Proof Verbalization as an Application of NLG

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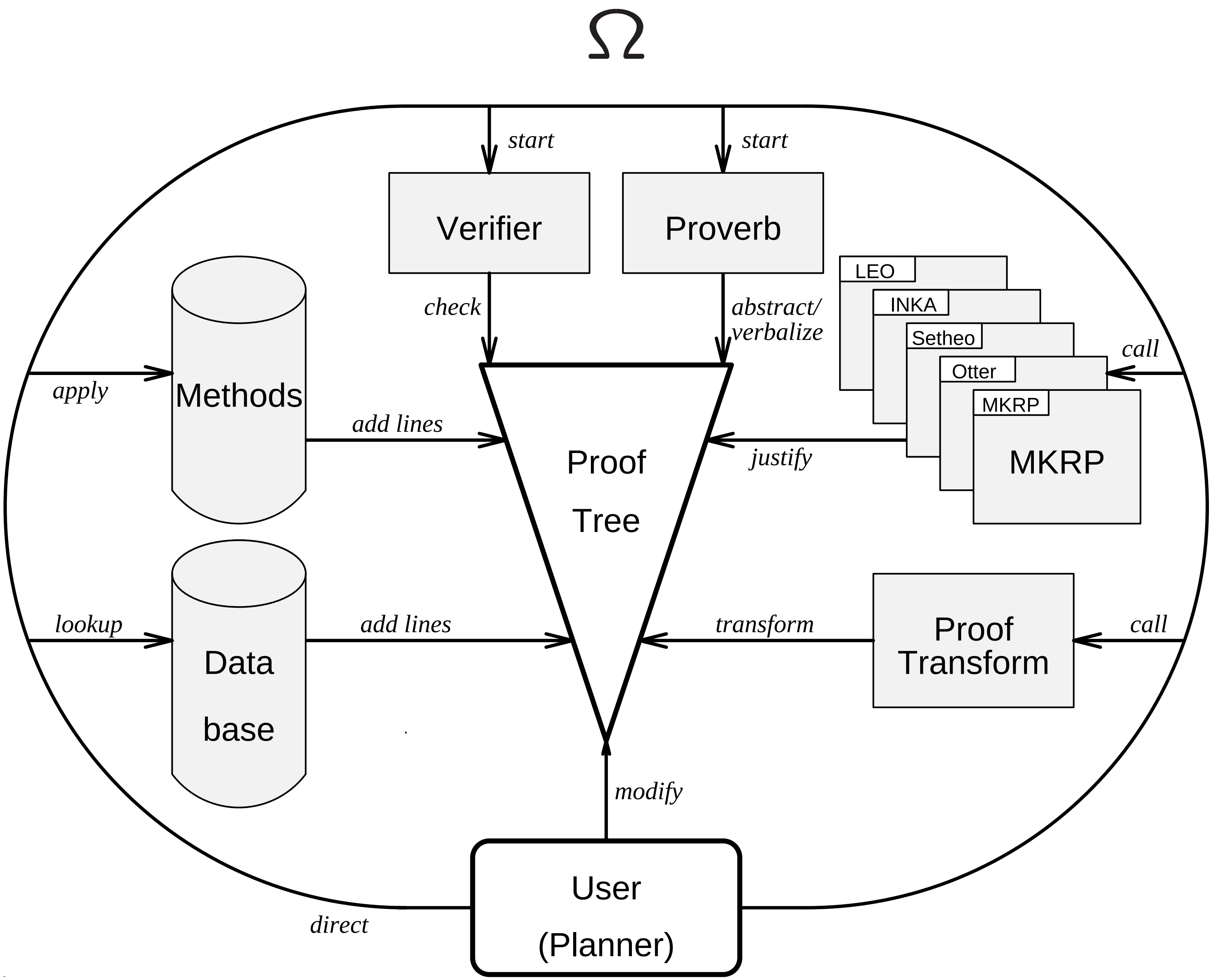
Goal: flexible sentence planning for argumentative text

Techniques:

- Combining *Text Structure* and *Upper Model*
- Aggregation



Ω
start
start
Verifier
Proverb
check
abstract/
verbalize
LEO
INKA
Setheo
Otter
MKRP
MKRP
call
apply
Methods
add lines
Proof
Tree
justify
lookup
Data
base
add lines
transform
Proof
Transform
call
modify
User
(Planner)
direct

Motivation

Theorem (Subgroup Criterion)
Let G be a group, $S \subset G$, if for all x, y in S, $y * x^{-1}$ is also in S, then the inverse of every element of S is also in S.

