Transformations of Imperfectly Nested loops

Induprakas Kodukula

Keshav Pingali

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Organization

- Linear framework
- Transformations
- Code generation and Completion
- Related Work and Conclusions

Imperfectly Nested Loops

Loop is perfectly nested if

- body is completely surrounded by all the loops

Loop is imperfectly nested if

- it is not perfectly nested

```
for I = 1 to N do
  S1: A[i] = sqrt (A[I])
  for J = I+1 to N do
    S2: A[J] = A[J] / A[I]
```

Existing Framework - perfectly nested loops

What is nice about it?

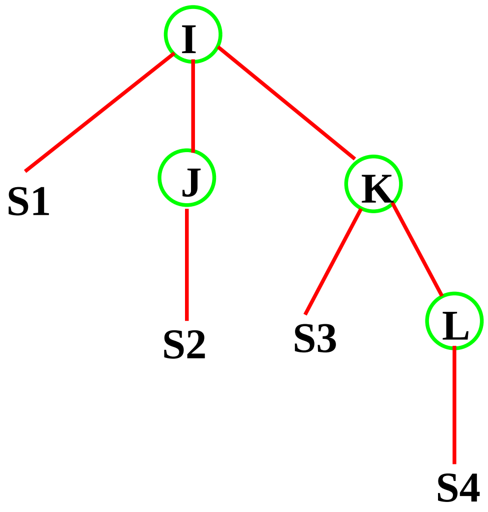
- Execution mapped to integer space modeling program execution order
- Transformations as matrices
- Composition
- Simple legality test
- Automatic code generation

Banerjee, Li and Pingali, Padua, Wolf and Lam,....

Imperfect Loop as AST

- Internal nodes $\equiv$ loops
- Edges $\equiv$ parent-child relationship
- Leaves $\equiv$ atomic statements

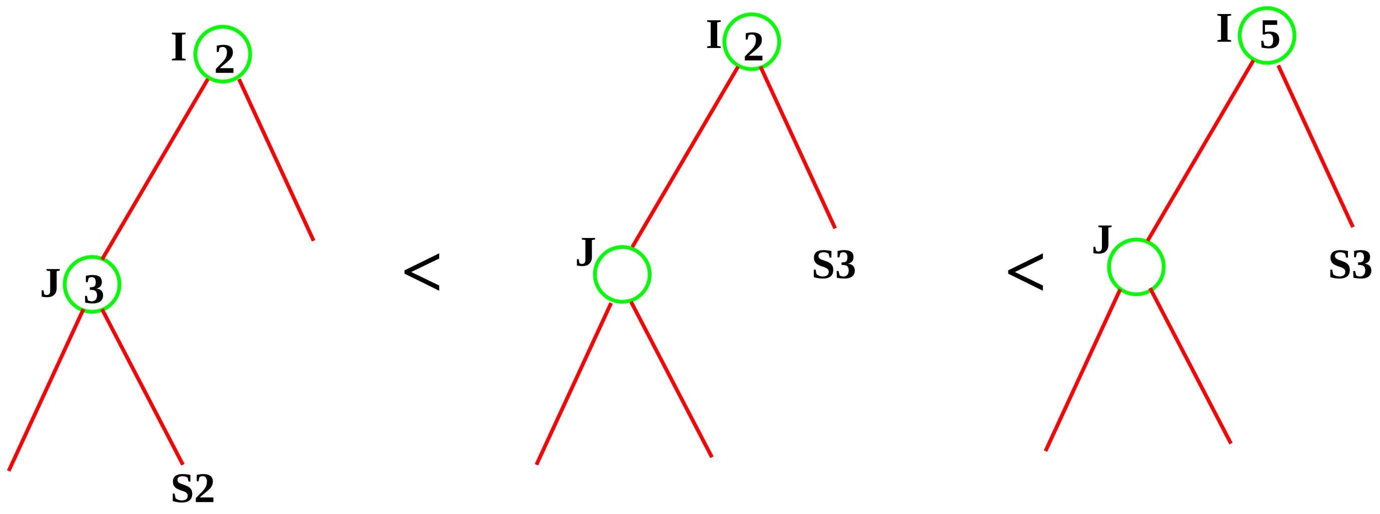
```
for I
    S1
    for J
        S2
    for K
        S3
        for L
            S4
```



