A Case for Source-Level Transformations in MATLAB

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Abstract

In this paper, we discuss various performance overheads in MATLAB codes and propose different program transformation strategies to overcome them. In particular, we demonstrate that high-level source-to-source transformations of MATLAB programs are effective in obtaining substantial performance gains regardless of whether programs are interpreted or later compiled into C or FORTRAN. We argue that automating such transformations provides a promising area of future research.

1 Introduction

MATLAB is a programming language and development environment which is popular in many application domains like signal processing and computational finance that involve matrix computations. There are many reasons for its popularity. First, MATLAB is a relatively high-level, untyped language in which matrices are a built-in data type with a rich set of primitive operations. Second, MATLAB programs are interpreted, making MATLAB well-suited for prototyping code. (A compiler that translates MATLAB code to C is available from MathWorks.) Third, the *mex-file* facility makes it easy to invoke compiled C or FORTRAN functions from the MATLAB interpreter. Finally, a full set of numerical and graphics libraries for many applications domains like computational finance and signal processing is available.

In general, there are overheads in the interpretation of programs in high-level untyped languages like MATLAB that are not there in executing code compiled from more conventional general-purpose languages like C or FORTRAN. The most important of these overheads in MATLAB are the following.

- *Type and shape checking/dispatch*: A variable can be introduced in MATLAB programs without declaring its type or shape. Therefore, execution of the MATLAB statement `C = A*B;` can require a computation as simple as the product of two doubles or as complicated as the product of two matrices `A` and `B` containing complex entries. The interpreter must check types and shapes of operands in expressions for compatibility, and dispatch to the right routine for carrying out the appropriate operation. In traditional compiled languages, by contrast, the types and shapes of matrices would be known to the compiler, so at runtime it

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is only necessary to check that the number of columns of `A` is equal to the number of rows of `B`.

- *Dynamic resizing*: Since matrix sizes are not declared by the programmer, the MATLAB interpreter allocates storage for a matrix on demand. If `x` is a vector and an attempt is made to write to element `x(i)` where `i` is outside the current bounds of the vector, MATLAB allocates new storage for a larger vector and copies over elements from the old vector into the new storage. In FORTRAN or C, it is the responsibility of the programmer to allocate a large enough matrix, and any attempt to write into a location outside the index space of the matrix is an error.

- *Array bounds checking*: The interpreter must check indexed accesses of array elements to ensure that the access is within array bounds. In conventional languages, static analysis can use array declarations to eliminate bounds checks in many cases.

Techniques for reducing such interpretive overheads may be useful not only for MATLAB programs but for programs in other domain-specific languages many of which are also high-level, untyped, interpreted languages.

One approach is to translate MATLAB programs into programs in a conventional language like C or FORTRAN, and attempt to eliminate these overheads when compiling the C/FORTRAN code down to machine code. Several projects in both academia and industry have taken this approach [2, 4, 5, 9, 13, 18, 19, 20]. The mex-file interface described earlier can be used to invoke compiled routines from the interpreter, permitting the programmer to use the familiar MATLAB execution environment when running compiled code. However, as MATLAB lacks variable declarations, generation of efficient C or FORTRAN requires inference of types, shapes, and sizes. Unfortunately, compiler techniques to automatically inference these properties without additional user input have had limited success, as we discuss later in this paper.

In this paper, we will make the case for a very different approach to reducing these overheads — by using source-level transformations of MATLAB code. The MATLAB community has developed a number of programmer tricks [12] to enhance performance of MATLAB codes. Surprisingly, these ideas have never been studied in the context of compilation. The conventional compiler approach, described above, replaces matrix operations with loops and indexed array accesses which are then optimized using standard compiler technology. The source-level approach advocated in this paper has the opposite effect since it replaces loops and indexed accesses by high-level matrix operations! This is somewhat counter-intuitive because type and shape checks, dynamic resizing and array bounds checks are not explicit in high-level matrix programs, so it is not obvious how these overheads are reduced by source-level transformations. Nevertheless, we show that such source-level transformations are beneficial regardless of whether the transformed code is executed by the MATLAB interpreter or compiled to C or FORTRAN.

The rest of this paper is organized as follows. A more detailed discussion of the overheads of interpreting MATLAB programs is given in Section 2. Section 3 discusses how the MCC MATLAB to C compiler attempts to eliminate these overheads. We also show the effect of using various compiler flags in MCC such as the `-i` flag to eliminate array bounds checks. Section 4 describes the source-level transformations of interest to us and evaluates their effect on performance if the resulting code is interpreted by the MATLAB interpreter. Section 5 argues that source-level transformations are useful even if the transformed code is compiled to native code by the MCC compiler. Section 6 compares our work with previous work. Section 7 gives a sketch of how these transformations can be performed automatically by a restructuring compiler; details of an implementation can be found in a companion paper [14].