Initial Clause Set:
C1 = $\{+(u*u^{-1} = e)\}$ C2 = $\{+(e*w = w)\}$ C3 = $\{-(x \in S), -(y \in S), -(x*y^{-1} = z), +(z \in S)\}$
C4 = $\{+(v \in S)\}$ C5 = $\{-(q^{-1} \in S)\}$

Resolution Steps:
C3,4 & C3,1 → R1: $\{-(x \in S), -(y \in S), -(x * y^{-1} = z), -(y' \in S), -(z * y'^{-1} = z'), +(z' \in S)\}$
R1,1 & C4,1 → R2: $\{-(y \in S), -(v * y^{-1} = z), -(y' \in S), -(z * y'^{-1} = z'), +(z' \in S)\}$
R2,1 & C4,1 → R3: $\{-(v * v^{-1} = z), -(y' \in S), -(z * y'^{-1} = z'), +(z' \in S)\}$
R3,2 & C4,1 → R4: $\{-(v * v^{-1} = z), -(z * v^{-1} = z'), +(z' \in S)\}$
R4,1 & C1,1 → R5: $\{-(e * v^{-1} = z'), +(z' \in S)\}$
R5,1 & C2,1 → R6: $\{+(v^{-1} \in S)\}$
R6,1 & C5,1 → R7: $\square$

Proof:
Let a be in S. According to the definition of inverse element, $a * a^{-1} = e$. According to our hypothesis, e in S. $e * a^{-1} = a^{-1}$ according to the definition of unit. Again according to our hypothesis, a^{-1} is in S.

Previous Work

Machine Oriented Proofs

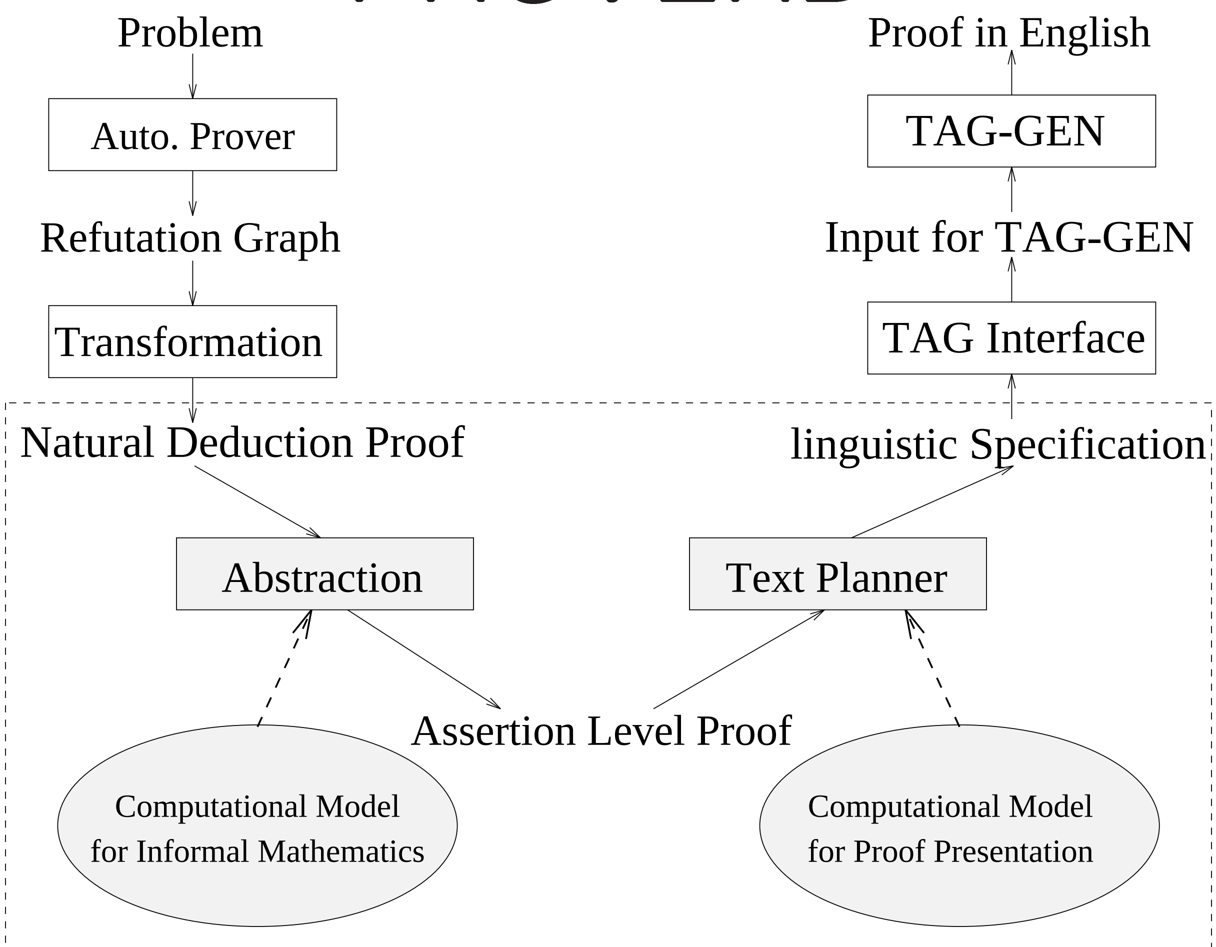
↓ [Andrews 80, Miller 83, Pfenning 87, Lingenfelder 90]