Desired - Encode statement execution

Want a vector encoding which:

- satisfies uniqueness
- preserves execution order as lexicographic order



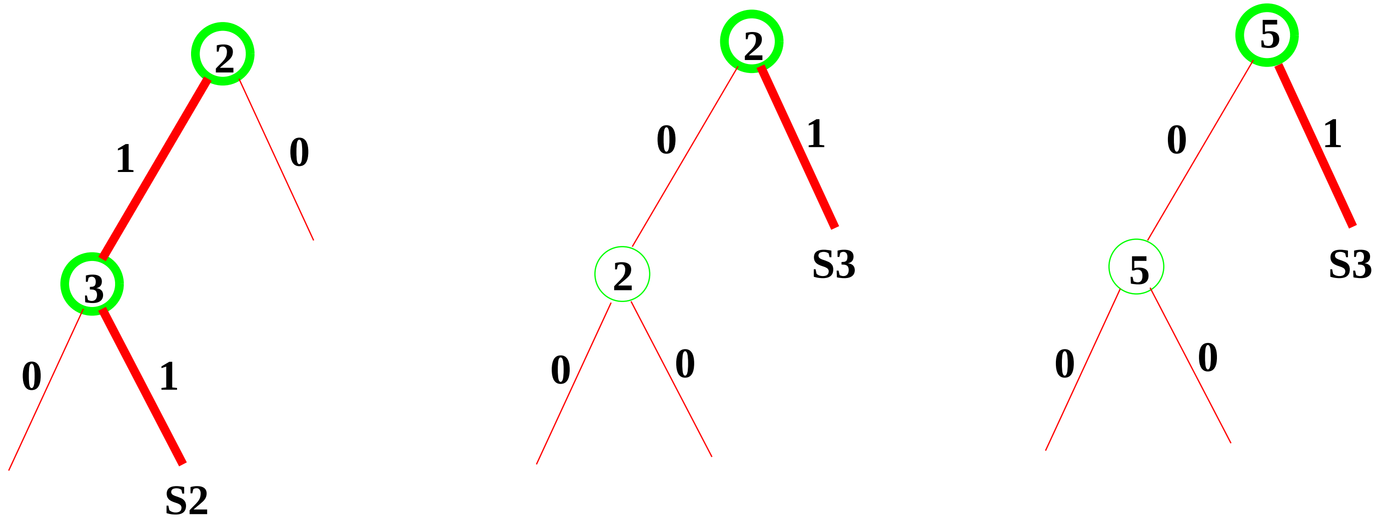
Rules to label the AST

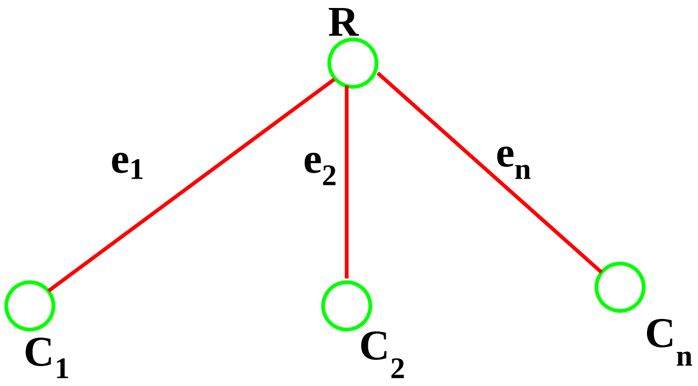
$\mathfrak{M}_e =$

- 1 if e is marked
- 0 otherwise

$\mathfrak{M}_n =$

- loopvar value if n is marked
- loopvar value of first marked ancestor otherwise