2 Interpretation Cost of MATLAB programs

Our work-load in this paper is the FALCON benchmark set from the University of Illinois, Urbana [3]. This is a set of 12 programs from the problem domain of computational science, and it contains iterative linear solvers (CG,QMR), finite-difference solvers for pdes (CN,SOR), preconditioner computation for iterative linear solvers (IC), etc. These benchmarks are described in Table 1. In terms of MATLAB behavior, De Rose [3] groups the programs into three separate categories. *Library-*

intensive programs (CG, Mei, QMR, SOR) operate upon entire matrices via high-level operations or routines. These codes contain few, if any, indexed accesses into arrays. *Elementary-operation-intensive* programs (CN, Di, FD, Ga, IC) operate upon elements of matrices via loops. Virtually the entire execution time is spent within loop nests operating on array elements. *Memory-intensive* programs (AQ, EC, RK) require considerable memory management overhead in the form of dynamic resizing.

To measure the overhead of MATLAB interpretation, we would have liked to execute our benchmark suite on a suitably instrumented MATLAB interpreter. Unfortunately, the MathWorks interpreter is proprietary code, so we did not have access to the source. We considered using publicly available MATLAB-like interpreters and even instrumented one of them (Octave [6]), but we found that they were sufficiently different from the MathWorks interpreter that we could not draw meaningful conclusions about the performance of the MathWorks interpreter from experiments on other interpreters. For example, Octave is written in C++ and it uses the type-dispatch mechanism of C++ to implement the type checking and type dispatch required to execute MATLAB matrix operations, so there is no direct way to measure this overhead. Therefore, we had to make do with measuring the effect of program transformations on overall performance. In this section, we describe the interpretive overheads in more detail.

2.1 Type and Shape Checking

Unlike in C or FORTRAN, array variables in MATLAB programs can be introduced without type declarations. Furthermore, a single variable can name matrices of different types, shapes and sizes in different parts of the program. The complexity that this introduces in the interpreter can be appreciated by considering the assignment `C = A*B`. The type and shape of `A` and `B` determine what computation is performed, so `A*B` may refer to scalar-scalar multiplication, scalar-matrix multiplication, or matrix-matrix multiplication where neither, either, or both of the arguments are complex. Each possible combination specifies a different kind of computation. Furthermore, the interpreter may also test for special cases where, for example, one of the arguments is a row or column vector. While a vector could be treated just as any other matrix, more efficient underlying implementations exist for multiplying a matrix with a vector. Finally, the interpreter may also have to test for legality; in the case of matrix-matrix multiplication, the second dimension of `A` and the first dimension of `B` must conform.

The MATLAB interpreter tests for all of the above possibilities each time it encounters the `*` operator. However, examining the context in which the expression occurs may reveal that the tests are redundant or even completely unnecessary. Consider, for example, the `*` operator in the code:

```
for i = 1:n
  y = y + a*x(i);
end
```

In this case, the cost of the checks is magnified in the interpreter as they are performed in each iteration of the loop. However, note that `x(i)` must be scalar, so no test is needed for it. Without additional information, the type and shape of `a` must be checked, but since `a` is not modified within the loop, these properties need to be tested just once, before the loop is executed.

2.2 Dynamic Resizing

In C or FORTRAN, storage for an array is allocated before its elements are computed. Since there are no variable declarations in MATLAB, storage for matrices and vectors is allocated incrementally during program execution. An attempt to write into a matrix element outside the bounds of the matrix causes the system to reallocate storage for the entire matrix, copying over all elements from the old storage to the newly allocated space. In loops, such memory management overheads can become prohibitively expensive. Consider the following code:

```
for i = 1 : 10000
  x(i)= i;
end
```

If `x` is initially undefined, the interpreter “grows” the vector incrementally during loop execution. On a Sparc 20, the MATLAB interpreter requires 14.2 seconds to execute this loop. However, it is clear before the execution of the loop that `x` will grow to

Benchmark		Flops	Lines of Code
AQ	Adaptive Quadrature Using Simpson's Rule	3.6×10^5	87
CG	Conjugate Gradient method	3.7×10^7	36
CN	Crank-Nicholson solution to the heat equation	2.2×10^6	29
Di	Dirichlet solution to Laplace's equation	1.9×10^6	39
FD	Finite Difference solution to the wave equation	2.3×10^6	28
Ga	Galerkin method to solve the Poisson equation	1.3×10^6	48
IC	Incomplete Cholesky Factorization	7.6×10^5	33
Mei	Generation of 3D-surface	1.7×10^7	28
EC	Two body problem using Euler-Cromer method	3.3×10^5	26
RK	Two body problem using 4th order Runge-Kutta	4.4×10^5	66
QMR	Quasi-Minimal Residual method	1.2×10^8	91
SOR	Successive Over-relaxation method	8.9×10^7	29

Table 1: Falcon Benchmark Suite

10,000 elements. If a vector `x` of this length is preallocated before the loop begins, the loop executes in 0.37 seconds. In other words, repeated reallocation in the loop slows the loop down by a factor of 40 in this case. Note that the MATLAB interpreter only allocates the minimal amount of memory each time. That is, if the vector is not preallocated before the loop, then, on each iteration, the vector is reallocated into a memory block one element larger.