Natural Deduction Proofs (ND)

↓ [Chester 76, McDonald 83, Edgar & Pelletier 93]

Natural Language Proofs (NL)

Reconstructive Explanation in *PROVERB*



The System *PROVERB*

Macroplanner: choice of content and order of the information to be conveyed

Microplanner: sentence scoping and planning of the internal structure of the sentences

Realizer: realization of the surface text by TAG-GEN

*PROVERB*s Macroplanner

Task: content determination

Input: an assertion level proof

Output: an ordered sequence of *proof communicative acts* (PCAs)

Methods:

- goal directed hierarchical planning (top-down)
- focus guided local organisation (bottom-up)

Case-Implicit

- Proof: $\dfrac{F \vee G, \overset{F}{Q}, \overset{G}{Q}}{Q}\text{CASE}$

- Acts:

  1. Subproof
  2. (`CASE-FIRST, Assumptions:` F)
  3. Subproof
  4. (`CASE-SECOND, Assumptions:` G)
  5. Subproof

- Features: (top-down compulsory implicit)

Local Organization

$$\dfrac{\dfrac{[1]:\ P(a,b)}{[3]:\ Q(a,b)} \qquad \dfrac{[1]:\ P(a,b),\ [2]:\ S(c)}{[4]:\ R(b,c)}}{[5]:\ Q(a,b) \wedge R(b,c)}$$

- local focus = [1]
  focal centers = $\{a, b\}$

- next node = [3],
  since [3] does not introduce any new objects and $\{b\} \subset \{a, b\}$

The Need for a Microplanner

- first version of *PROVERB* without a microplanner:
  - no paraphrasing:
    Since A, B.
    [A leads to B.]
  - rigid recursive verbalization:
    $\mathit{Set}(F) \land \mathit{Subset}(F, G)$
    F is a set and F is a subset of G.
    [F is a set and a subset of G.]
    [The set F is a subset of G.]
- only microplanning technique: derivation reference choice
  $\Rightarrow$ *preverbal message* (PM)

Example

(1) Let F be a group and U be a subgroup of F and 1 be a unit element of F and 1_U be a unit element of U.

(2) According to the definition of unit element $1_U \in U$.

(3) Therefore there is an X, $X \in U$.

(4) Now suppose that u_1 is such an X.

(5) According to the definition of unit element $u_1 * 1_U = u_1$.

(6) Since U is a subgroup of F, $U \subset F$.

(7) Therefore $u_1 \in F$.

(8) Similarly $1_U \in F$, since $1_U \in U$.

(9) Since F is a group, F is a semigroup.

(10) Since $u_1 * 1_U = u_1$, 1_U is a solution of the equation $u_1 * X = u_1$.

(11) Since 1 is a unit element of F, $u_1 * 1 = u_1$.

(12) Since 1 is a unit element of F, $1 \in F$.

(13) Since $u_1 \in F$, 1 is a solution of the equation $u_1 * X = u_1$.

(14) Since F is a group, $1_U = 1$ by the uniqueness of solution.

(15) This conclusion is independent of the choice of the element u_1.

*PROVERB*s Microplanner

Task: sentence scoping and sentence organisation

Input: a sequence of PCAs

Output: a *Text Structure*

Methods:

- progressive refinement of the Text Structure
- operations on the Text Structure