$\mathfrak{L}(R) =$

- for leaf nodes

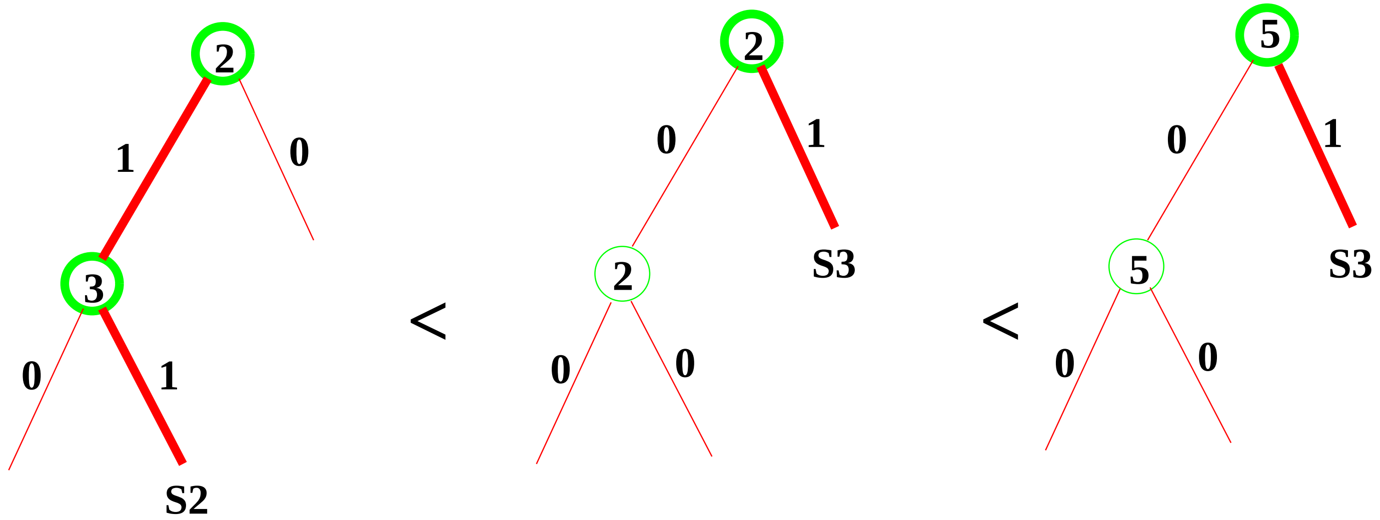
  ϵ

- For internal nodes

  $$\mathfrak{M}_R // \mathfrak{M}(e_n) // \ldots // \mathfrak{M}(e_2) // \mathfrak{M}(e_1) // \mathfrak{L}(C_n) // \ldots // \mathfrak{L}(C_2) // \mathfrak{L}(C_1)$$

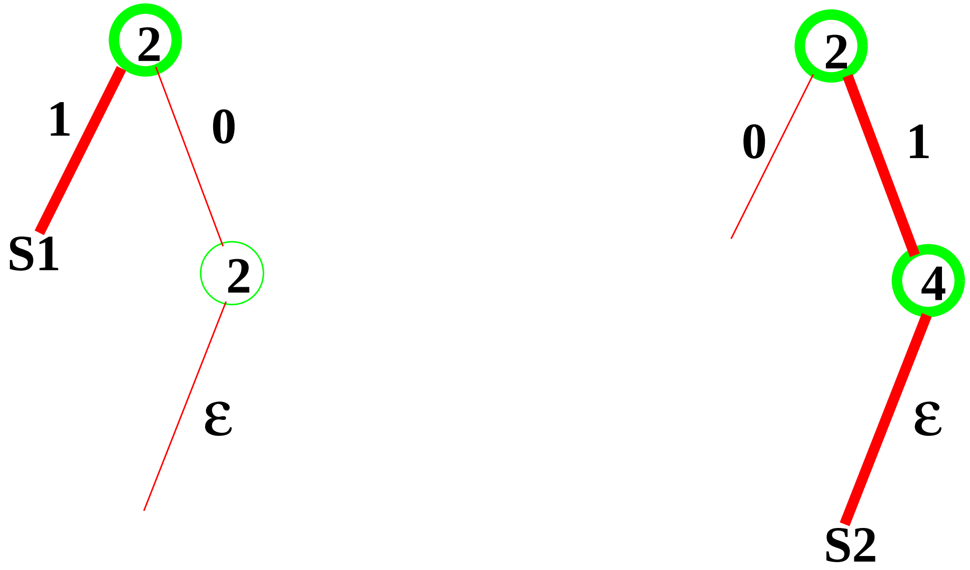
Labeled AST $\equiv \mathfrak{L}(root) \equiv$ Instance Vector