Interestingly, this overhead in the MATLAB interpreter is not nearly as significant for two dimensional arrays. Consider:

```
for i = 1 : 100
  for j = 1 : 100
    y(i,j)= i;
  end
end
```

Again, we have an array eventually resized to hold 10,000 elements. However, in this case, reallocation is not done on each iteration. In the first iteration of the `i` loop, each iteration of the `j` loop resizes `y` by an additional column. In subsequent iterations of the `i` loop, only the first iteration of the `j` loop causes resizing. That iteration resizes `y` to an $i \times 100$ array. No other iteration triggers resizing. When `y` is initially undefined, the interpreter requires 0.67 seconds to execute the above loop. With `y` already allocated, the interpreter requires 0.48 seconds.

While it is clear that the two dimensional case requires fewer memory reallocations, it is also true that it requires less data copying. In the one dimensional case, iteration `i` requires copying of $i-1$ data elements. An entire n^2 element vector (where $n = 100$ in our example) requires $O(n^4)$ copies. On the other hand, for an equivalent size $n \times n$ array, $O(n^2)$ copies are required for the first row, and $(i-1) * n$ copies are required for each subsequent row `i`. Thus, the total number copies for the two dimensional array is $O(n^3)$, asymptotically smaller than the one dimensional case.

We conclude that the overhead of dynamic resizing is most important when vectors are resized within loops.

2.3 Array Bounds Checking

Indexed accesses into arrays are another source of run-time checks in MATLAB. Consider the following code:

```
x(i) = y(i);
```

The index `i` is checked to see if it is within the bounds of `x` and `y`. If it is not within the bounds of `x`, it triggers resizing as explained above. If it is not within the bounds of `y`, an error is reported.

As with type and shape checks, array bounds checks are often redundant, as in the code:

```
for i = 2:n-1
  x(i) = x(i-1)+x(i+1);
end
```

As before, the loop magnifies the overhead in the interpreter since three checks are performed in each iteration. Clearly, the three checks performed on the inner statement are redundant and can be collapsed to one. The array `x` must contain at least `i+1` elements for the statement to be legally executed. The other two checks are subsumed by this check. Furthermore, this remaining check need not be performed each iteration. If `x` does not contain at least `n` elements, it is clear that the loop cannot execute correctly.

3 Conventional Compilation

In this section, we examine the standard approach to compiling away the interpretive overheads of MATLAB programs. The key idea is to translate MATLAB code into C or FORTRAN programs, making type checking, dynamic resizing, etc. explicit and therefore amenable to optimization. We describe how the commercial MathWorks MCC compiler removes these overheads and quantify its effectiveness.

3.1 Type Inference

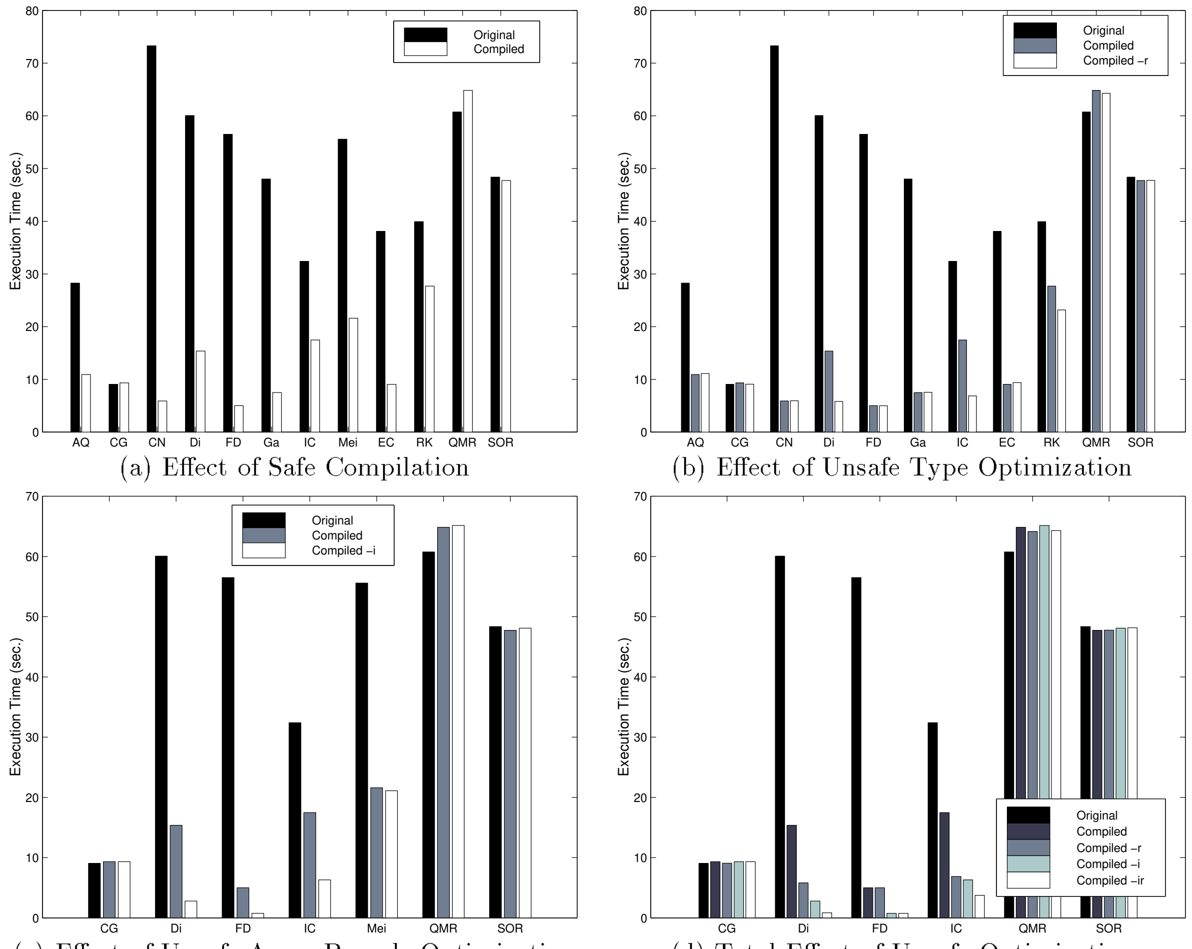
The MCC compiler attempts to eliminate type and shape checks through the use of type and shape inference. The most sophisticated type inference algorithm in the literature is the one in the FALCON compiler of De Rose et al. [3, 4]. The algorithm used in MCC is unpublished, but it appears to be similar. The high level idea is to generate a system of type equations relating the types and shapes of different variables and solve this system to determine what these types and shapes can be. In particular, type and shape information of inputs to expressions can be used to determine type and shape information of outputs.

