CS421 Assignment 3: 1997

Due Date: October 17, 1997

1. When the upper bandwidth $q = 2$ and lower bandwidth $p = 2$ the band matrix is called *pentadiagonal* matrix. A pentadiagonal matrix can be stored in five vectors e_1, e_2, d, f_1 and f_2 which contain the subdiagonal, second subdiagonal, diagonal, superdiagonal and second superdiagonal respectively. Write a Matlab function to factor, without pivoting, a pentadiagonal matrix stored in five vectors. The factorization matrices should also be stored in vector forms. Write a Matlab function to solve a pentadiagonal linear system using the computed factorization. Devise some test problems to verify that your functions work.

2. A $n \times n$ matrix P is an *orthogonal projector* if it is both idempotent ($P^2 = P$) and symmetric ($P = P^T$). Such a matrix projects any given n-vector onto a subspace (namely, the column space of P) but leaves unchanged any vector that is already in that subspace.

   (a) Suppose that Q is an $n \times k$ matrix whose columns form an orthonormal basis for a subspace S of $\Re^n$. Show that QQ^T is an orthogonal projector onto S.

   (b) If A is a matrix with linearly independent columns, show that $A(A^TA)^{-1}A^T$ is an orthogonal projector onto the column space of A. How does this result relate to linear least squares problem?

   (c) If P is an orthogonal projector onto a subspace S, show that $I - P$ is an orthogonal projector onto the orthogonal complement of S. (The orthogonal complement of $S \subseteq \Re^n$ is defined by $S^\perp = \{y \in \Re^n : y^Tx = 0 \text{ for all } x \in S\}$).

3. A planet follows an elliptical orbit, which can be represented in a Cartesian (x, y) coordinate system by the equation

$$ay^2 + bxy + cx + dy + e = x^2.$$

(a) Use Matlab \ for linear least squares to determine the orbital parameters a, b, c, d, e given the following observations of the planet's position:

x	1.02	0.95	0.87	0.77	0.67	0.56	0.44	0.30	0.16	0.01
y	0.39	0.32	0.27	0.22	0.18	0.15	0.13	0.12	0.13	0.15

In addition to printing the values for the orbital parameters, plot the resulting orbit and the given data points in the (x, y) plane. You may find the Matlab functions **plot, linspace, roots, imag** useful. Note also that componentwise operations for vectors exist in Matlab, e.g., $x. * y$ denotes element-by-element multiplication. Type help *arith* in Matlab to find out more.

(b) What is the 2-norm condition number of the coefficient matrix A for this least squares problem? To see what effect this has on the solution, perturb the input data slightly by adding to each coordinate of each data point a random number uniformly distributed on the interval $[-0.005, 0.005]$ and solve the least squares problem with the perturbed data. Compare the new values for the parameter with those previously computed. What effect does this difference have on the plot of the orbit? Can you explain this behavior? You can use Matlab function **rand** to generate the perturbation and **cond** to compute the condition number.