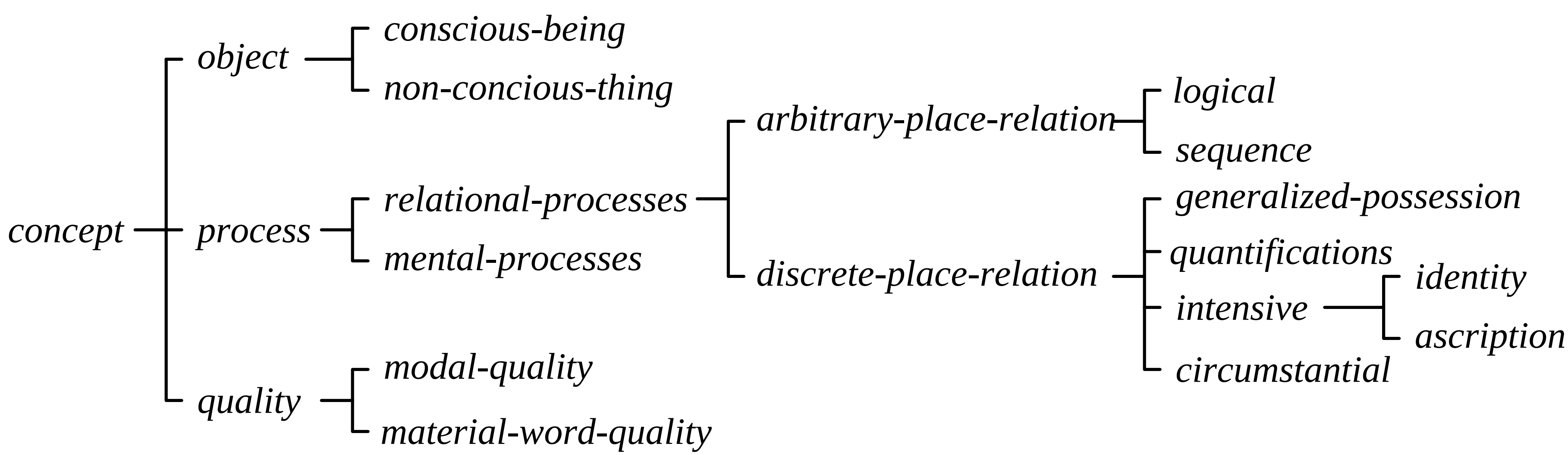
Text Structure

A Text Structure [Meteer, 91] contains information about:

- constituency
- structural relations between constituents
- semantic categories of the constituents

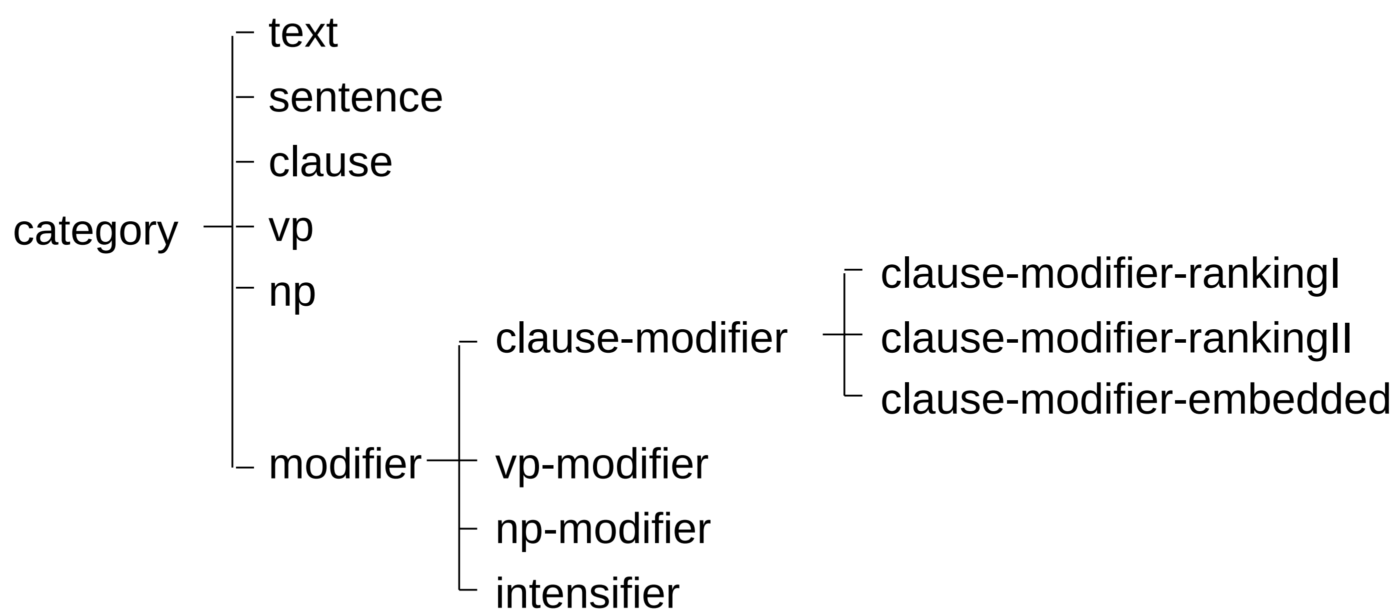
*PROVERB*s Upper Model

- adopted from [Bateman *et al.*, 90]