MCC operates at the level of single MATLAB functions, or *m-files*. When MCC performs inference on a function, the result is a forward propagation algorithm in which the types and shapes of parameters and initialized local variables in a program are used to determine the types of intermediate and output variables. This, however, requires that type and shape information be known for parameter variables; this is very difficult to infer automatically. MCC's strategy is to generate two versions of compiled code for each function: one that assumes that all inputs are real and one that assumes all are complex. The compiler then inserts an initial run-time test at the beginning of execution to determine if, in fact, all inputs are real.

Figure 1a demonstrates the effectiveness of the MCC compiler on the Falcon benchmarks on a Sun Sparc 20 machine. For this set of measurements, no additional compile-time flags were used; we will consider the effect of the two available optimization flags below. Note that the compiler is most effective on loop-nest intensive codes. On codes that are not loop intensive and predominantly utilize high-level operations, such as CG, QMR and SOR, the compiler shows no performance benefit. Surprisingly, the compiled code actually shows a slight slowdown in two of these cases. In these cases, type checking and type dispatch overhead is relatively insignificant compared to the time taken by the actual computation.

Users may obtain better performance from MCC by directing it to perform additional optimizations through the use of compilation flags. These optimizations, unlike the default ones, are potentially unsafe since they may by illegal for some programs. One unsafe optimization, triggered by a `-r` option to the compiler, eliminates all tests for complex types and as a result, generates code that assume all computation is on real numbers. Obviously, this will not produce correct results for programs operating on complex numbers. In the Falcon benchmarks, however, this optimization is applicable on all but one of the benchmarks (Mei). Figure 1b illustrates the effect of this optimization on the remaining eleven benchmarks.

There are noticeable performance gains in only three benchmarks (Di, IC, RK). As mentioned above, the default behavior of MCC is to generate two versions of compiled code in which one version assumes that all parameters are real. In this version, the compiler is usually able to determine that all intermediate and output variables are real as well. If this is the case, the compiler will eliminate all checks for complex values. The only additional advantage brought by the `-r` option is to eliminate the initial test on input values which, as seen in Figure 1b, is negligible. However, real input variables may not necessarily imply real intermediate or output variables. Certain operations, such as a square root, may produce complex values from real ones. In the presence of such operations, type inference will fail to eliminate all type checks. For example, in the In-



(a) Effect of Safe Compilation

(b) Effect of Unsafe Type Optimization

(c) Effect of Unsafe Array Bounds Optimization

(d) Total Effect of Unsafe Optimization

Figure 1: MCC Compilation

complete Cholesky benchmark, each column of the matrix is scaled by the square root of the diagonal element. Even though the input matrix is real, MCC is unable to infer that the result matrix will also be real. In these cases, the `-r` option, if applicable, can noticeably enhance performance.

3.2 Array Bounds Optimization

The MCC compiler does not optimize away any array bounds checks. However, the programmer can trigger bounds check elimination by using another compilation flag (the `-i` flag). As described earlier, this is not safe even for correct programs because out-of-bounds writes to an array are used to trigger dynamic resizing. Furthermore, if the code has an out-of-bounds read access, the compiled code generated by using this flag may produce either incorrect results or catastrophic errors.

In the Falcon benchmarks, bounds check elimination is valid in seven of the twelve programs (in the remainder, the interpreter either halts with an error or produces incorrect results). The effect of using the `-i` compiler flag on these programs is shown in Figure 1c. There are substantial improvements in three of the seven benchmarks for which this flag is legal (Di, FD, IC). The remaining benchmarks predominantly utilize higher-level functions and contain few, if any, subscripted references to arrays.