Putting it all together



$$\begin{bmatrix}2\\0\\1\\3\\1\\0\end{bmatrix} < \begin{bmatrix}2\\1\\0\\2\\0\\0\end{bmatrix} < \begin{bmatrix}5\\1\\0\\5\\0\\0\end{bmatrix}$$

Optimizing single edges



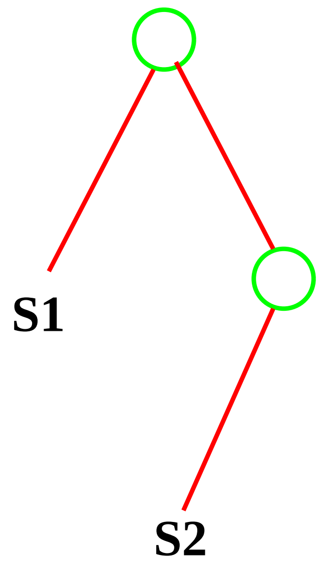
$$\begin{bmatrix} 2 \\ 0 \\ 1 \\ 2 \end{bmatrix} \qquad \begin{bmatrix} 2 \\ 1 \\ 0 \\ 4 \end{bmatrix}$$

Properties of Instance Vectors

- Program execution order $\equiv$ lexicographic order
- Flow dependence = $IV_{read} - IV_{write}$, etc
- Distance/direction vectors are lexicographically positive

Computing dependences

```
for I = 1..N
     S1: A[I]= sqrt (A[I])
     for J = I+1..N
          S2: A[J] = A[J]/A[I]
```



Flow S1→S2 on A[I]: $\begin{bmatrix} I_r \\ 1 \\ 0 \\ J_r \end{bmatrix} (read) - \begin{bmatrix} I_w \\ 0 \\ 1 \\ I_w \end{bmatrix} (write)$

```
for I = 1 to N do
  S1:  A[I]  = sqrt (A[I]);
  for J = I+1 to N do
    S2: A[J] = A[J] /  A[I] ;
```

$$1 \leq I_r \leq N, 1 \leq I_w \leq N, I_r < J_r$$
$$I_w \leq I_r$$
$$I_r = I_w$$

$$\Delta 1 = I_r - I_w, \Delta 2 = J_r - I_w$$

$$\begin{bmatrix} I_r \\ 1 \\ 0 \\ J_r \end{bmatrix} - \begin{bmatrix} I_w \\ 0 \\ 1 \\ I_w \end{bmatrix} = \begin{bmatrix} I_r - I_w \\ 1 \\ -1 \\ J_r - I_w \end{bmatrix} = \begin{bmatrix} \Delta 1 \\ 1 \\ -1 \\ \Delta 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \\ + \end{bmatrix}$$

- Model
- Transformations
- Code generation and Completion
- Related Work and Conclusions

Transformations

- Following transformations:
  - reversal, scaling, skewing and permutation
  - Distribution and Jamming
  - Statement alignment and reordering
- Individual transformations $\equiv$ Matrices
- Legality: $T * d > 0$ (similar to perfectly nested loop case)

Permutation and reordering

```
for I = 1 to N do
    S1: A[I] = sqrt (A[I]);
    for J = I+1 to N do
        S2: A[J] = A[J] / A[I];
```

$$\Downarrow$$

```
for J = 1 to N do
    for I = 1 to J-1 do
        S2: A[J] = A[J] / A[I];
    S1: A[J] = sqrt (A[J]);
```

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} I \\ 1 \\ 0 \\ J \end{bmatrix} = \begin{bmatrix} J \\ 0 \\ 1 \\ I \end{bmatrix}$$

Legality

T is legal w.r.t d if $T * d > 0$ (lexicographically positive)

T must be legal w.r.t all dependences in the initial code

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ -1 & 1 & 0 \\ + & 0 & 1 \end{bmatrix} = \begin{bmatrix} + & 0 & 1 \\ -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

- Can prove that all six versions of Cholesky are equivalent to each other

```
for I = 1 to N do
  S1: = .... A[I-4,I+2]
  for J = I to N do
    S2:  A[I,J]  = ...
```

