CS 421 Numerical Analysis Prelim

October 10, 1997, in class

1. Consider the following statements. In each case indicate whether the statement is **true** or **false**. If it is true, give a proof or explanation. If it is false, give a counter-example or explain. No more than a few lines are necessary. (30 Points)

   (a) Assume that A is a n-by-n nonsingular matrix. Then $C = A^T A$ is symmetric positive definite.

   (b) The relative error of a floating point arithmetic operation is bounded by the smallest positive number in the floating point system.

   (c) If a n-by-n matrix A is orthogonal, then $Ax = b$ can be solved in $O(n^2)$ flops.

   (d) Let $\hat{x}$ be the computed solution for $Ax = b$ where A is a n-by-n nonsingular matrix. If the residual $r = b - A\hat{x}$ is small, then $\hat{x}$ is accurate.

   (e) If A is singular, then Gaussian elimination with partial pivoting can still produce $PA = LU$ where P is a permutation matrix, L is a lower triangular and U is upper triangular.

2. Assume that A is a n-by-n symmetric positive definite matrix whose elements are zero everywhere except on the diagonal and the last column and row. Let $d \in \Re^n$ denote the diagonal and $e \in \Re^{n-1}$ denote the first $(n-1)$ elements of the last row, e.g., when $n = 5$,

$$A = \begin{bmatrix} d_1 & 0 & 0 & 0 & e_1 \\ 0 & d_2 & 0 & 0 & e_2 \\ 0 & 0 & d_3 & 0 & e_3 \\ 0 & 0 & 0 & d_4 & e_3 \\ e_1 & e_2 & e_3 & e_4 & d_5 \end{bmatrix}.$$

   Write an efficient Matlab function which takes d and e as inputs and produces the Cholesky factorization of A. What is the leading term in the flops count? (15 Points)

3. Assume that $A \in \Re^{m\times n}$, $m \geq n$, $b_i \in \Re^m$, $i = 1, \cdots, p$. Suppose that A has full column rank n and the QR factorization $A = QR$ has been obtained, where Q is an m-by-m orthogonal matrix and R is m-by-n upper triangular. Explain how you can solve

$$\min_{x_1\in\Re^n,\cdots,x_p\in\Re^n} \|Ax_1 - b_1\|_2 + \cdots + \|Ax_p - b_p\|_2,$$

efficiently. Write a Matlab segment to describe your method. (15 Points)