3.3 Discussion

The overall effect of compilation, using compiler flags to eliminate type and shape checks as well as array out of bounds checks where legal, is shown in

Figure 1d. Eliminating type checks by using the `-r` flag is useful in the Di, IC and RK benchmarks while elimination of array bounds checks by using the `-i` flag is most effective in the Di, FD and IC benchmarks. Note that when a compiler flag is unsafe for a program, it may still be possible to apply the corresponding optimization to just a portion of the program. In the Galerkin benchmark, for example, dynamic resizing occurs within the function, and so the `-i` option will generate erroneous code. However, it does not occur within the innermost loop, where array bounds checks are most expensive. Performance measurements at this finer level of granularity require access to the MathWorks interpreter and compiler.

4 Source-Level Optimizations

In this section, we discuss source-level transformations of MATLAB programs and show how they can be used to reduce interpretive overhead. At first glance, this may seem to be a counterintuitive idea since the language provides no direct means of instructing an interpreter when and when not to perform various checks. *The key insight is that these overheads are most significant in loops, so loops can be transformed to eliminate interpretive overhead.* In this section, we discuss three different source-level transformations and show how they improve the efficiency of our work-load. In the next section, we compare these performance improvements with the performance improvements obtained by the MCC compiler.

4.1 Vectorization

Vectorization transforms loop programs into high-level matrix operations. This is similar to vectorization for vector supercomputers; in both cases, the key is to map a sequence of operations on array elements into one or more high-level operations on entire arrays. On vector hardware, these array operations can be executed more efficiently than loops with scalar operations. A similar gain in efficiency is possible in MATLAB interpretation. Loops slow down MATLAB programs by magnifying the overhead of statements contained with in the loop. Any type checks or array bounds checks performed on a statement within the loop will be repeated for every iteration even though multiple checks may be redundant. However, for higher level MATLAB operations that act on entire matrices and vectors, these checks are performed only once. Hence, the performance benefit of high level operations can be very large.

Consider the execution profile of the Galerkin benchmark in Figure 3a. This loop nest represents a small portion of the program but it is clearly the bottleneck in performance since 97% of the execution time of the entire program is spent within this loop nest. However, the entire loop nest can be vectorized by realizing that it is actually performing a vector-matrix-vector multiplication. When this loop nest is replaced by equivalent matrix-vector operations, the resulting profile is as shown in Figure 3b. This transformation enhances the performance of the loop nest by a factor of 250, and the performance of the entire benchmark is increased 100-fold!

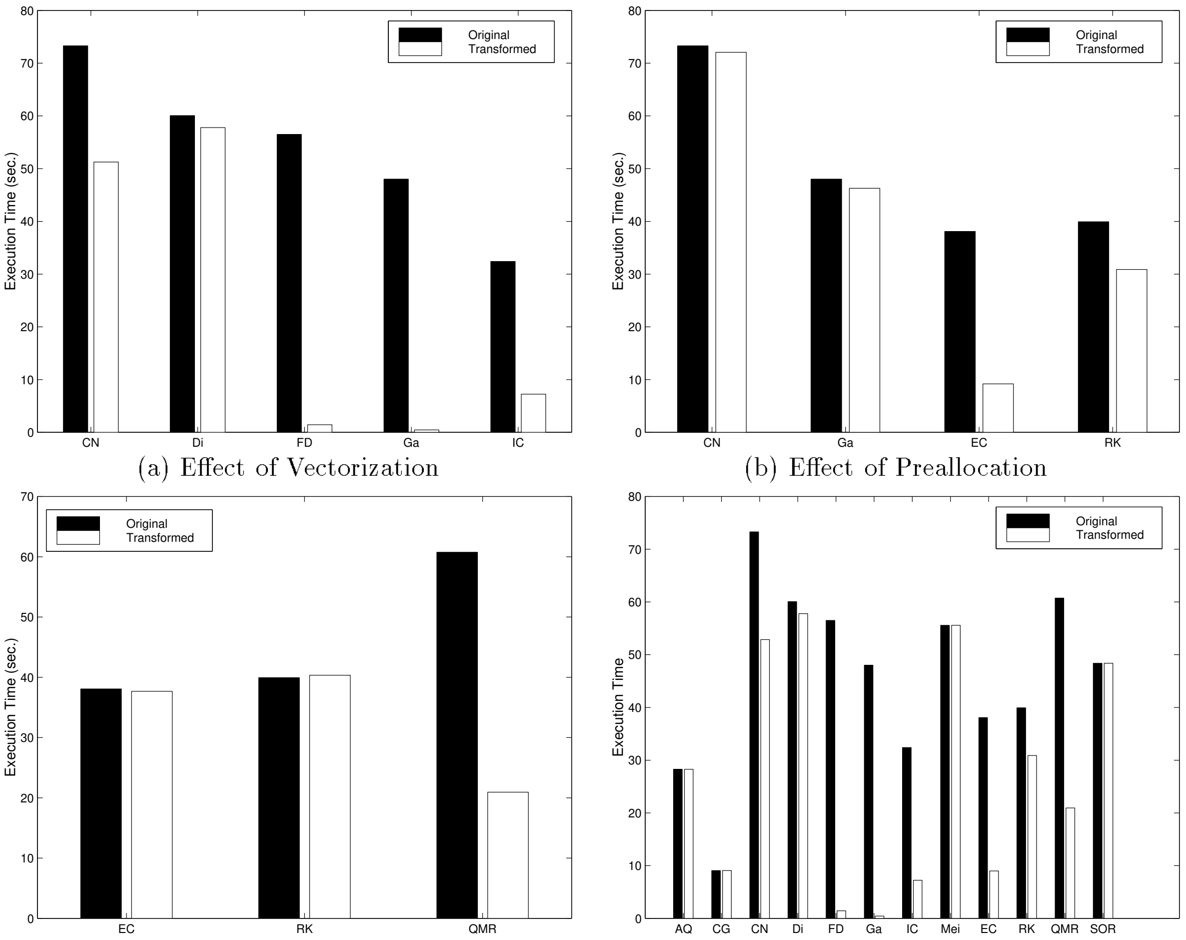
Of course, vectorization is not always applicable. Many programs such as CG and QMR in the Falcon suite already extensively utilize higher-level operations and contain no for-loops. In other programs, such as the Dirichlet code in Figure 4, expensive for-loops exist but cannot be mapped to higher-level operations. In this case, the dependences due to `U` prevent either of the for-loops from being vectorized.

