

One's digits and remainders

When we add, subtract, or multiply two numbers, the one's digit of the result only depends on the one's digit of the operands. For example, we don't need to figure out what 2314×1117 is to know that the one's digit must be 8.

1. What is the one's digit of the result of 2883×799912 ?
2. What is the one's digit of the result of 3487623×11117 ?
3. What is the one's digit of the result of $(21783 + 9838) \times 8917$?
4. If the remainder when two numbers are divided by 10 is 7, what is the remainder of their product when divided by 10?

There is nothing special about the number 10. The same thing works when we write a number in any number base. The one's digit of a number in some base N is the same as the remainder when we divide by N . So to know what the one's digits (the remainder) of some computation is, we only need to know the remainder for all the numbers involved in the computation!

5. What does the base-8 number 41 represent in base 10? Show how to multiply this number by itself *in base 8*. Notice that the one's digit must be 1!
6. Consider the base-2 numbers 100101_2 and 10101_2 . What is the one's digit of their product? Is the product even or odd?
7. Consider the base-8 numbers 45_8 and 32_8 . What is the one's digit of their product, written in base 8?
8. What is the remainder when we divide $78 \times 34 \times 12$ by 11?
9. What is the remainder when we divide $(99 + 2) \times 17$ by 9?
10. Suppose we take the product of two numbers whose remainder when divided by 12 is 7. When their product is divided by 12, what is the remainder?

Exponentiation is just repeated multiplication, so the same thing works for exponentiation. For example, when raising numbers to a power base 10, the one's digit always follows a repeating pattern of length 4 (or some divisor of 4). For example, $7^1 = 7$, $7^2 = 49$, $7^3 = 343$, $7^4 = 2401$. So the pattern is 7, 9, 3, 1, 7, 9, 3, 1, ...

11. What is the one's digit of 11^{10} ?

12. What is the one's digit of 9^{10} ?

13. What is the one's digit of 99^{10} ?

14. What is the one's digit of 2^{2016} ?

15. What is the remainder when 13^{99} is divided by 8?

16. What is the remainder when 2^{601} is divided by 7?