

Example: Reversing a String

- Precise Specification:**
 - Returns: reverse of s
- Solving with recursion
 - Suppose we can reverse a smaller string (e.g. less one character)
 - Can we use that solution to reverse whole string?
- Often easy to understand first without Python
 - Then sit down and code

Example: Reversing a String

```
def reverse(s):
    """Returns: reverse of s

    Precondition: s a string"""
    # {s is empty}
    if s == "":
        return s

    # {s at least one char}
    # (reverse of s[1:])+s[0]
    return reverse(s[1:])+s[0]
```

- 1. Precise specification?
- 2. Base case: correct?
- 3. Recursive case: progress to termination?
- 4. Recursive case: correct?

Example: Palindromes

- String with ≥ 2 characters is a palindrome if:
 - its first and last characters are equal, and
 - the rest of the characters form a palindrome
- Example:**

AMANAPLANACANALPANAMA
 (A and P have to be the same, MANAPLANACANAL has to be a palindrome)
- Precise Specification:**

```
def ispalindrome(s):
    """Returns: True if s is a palindrome"""
```

Example: Palindromes

- String with ≥ 2 characters is a palindrome if:
 - its first and last characters are equal, and
 - the rest of the characters form a palindrome
- Recursive Function:**

```
def ispalindrome(s):
    """Returns: True if s is a palindrome"""
    if len(s) < 2:
        return True
    // {s has at least two characters}
    return s[0] == s[-1] and ispalindrome(s[1:-1])
```

Example: More Palindromes

```
def ispalindrome2(s):
    """Returns: True if s is a palindrome
    Case of characters is ignored"""
    if len(s) < 2:
        return True

    // {s has at least two characters}
    return (equals_ignore_case(s[0],s[-1])
            and ispalindrome2(s[1:-1]))

def equals_ignore_case(a, b):
    """Returns: True if a and b are same ignoring case"""
    return a.upper() == b.upper()
```

Example: More Palindromes

```
def ispalindrome3(s):
    """Returns: True if s is a palindrome
    Case of characters and non-letters ignored."""
    return ispalindrome2(depunct(s))

def depunct(s):
    """Returns: s with non-letters removed"""
    if s == "":
        return s
    # use string.letters to isolate letters
    return s[0]+deblank(s[1:]) if s[0] in string.letters
    else deblank(s[1:])
```

How to Break Up a Recursive Function?

```
def commafy(s):
    """Returns: string with commas every 3 digits
    e.g. commafy('5341267') = '5,341,267'
    Precondition: s represents a non-negative int"""
```

Approach 1

Approach 2

How to Break Up a Recursive Solution?

```
def commafy(s):
    """Returns: string with commas every 3 digits
    e.g. commafy('5341267') = '5,341,267'
    Precondition: s represents a non-negative int"""
    # No commas if too few digits.
    if len(s) < 3:
        return s Base case
    # Add the comma before last 3 digits
    return commafy(s[:-3]) + ',' + s[-3:] Recursive case
```

How to Break Up a Recursive Function?

```
def exp(b, c):
    """Returns: b^c
    Precondition: b a float, c ≥ 0 an int"""
```

Approach 1

$$12^{256} = 12 \times (12^{255})$$

Recursive

$$b^c = b \times (b^{c-1})$$

Approach 2

$$12^{256} = (12^{128}) \times (12^{128})$$

Recursive Recursive

$$b^c = (b \times b)^{c/2} \text{ if } c \text{ even}$$

Raising a Number to an Exponent

```
def exp(b, c):
    """Returns: b^c
    Precondition: b a float,
    c ≥ 0 an int"""
    # b^0 is 1
    if c == 0:
        return 1
    # c > 0
    if c % 2 == 0:
        return exp(b*b, c/2)
    return b*exp(b*b, c/2)
```

c	# of calls
0	0
1	1
2	2
4	3
8	4
16	5
32	6
2^n	n + 1

32768 is 215
b³²⁷⁶⁸ needs only 215 calls!

Hilbert's Space Filling Curve

Hilbert's Space Filling Curve

Basic Idea

- Given a box
- Draw $2^n \times 2^n$ grid in box
- Trace the curve
- As n goes to ∞ , curve fills box