Vectorization is applicable to five of the twelve Falcon benchmarks. The effects on each of these five programs is shown in Figure 2a. In two programs (FD, Ga), the effects are dramatic since they result in more than 30-fold improvements. In these cases, vectorizable loops were responsible for nearly all the original execution time. In one case (Di), the effect is minor. Here, the vectorizable loop took only a minor portion of the original execution time.

4.2 Preallocation

As discussed in Section 2.2, resizing of arrays can result in significant memory management overhead due to repeated reallocation and copying. In many cases, the final size of the array can be easily inferred. When this is the case, it is often safe to preallocate the entire array at once. Consider the original code for the Euler-Cromer program in Figure 5a.

87% of program execution time is spent in the lines



(a) Effect of Vectorization

(b) Effect of Preallocation

(c) Effect of Expression Optimization

(d) Total Effect of Source-level Optimizations

Figure 2: Source-to-Source Optimizations

shown. In this case, each of the arrays shown above is undefined prior to execution of this loop, so the array is resized on each iteration of the loop. However, it is clear in this case that ultimately, each array will be of length **nstep**. There is no way to declare the size of a matrix in MATLAB, but an indirect way to accomplish the same goal is to use the **zeros** operator that creates an array of a desired size and initializes its values to 0. Therefore, statements of the following form can be used to avoid resizing:

```
rplot = zeros(1,nstep);
```

When preallocation is done for all arrays, we obtain the profile shown in Figure 5b. Each of these statements is now significantly faster (by a factor of more than seven). As a result, the entire benchmark is faster by a factor of roughly four.

Unfortunately, simple preallocation as above cannot eliminate all instances of dynamic resizing. For example, in the AQ code, the final size of the array cannot be determined a priori. While more complex strategies such as preallocating an estimated size or explicitly growing the array in an exponential manner may reduce this cost, we have not attempted to do so in this paper.

Finally, even when preallocation can be done, the performance benefits will differ from case to case. The Euler-Cromer code represents a relatively extreme case since several one dimensional arrays are resized, and the final size of each (over 6,000) is fairly large. As mentioned in Section 2.2, the resizing overhead is significantly less with two dimensional arrays.

Dynamic array resizing occurs in five of the twelve Falcon benchmarks. In four of these cases (AQ is

a) Original Galerkin Code:

```
             39: for i=1:N
  0.24s,  1% 40:   xtemp = cos((i-1)*pi*x/L);
  0.23s,  1% 41:   for j=1:N
 16.36s, 88% 42:     phi(k) = phi(k) + a(i,j)*xtemp*cos((j-1)*pi*y/L);
  1.26s,  7% 43:   end
  0.03s,  0% 44: end
```

b) Transformed Galerkin Code:

```
  0.01s,  1% 39: xtemp_se = cos((0:N-1)*pi*x/L);
  0.06s,  9% 40: phi(k) = phi(k) + xtemp_se*a*cos((0:N-1)*pi*y/L)';
```

Figure 3: Effect of Vectorization on Galerkin Benchmark

Original Dirichlet Code:

```
             53: for j=2:(m-1),
  0.72s,  1% 54:   for i=2:(n-1),
 33.91s, 53% 55:     relx = w*(U(i,j+1)+U(i,j-1)+U(i+1,j)+ U(i-1,j)-4*U(i,j));
 20.39s, 32% 56:     U(i,j) = U(i,j) + relx;
  6.13s, 10% 57:     if (err<=abs(relx))
  0.06s,  0% 58:       err=abs(relx);
  0.02s,  0% 59:     end
  2.13s,  3% 60:   end
  0.05s,  0% 61: end
```

