CS 421 Assignment 1: 1997

Due Date: 12 September 1997

1. The Lehmer matrix of order n is defined by

$$a_{ij} = \begin{cases} \frac{i}{j}, & j \geq i, \\ \frac{j}{i}, & j \leq i. \end{cases}$$

   Write an M-file **LEHMER**(N) to generate this matrix. What interesting properties can you find of the matrix? For example, look at LU(A), INV(A), EIG(A).

2. (Heath, Exercise 1.9)

   (a) The Earth surface area can be approximated by a sphere of radius r, i.e., $A = 4\pi r^2$. Using four-digit decimal arithmetic, compute the surface area of the Earth with $r = 6470$ km.

   (b) Using the same formula and precision, compute the difference in surface area if the value for the radius is increased by 1 km.

   (c) Since $dA/dr = 8\pi r$, the change in surface area is approximated by $8\pi rh$, where h is the change in radius. Use this formula, still with four-digit arithmetic, to compute the difference in surface area due to an increase of 1 km in radius. How does the value obtained using this approximate formula compare with that obtained from the "exact" formula in (b)?

   (d) Determine which answer is more nearly correct by repeating both computation using higher precision, say, six-digit decimal arithmetic.

   (e) Explain the results you obtained in parts (a)-(d).

   (f) Try this problem in Matlab. How small must the change in h radius be for the same phenomenon to occur? (Matlab only has double precision available.)

3. (Heath, computer problems 1.16)

   Write a program to generate the first n terms in the sequence given by the difference equation

$$x_{k+1} = 2.25x_k - 0.5x_{k-1}$$

   with starting values

$$x_1 = \frac{1}{3} \quad \text{and} \quad x_2 = \frac{1}{12}.$$

   Using $n = 60$, make a semilog plot of the values you obtain as a function of k. The exact solution of the difference equation is given by

$$x_k = \frac{4^{1-k}}{3},$$

   which decreases monotonically as k increases. Does your graph confirm this theoretically expected behavior? Can you explain your results? (*Hint: the general solution to the difference equation is* $x_k = \alpha(\frac{1}{4})^k + \beta 2^k$.)