Cayuga Heights 5th Grade Math Club
Problems for October 16, 2015

When a shape is changed in size, its different dimensions (length, area, and volume) change in different ways. Suppose a big shape is just like a small shape except that its linear dimensions (height, width, depth) are all M times bigger for some number M. All linear dimensions are *scaled* by M in the big shape. However, all areas in the big shape are M^2 times bigger. A square of side length 3 has 9 times the area of a square of side 1, and the same thing applies to circles too. It even works for three-dimensional shapes. A sphere of diameter 2 has 4 times the surface area as a sphere of diameter 1, and if cut through the center, the area of the cross-section is also 4 times as big.

What about the volume of shapes? Inside a cube of side length 2, you can fit 8 cubes of side length 1. In general, volume *scales* as M^3, because *three* linear dimensions are being increased at once. Assuming that the two cubes are made of the same material, the larger cube will weigh 8 times as much. The same thing holds true for any other shape.

1. Jack painted a square 10 feet on a side, and painted a 1-inch side black border around it. It took 1 bucket of paint to paint the center part, and 0.1 buckets of paint to paint the border. Now he has moved on to a 40-foot square with a 1-inch border. How much paint will it take to paint the center of the new square, and how much to paint the border?

2. Ananya has a gold cube 4 inches on a side. She melts the gold down into 64 little cubes to give to all her math club friends. How big are the new cubes on a side?

3. Anna hammers out her gold cube into a thin layer of *gold leaf* and uses it to cover a sphere 6 inches in diameter. But she wants a sphere that looks much bigger than Ananya's original cube. How many little cubes will she need to cover a sphere that is 3 feet in diameter?

4. **Giants!** Suppose a 6-foot tall person weighs 150 pounds and can lift 250 pounds with their legs. How much would a 12-foot-tall giant weigh who is otherwise similar in shape to a normal person, assuming they are made out of the same stuff?

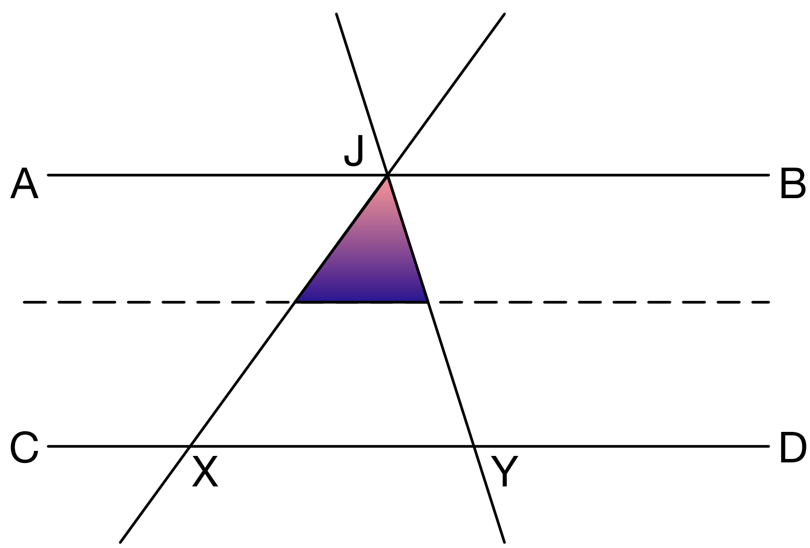
5. Following up on the previous question, the strength of muscles is proportional to the *area of the cross-section* of the muscle. Suppose the area of the cross-section of a human leg is 30 square inches.

   (a) What would be the area of the cross-section of a giant's leg?

   (b) How much could the giant lift with its legs?

   (c) Why can't giants stand up?

6. Suppose two lines from A to B (call this line AB) and from C to D (call it CD) are *parallel*, and there are two other lines that intersect line AB at a common point J, and intersect line CD at points X and Y. Now we add a third dashed line halfway between AB and CD, creating a small triangle that is shaded in the diagram.



   (a) Is the big triangle JXY *similar* to the small shaded triangle?

   (b) If the area of the big triangle is 12 cm^2, what is the area of the small shaded triangle?