terminology

- **assertion**: true-false statement placed in a program to *assert* that it is true at that point
  - Can either be a **comment**, or an **assert** command
- **precondition**: assertion placed before a statement
  - Same idea as **function precondition**, but more general
- **postcondition**: assertion placed after a statement
- **loop invariant**: assertion supposed to be true before and after each iteration of the loop
  - Distinct from **attribute invariant**
- **iteration of a loop**: one execution of its repetend

Some Important Terminology

- **assertion**: true-false statement placed in a program to *assert* that it is true at that point
  - Can either be a **comment**, or an **assert** comma
- **precondition**: assertion placed
  - Same idea as **fun** general
- **postc** d after a statement
- **l** ssertion supposed to be true before an er each iteration of the loop
  - Distinct from **attribute invariant**
- **iteration of a loop**: one execution of its repetend

Gives methodology for designing loops

Assertions versus Asserts

- Assertions **prevent bugs**
  - Help you keep track of what you are doing
- Also **track down bugs**
  - Make it easier to check belief/code mismatches
- Do not confuse w/ asserts
  - All asserts are assertions
  - But reverse is not true
  - Cannot always convert a comment to an assert

 # x is the sum of 1..n

Comment form of the assertion.

x	?	n	1
x	?	n	3
x	?	n	0

Preconditions & Postconditions

precondition

```
# x  = sum of 1..n-1
x = x + n
n = n + 1
# x =  sum of 1..n-1
```

postcondition

n

1 2 3 4 5 6 7 8

x contains the sum of these (6)

n

1 2 3 4 5 6 7 8

x contains the sum of these (10)

- **Precondition:** assertion placed before a segment
- **Postcondition:** assertion placed after a segment

Relationship Between Two

If precondition is true, then postcondition will be true

Solving a Problem

precondition

```
# x  = sum of 1..n

n = n + 1
# x =  sum of 1..n
```

postcondition

What statement do you put here to make the postcondition true?

A: x = x + 1
B: x = x + n
C: x = x + n+1
D: None of the above
E: I don't know

Solving a Problem

precondition

```
# x  = sum of 1..n

n = n + 1
# x =  sum of 1..n
```

postcondition

What statement do you put here to make the postcondition true?

A: x = x + 1
B: x = x + n
C: x = x + n+1
D: None of the above
E: I don't know

Remember the new value of n

Invariants: Assertions That Do Not Change

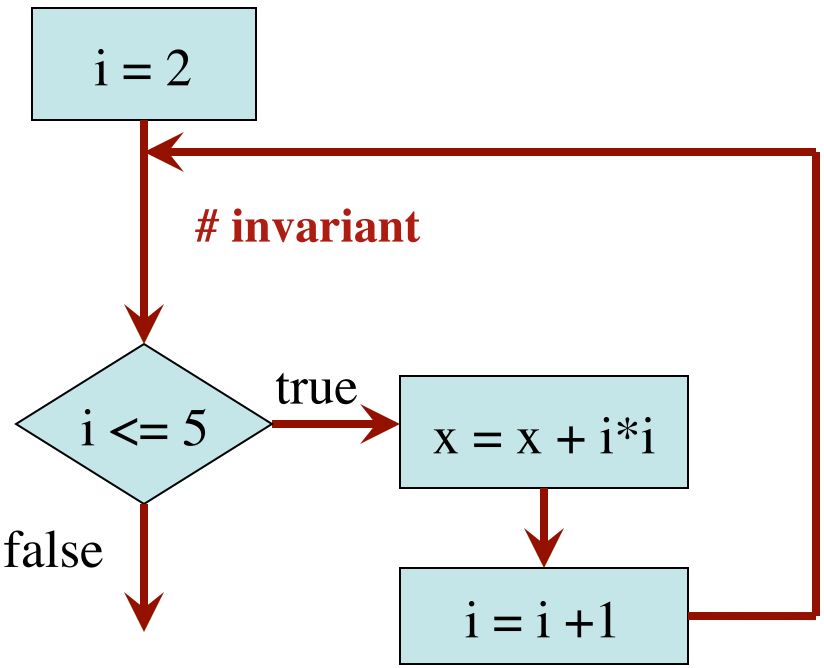
- **Loop Invariant**: an assertion that is true before and after each iteration (execution of repetend)

```
x = 0; i = 2
while i <= 5:
    x = x + i*i
    i = i +1
# x = sum of squares of 2..5
```