Figure 4: Dirichlet Benchmark

a) Original Euler-Cromer Code:

```
            18: for istep=1:nstep
 8.75s, 18% 19:   rplot(istep) = norm(r);
 8.05s, 17% 20:   thplot(istep) = atan2(r(2),r(1));
 6.99s, 15% 21:   tplot(istep) = time;
 8.89s, 19% 22:   kinetic(istep) = .5*mass*norm(v)^2;
 8.49s, 18% 23:   potential(istep) = - GM*mass/norm(r);
                  ...
 0.15s,  0% 29: end
```

b) Transformed Euler-Cromer Code:

```
            18: for istep=1:nstep
 1.12s, 10% 19:   rplot(istep) = norm(r);
 0.85s,  7% 20:   thplot(istep) = atan2(r(2),r(1));
 0.24s,  2% 21:   tplot(istep) = time;
 1.78s, 15% 22:   kinetic(istep) = .5*mass*norm(v)\^{}2;
 1.44s, 13% 23:   potential(istep) = - GM*mass/norm(r);
                  ...
 0.12s,  1% 29: end
```

Figure 5: Effect of Preallocation on Euler-Cromer Benchmark

a) Original QMR Code:

```
19.10s, 70% 50: w_tld = ( A'*q ) - ( beta*w );
```

b) Transformed QMR Code:

```
0.80s,  9% 50: w_tld = ( q'*A )' - ( beta*w );
```

Figure 6: Effect of Expression Optimization on QMR Benchmark

the exception), the eventual size of resized arrays is easily determined. Figure 2b highlights the effect of preallocation on these four benchmarks.

4.3 Expression Optimization

Finally, we consider a source-level transformation not directly motivated by MATLAB overheads. Instead, we are motivated by the naivete of MATLAB's evaluation process. Unlike an optimizing compiler, the MATLAB interpreter does not consider the best manner in which to compute an expression. Instead, it blindly computes it in the most straightforward manner. Consider the profile information from the QMR benchmark in Figure 6a.

In an eighty line program, this single statement requires 70% of the entire execution time. Closer examination of this program reveals that `beta` is a scalar, `w_tld`, `q`, and `w` are column vectors, and `A` is a two-dimensional matrix. Thus, the subexpression $\mathtt{A}^{\mathbf{T}}\mathtt{*q}$ is clearly the most expensive to compute, requiring $O(n^2)$ work to perform a matrix-vector product. However, the MATLAB interpreter will also compute a temporary matrix for the value $\mathtt{A}^{\mathbf{T}}$, requiring an additional n^2 space and and copy operations, before it computes the product. Clearly, a temporary matrix should not be necessary to perform the computation. Unfortunately, the MATLAB language does not provide a way of expressing this computation as a single operation, thus forcing the evaluation of the subexpression. While a source-level transformation cannot directly avoid this, it can reduce the cost by realizing that $(\mathtt{q}^{\mathbf{T}}\mathtt{*A})^{\mathbf{T}}$ is an equivalent and less expensive expression. Although this expression requires two transpose operations, in both cases vectors are transposed instead of matrices. Note the profile of the transformed code in Figure 6b.

The result is a better that twenty-fold increase on that single statement and a three-fold increase on the entire benchmark. These kinds of transformations that exploit the semantics of matrix operations are not feasible at the C/FORTRAN level.

5 Comparison of Source-level Transformations and Compilation

In this section, we compare the separate and the combined effects of source-level and compiler optimizations. Figure 7 shows the performance benefits realized by different combinations of optimizations. The first three sets of bars represent the original MATLAB code, MCC compiled code with no unsafe optimizations, and MCC compiled code with all unsafe optimizations legal for that particular benchmark activated. The second three sets of bars represent the same measurements taken on source-level transformed code.

There are a number of interesting observations to be made. First, the performance improvement from source-level optimizations is quite comparable with performance improvement from MCC compiler optimizations. In four cases (CG, EC, RK, SOR), source-level optimizations have a roughly similar effect on performance when compared to safe MCC optimizations. In four other cases (FD, Ga, IC, QMR), source-level optimizations are better by a factor of two or more. In two of these cases (Ga, QMR), source-level optimizations outperform even unsafe optimizations by a wide margin! On the other hand, the remaining four cases (AQ, CN, Di, Mei) profit much more from compilation than source-level optimizations. These codes all contained expensive loops performing scalar operations that could not be eliminated by vectorization.

Second, source-level optimizations are best viewed as being complementary to compiler optimizations.