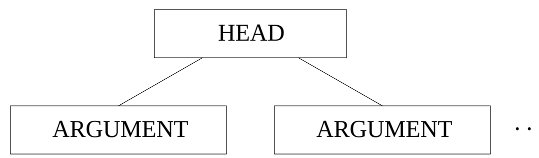
*PROVERB*s Textual Semantic Categories

- adopted from [Panaget, 94]



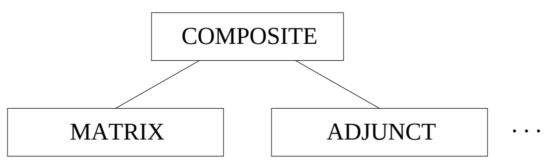
Resource Trees

- Text Structure build up by *resource trees*
- resource trees consist of basic tree types:

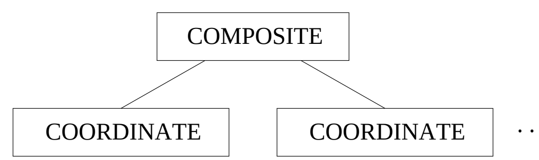


Peter likes Mary

Kernel tree



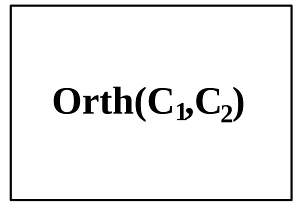
pretty Mary



Peter and Mary

Composite trees

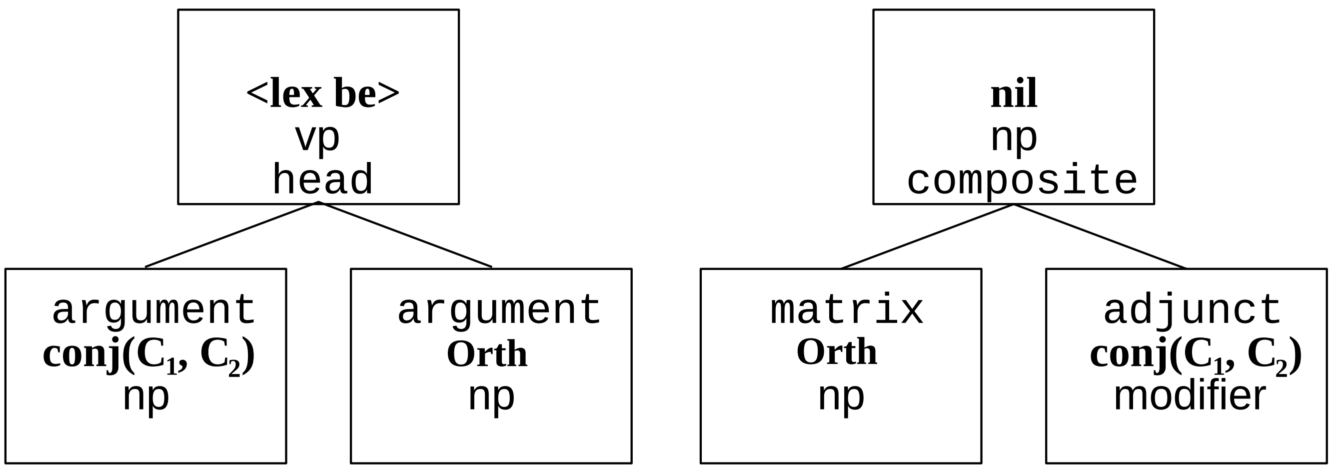
Paraphrasing



quality-relation(`Orth`, `C1`, `C2`)

process-relation(`Orth`, `C1`, `C2`)