Invariant:

x = sum of squares of 2..i-1

in terms of the range of integers that have been processed so far



The loop processes the range 2..5

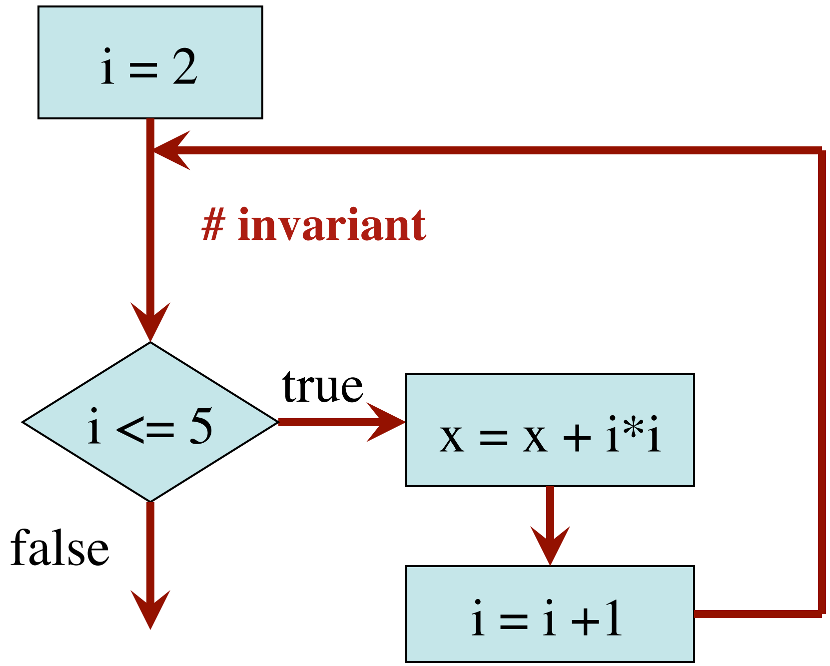
Invariants: Assertions That Do Not Change

```
x = 0; i = 2
# Inv: x = sum of squares of 2..i-1
while i <= 5:
    x = x + i*i
    i= i +1
# Post: x = sum of squares of 2..5
```

Integers that have been processed:	2, 3, 4, 5
Range 2..i-1:	2..5

Invariant was always true just before test of loop condition. So it's true when loop terminates

