CS421 Assignment 2: Fall 1997

Due Date: September 26, 1997

1. Write a *column oriented* upper triangular solver x =**UTriSol**(U, b) in MATLAB for $Ux = b$ where U is an upper triangular matrix. Efficiency matters. Test your program by generating random upper triangular matrices and setting the right hand side vectors so that the solutions are known. Note that **rand**(m, n) generate a random matrix in $\Re^{m\times n}$ and **triu**(X) returns the upper triangular part of a matrix X.

2. Let y be an approximate solution to $Ax = b$. Prove the following results:
   (a) $(A + E)y = b$ where $E = ry^T/(y^Ty)$, $r = b - Ay$.
   (b) There is not a unique F such that $(A + F)y = b$.
   (c) E has a minimum 2-norm over all F satisfying $(A + F)y = b$.

   Hence explain how you would assess the backward stability of an approximate solution $\hat{x}$ to $Ax = b$ computed in floating-point arithmetic.

3. The $n \times n$ Pascal matrix $P_n = (p_{ij})$ is defined by

$$p_{ij} = \left\{ \begin{array}{ll} 1 & \text{if } i = 1 \text{ or } j = 1, \\ p_{i,j-1} + p_{i-1,j} & \text{if } 2 \le i \le n,\ 2 \le j \le n. \end{array} \right\}$$

   For $n = 4 : 4 : 20$ solve the linear system $Ax = b$ where $A = P_n$, $b = Ae$, and e equals $(1, 1, \cdots, 1)^T$. Use LU factorization with partial pivoting, i.e., $[P, L, U]$ =LU(A) in Matlab. For each n print
   (a) the computed solution, $\hat{x}$;
   (b) the relative residual, $\|b - A\hat{x}\|_1/(\|A\|_1\|\hat{x}\|_1)$;
   (c) the relative error, $\|x - \hat{x}\|_1/\|x\|_1$;
   (d) a condition number estimate, namely the reciprocal of **RCOND**(A);

(e) the time required to solve the linear system. [Note: t = clock; <code>; t=etime(clock,t) puts in t the time in seconds for <code> to execute.]

Use short e format for the output. Alternatively you can control the output format using the command **fprintf** (see help **fprintf**). Comment on your ouput. Are the numbers consistent with each other? How and why do the results vary with n? Are the times what you would expect?