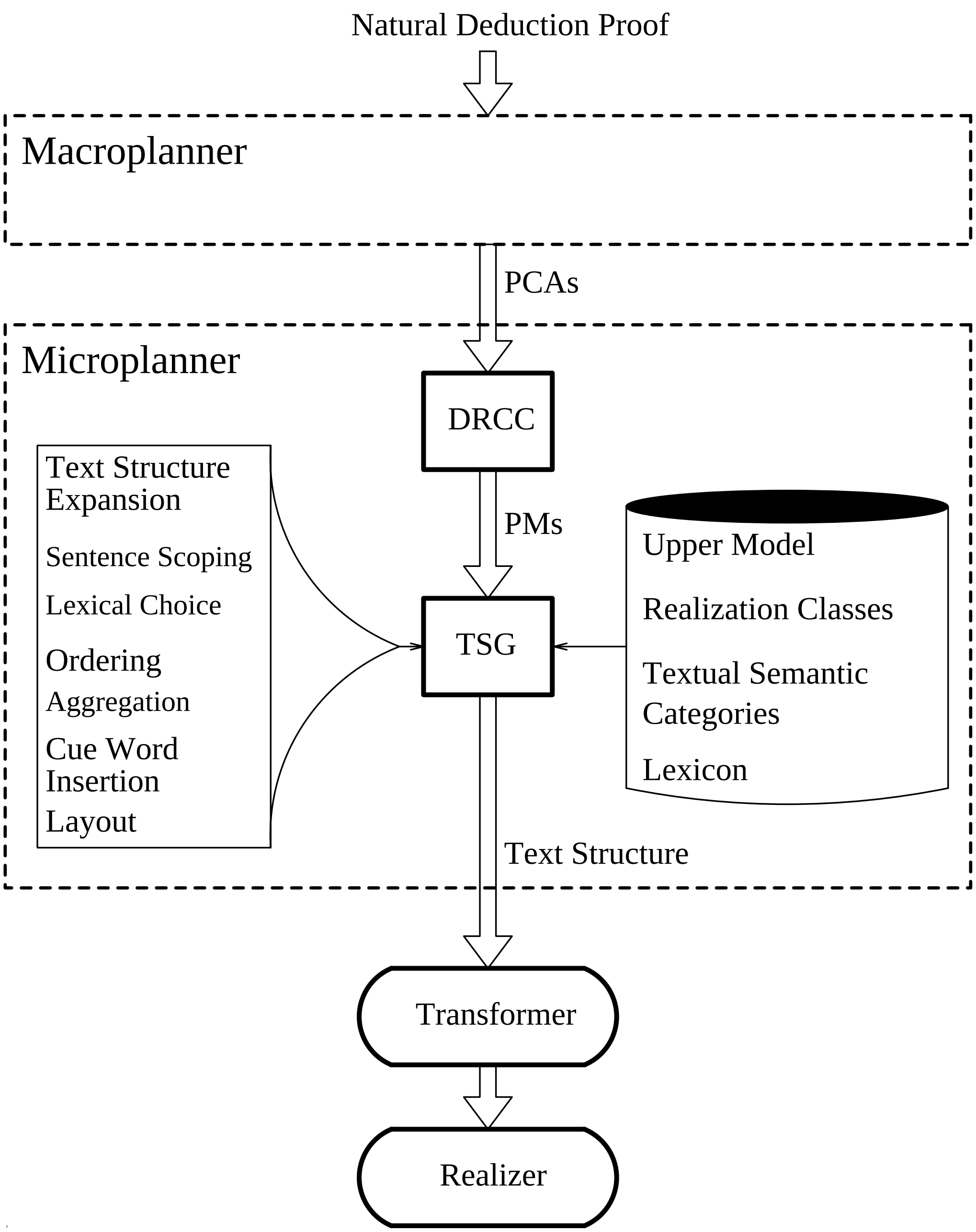
property-ascription(`Orth`, *conjunction*(`C1`, `C2`))