x	~~0~~ ~~4~~ ~~13~~ ~~29~~ 54
i	~~?~~ ~~2~~ ~~3~~ ~~4~~ ~~5~~ 6



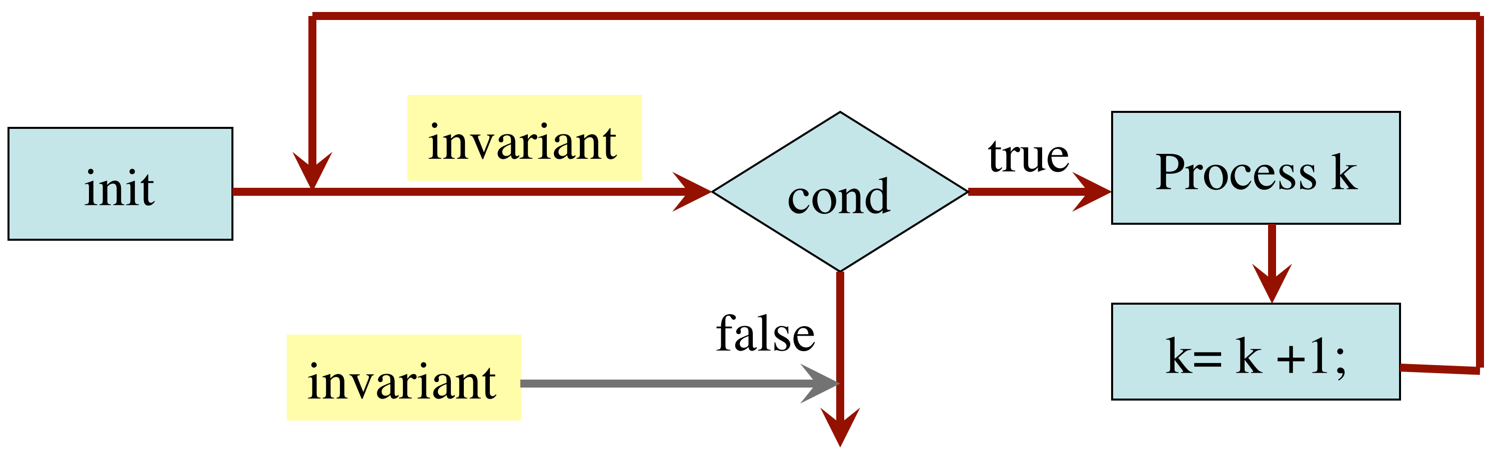
The loop processes the range 2..5

Designing Integer while-loops

```
# Process integers in a..b
# inv: integers in a..k-1 have been processed
k = a
while  k <= b:
    process integer k
    k = k + 1
# post: integers in a..b have been processed
```

Command to do something

Equivalent postcondition



Designing Integer **while-loops**

1. Recognize that a range of integers b..c has to be processed
2. Write the command and equivalent postcondition
3. Write the basic part of the for-loop
4. Write loop invariant
5. Figure out any initialization
6. Implement the repetend (process k)

```
# Process b..c
Initialize variables (if necessary) to make invariant true

# Invariant: range b..k-1 has been processed
while  k <= c:
    # Process k
    k = k + 1

# Postcondition: range b..c has been processed
```

Finding an Invariant

Command to do something

```
# Make b True if no int in 2..n-1 divides n, False otherwise
b = True
k = 2
# invariant: b is True if no int in 2..k-1 divides n, False otherwise
while k < n:
    # Process k;
    if n % k == 0:
        b = False
    k = k +1
# b is True if no int in 2..n-1 divides n, False otherwise
```

Equivalent postcondition

What is the invariant?

1 2 3 … k-1 k k+1 … n

Finding an Invariant

```
# set x to # adjacent equal pairs in s[0..len(s)-1]
```

for s = 'ebeee', x = 2

Command to do something

```
# invariant: ???
k = 0
while k < len(s):
    # Process k;
    k= k +1
# x = # adjacent equal pairs in s[0..len(s)-1]
```

Equivalent postcondition

k: next integer to process.
Which have been processed?

A: 0..k
B: 1..k
C: 0..k–1
D: 1..k–1
E: I don't know

What is the invariant?

A: x = no. adj. equal pairs in s[1..k]
B: x = no. adj. equal pairs in s[0..k]
C: x = no. adj. equal pairs in s[1..k–1]
D: x = no. adj. equal pairs in s[0..k–1]
E: I don't know

Finding an Invariant

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# set x to # adjacent equal pairs in s[0..len(s)-1]
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Command to do something

for s = 'ebeee', x = 2

```
# invariant: ???
k = 0
while k < len(s):
    # Process k;
    k= k +1
# x = # adjacent equal pairs in s[0..len(s)-1]
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Equivalent postcondition

k: next integer to process.
Which have been processed?

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What is the invariant?

A: x = no. adj. equal pairs in s[1..k]
B: x = no. adj. equal pairs in s[0..k]
C: x = no. adj. equal pairs in s[1..k–1]
D: x = no. adj. equal pairs in s[0..k–1]
E: I don't know

Be Careful!

```
# String s has at least 1 element
# Set c to largest element in s
c = ??
k = ??
# inv: c is largest element in s[0..k–1]
while k < len(s):
    # Process k
    k = k+1
#  c = largest char in s[0..len(s)–1]
```

Command to do something

Equivalent postcondition

1. What is the invariant?
2. How do we initialize c and k?

A: k = 0; c = s[0]

B: k = 1; c = s[0]

C: k = 1; c = s[1]

D: k = 0; c = s[1]

E: None of the above

An empty set of characters or integers has no maximum. Therefore, be sure that 0..k–1 is not empty. You must start with k = 1.

Be Careful!

```
# String s has at least 1 element
# Set c to largest element in s
c = ??
k = ??
# inv: c is largest element in s[0..k–1]
while k < len(s):
    # Process k
    k = k+1
#  c = largest char in s[0..len(s)–1]
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Command to do something

Equivalent postcondition

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A: k = 0; c = s[0]

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