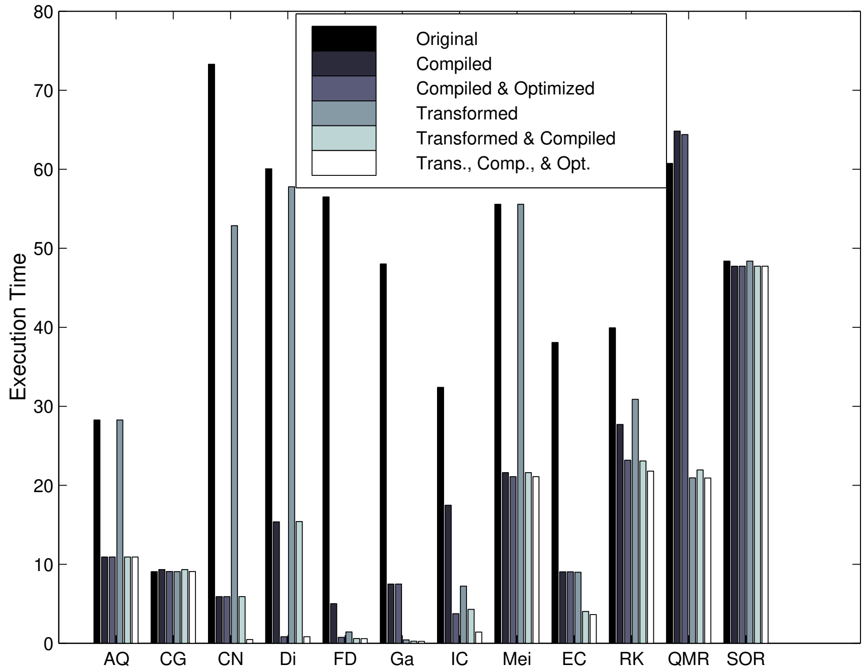


Figure 7: Source-Level Transformations and Compilation

The last two set of bars in Figure 7 illustrate the combined effect of source-level and compiler optimizations. For programs in which source-level transformations result in improved interpreted performance, these transformations result in improved compiled performance as well. Across all benchmarks, the combination of optimizations provides the best performance. In five cases (CN, FD, Ga, IC, EC), the combination significantly exceeds the effect of either source-level or compiler optimizations alone. Each of the different source-level transformations provides benefits not currently provided by MCC.

- When vectorization can generate high-level operations such as those in BLAS, as in Galerkin, efficient underlying libraries may be utilized by both interpreter and compiler. These libraries outperform code generated by a C or FORTRAN compiler on the corresponding loops.
- Although preallocation does not significantly effect the performance of every benchmark where it is applicable (see Figure 2b), it permits the otherwise unsafe compiler optimization of array bounds removal. The effect of this optimization after preallocation on, for example, Crank-Nicholson is dramatic.
- Algebraic optimizations such as the one used on QMR have no equivalent in MCC. These types of optimizations dependent on matrix properties cannot easily be applied at the lower level of a C compiler.

We conclude from these results that source-level transformations are key to obtaining good performance from MATLAB codes for both interpreted and compiled execution.

6 Related Work

As mentioned in the introduction, several MATLAB compilers have been developed or are under development. Each of these compilers translates MATLAB into a lower language such as FORTRAN, C, or C++.

There are two commercial MATLAB compilers. MCC [13] is from the The MathWorks, developers of MATLAB itself, and is the compiler studied in this paper. MCC can handle most features in MATLAB 5 and generates C code. MATCOM [9], originally developed at the Israel Institute of Technology and now offered by MathTools, also handles most features of MATLAB 5 and generates C++ code. Both MCC and MATCOM are capable of generating either stand-alone programs or mex-files that may be linked back into the MATLAB interpreter. As far as we are aware, these are the only two publicly available compilers and the only two capable of generating mex-files.

There are a handful of compilers under development in academia. Falcon [4] from University of Illinois, translates MATLAB 4 into FORTRAN-90. Menhir [2], from Irisa in France, focuses on a retargetable code generator capable of generating C or FORTRAN for sequential or parallel machines. MATCH [18], from Northwestern, uses MATLAB to directly target special purpose hardware. Three other compilers, CONLAB [5], from the University of Umea in Sweden, Otter [19], from Oregon State University, and one other from Northwestern University [20], explicitly target parallel machines by generating message passing code from MATLAB.

Of all of these compilers, only Falcon appears to consider source-level transformations [4, 11] along the lines of those described in this paper. However, these transformation must be applied interactively via a user tool and are not part of the automatic compilation process. Furthermore, the source-level transformations are limited to syntactic pattern match and replacement, so they do not provide a general solution for optimizations such as vectorization and preallocation. A similar type of idiom recognition appears to be performed by optimizing FORTRAN preprocessors such as KAPF [10] and VAST-2 [16]. These tools do attempt to detect matrix products in loop nests in order to generate BLAS operations. However, pattern matching is inherently limited in its ability to do this; neither processor is able to detect a vector-matrix-vector product written as in the Galerkin code shown in Section 4.1.

There has been compiler research on performing optimizations similar to the source-level transformations presented in this paper. Vectorization for vector supercomputers has been studied extensively over the past three decades, for example in [1, 21]. However, this work largely focuses on point-wise assignments and scalar operations between arrays and, occasionally, on reduction operations rather than the higher-level operations available in MATLAB. The problem of array bounds removal, similar to preallocation and directly applicable to MATLAB, has been studied in the context of conventional languages [7].

Finally, a handful of projects [8, 15, 17] have developed parallel toolkits for use with the MATLAB interpreter. These toolkits allow MATLAB programs to directly access message passing libraries for interprocessor communication. To gain any performance benefit, users must parallelize their MATLAB programs using provided MATLAB-level message passing constructs.

7 Conclusion and Future Work

Source-level transformations provide an effective means of obtaining performance for MATLAB programs, regardless of whether they are interpreted or later compiled. These transformations are capable of eliminating many inefficiencies that currently available MATLAB compilers are unable to optimize away.

We have implemented an automatic tool to perform source-level optimizations as part of the FALCON project, a joint project of Cornell University and the University of Illinois, Urbana. A detailed description of this tool can be found in a companion paper [14]. Incorporating both source-level and lower-level optimizations into an interpreter in a just-in-time manner is an ongoing effort.

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