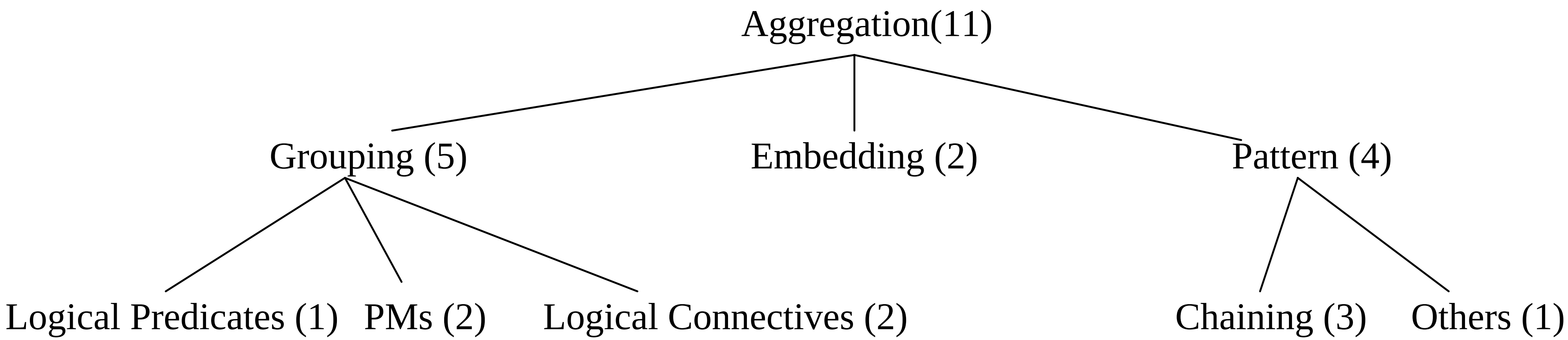
C_1 and C_2 are orthogonal

the orthogonality of C_1 and C_2

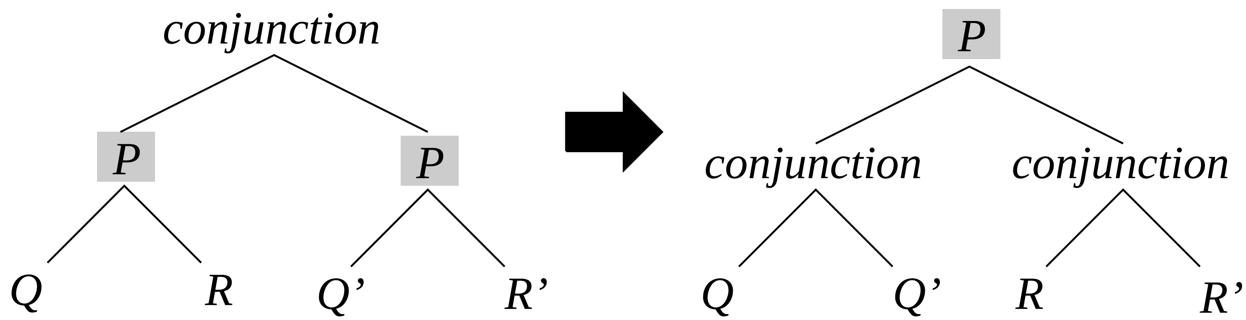
The Architecture of *PROVERB*



Aggregation



Predicate Grouping



$$\mathit{Set}(F) \wedge \mathit{Set}(G)$$

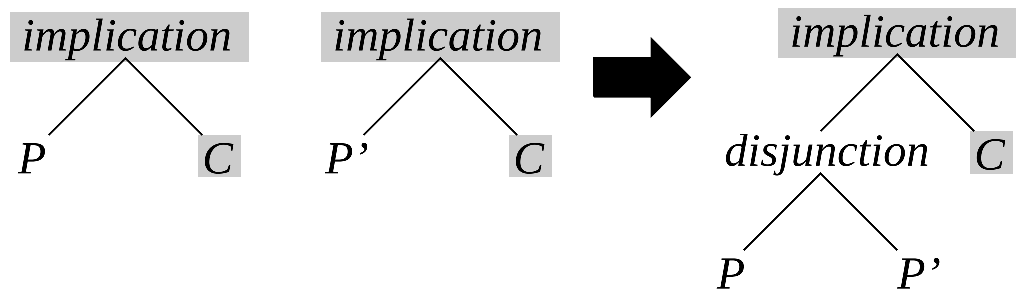
"F is a set. G is a set."

$$\Downarrow$$

$$\mathit{Set}(F \wedge G)$$

"F and G are sets."

Grouping of Implications



$$\mathit{conjunction}(\mathit{implication}(a < b, a \neq b), \mathit{implication}(a > b, a \neq b))$$

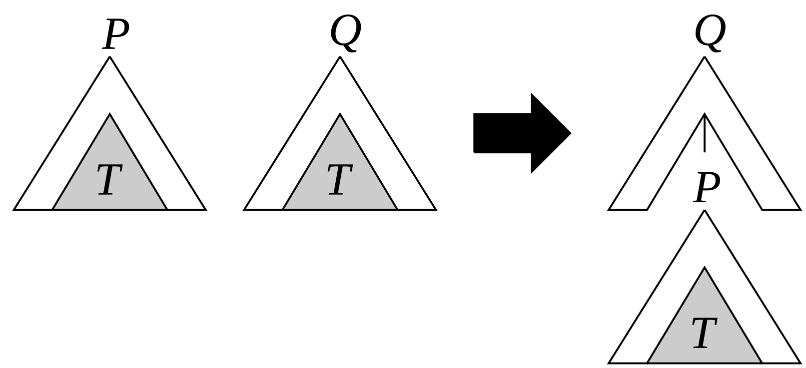
"If $a < b$ then $a \neq b$. If $a > b$ then $a \neq b$."

$$\Downarrow$$

$$\mathit{implication}(\mathit{disjunction}(a < b, a > b), a \neq b)$$

"If $a < b$ or $a > b$ then $a \neq b$."

Embedding



$$\textit{Set}(\boxed{F}) \wedge \textit{Subset}(\boxed{F}, G)$$

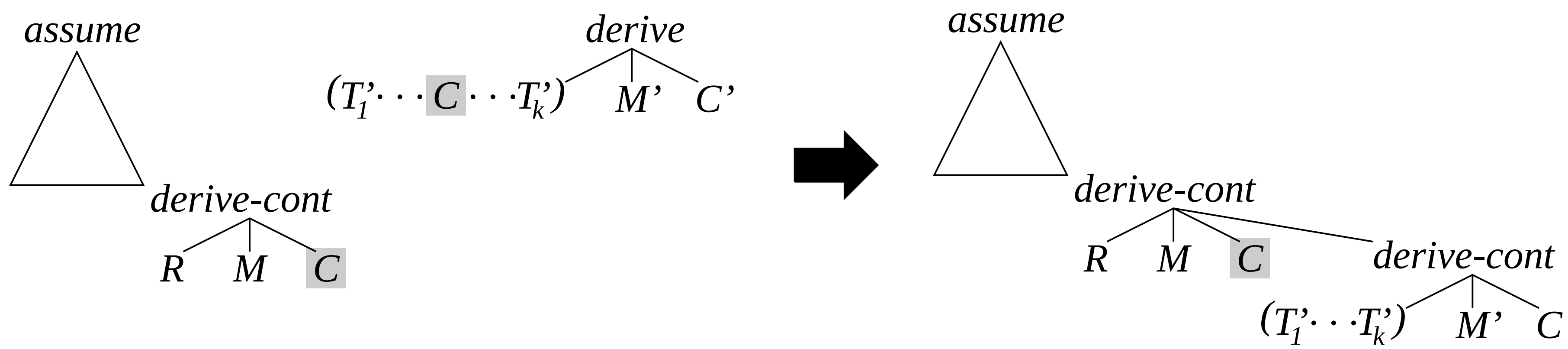
"F is a set. F is a subset of G."

$$\Downarrow$$

$$\textit{Subset}(\textit{Set}(\boxed{F}), G)$$

"The set F is a subset of G."

Pattern Based Aggregation



$\textit{derive}(\epsilon, \textit{DefTrans}, \sigma \subseteq \sigma^*)$

"$\sigma \subseteq \sigma^*$ by the definition of transitive closure."

$\textit{derive}(((x,y) \in \sigma, \sigma \subseteq \sigma^*), \textit{DefSubset}, (x,y) \in \sigma^*)$

"Since $(x,y) \in \sigma$ and $\sigma \subseteq \sigma^*$, $(x,y) \in \sigma^*$ by the definition of subset."

$\Downarrow$

$\textit{derive}(\epsilon, \textit{DefTrans}, \sigma \subseteq \sigma^*, \textit{derive-cont}(((x,y) \in \sigma), \textit{DefSubset}, (x,y) \in \sigma^*))$

"$\sigma \subseteq \sigma^*$ by the definition of transitive closure, thus establishing $(x,y) \in \sigma^*$ by the definition of subset, since $(x,y) \in \sigma$."

Example (cont'd)

Theorem:

Let F be a group, U be a subgroup of F, and 1 and 1_U be unit elements of F and U. Then $1_U = 1$.

Proof:

(1) Let F be a group, U be a subgroup of F, and 1 and 1_U be unit elements of F and U.

(2,3) $1_U \in U$ by the definition of unit element, which leads to the existence of an x, such that $x \in U$. (4) Let u_1 be such an x.

(5) $u_1 * 1_U = u_1$ by the definition of unit element. (6,7) Since U is a subgroup of F, $U \subset F$, which implies $u_1 \in F$. (8) Similarly, since $1_U \in U$, $1_U \in F$. (9) Since F is a group, F is a semigroup. (10) Since $u_1 * 1_U = u_1$, 1_U is a solution of the equation $u_1 * x = u_1$.

(11,12) Since 1 is a unit element of F, $u_1 * 1 = u_1$ and $1 \in F$. (13) Since $u_1 \in F$, 1 is a solution of the equation $u_1 * x = u_1$. (14) Since F is a group, $1_U = 1$ by the uniqueness of the solution. (15) This conclusion is independent of the choice of the element u_1. ■

Conclusion

- Microplanning techniques necessary for mathematical proofs
- Text Structure combined with Upper Model and textual semantic categories
- Aggregation rules defined in terms of Upper Model concepts