Flow dependence S2⟶S1:

$$\begin{bmatrix} I_r \\ 0 \\ 1 \\ I_r \end{bmatrix} - \begin{bmatrix} I_w \\ 1 \\ 0 \\ J_r \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 1 \\ -2 \end{bmatrix}$$

Want parallel outer loop, i.e.

first entry of transformed dependence vector equal to 0

Statement Alignment

$$\begin{bmatrix} 1 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 4 \\ -1 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \\ -2 \end{bmatrix}$$

```
for I = -3 to N do
  if (I >= 1) then
    for J = I to N do
      S2:  A[I,J]  = ...
  if (I <= (N-4)) then
    S1: = .... A[I,I+6]
```

Loop Skewing

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 4 \\ -1 \\ 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -1 \\ -2 \end{bmatrix}$$

```
for I = 3 to 3*N do
  for J = ⌈ I/3 ⌉ to min(⌊ I/2 ⌋, N) do
    S2:  A[I-2*J,J]  = ...
  if (I mod 3 == 0) then
    S1: = .... A[(I/3)-4,(I/3)+2]
```

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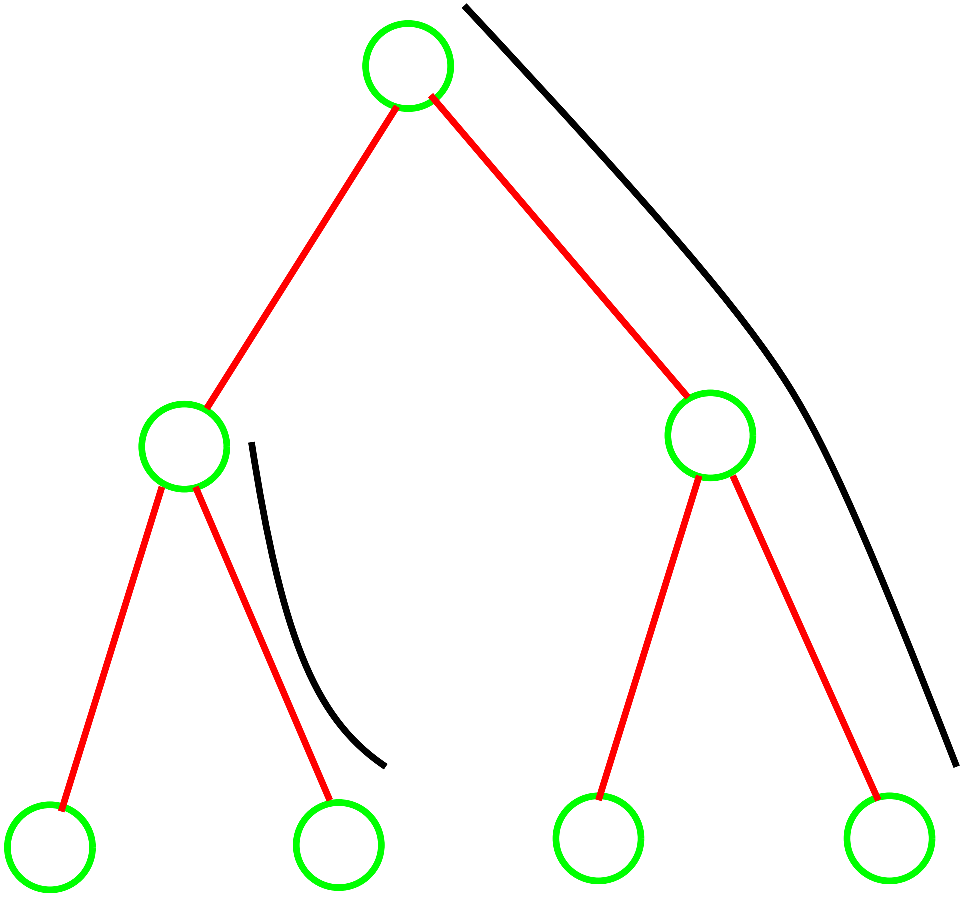
Multiple transformations on disjoint paths



Code generation

Input:

- Input program and matrix of transformation

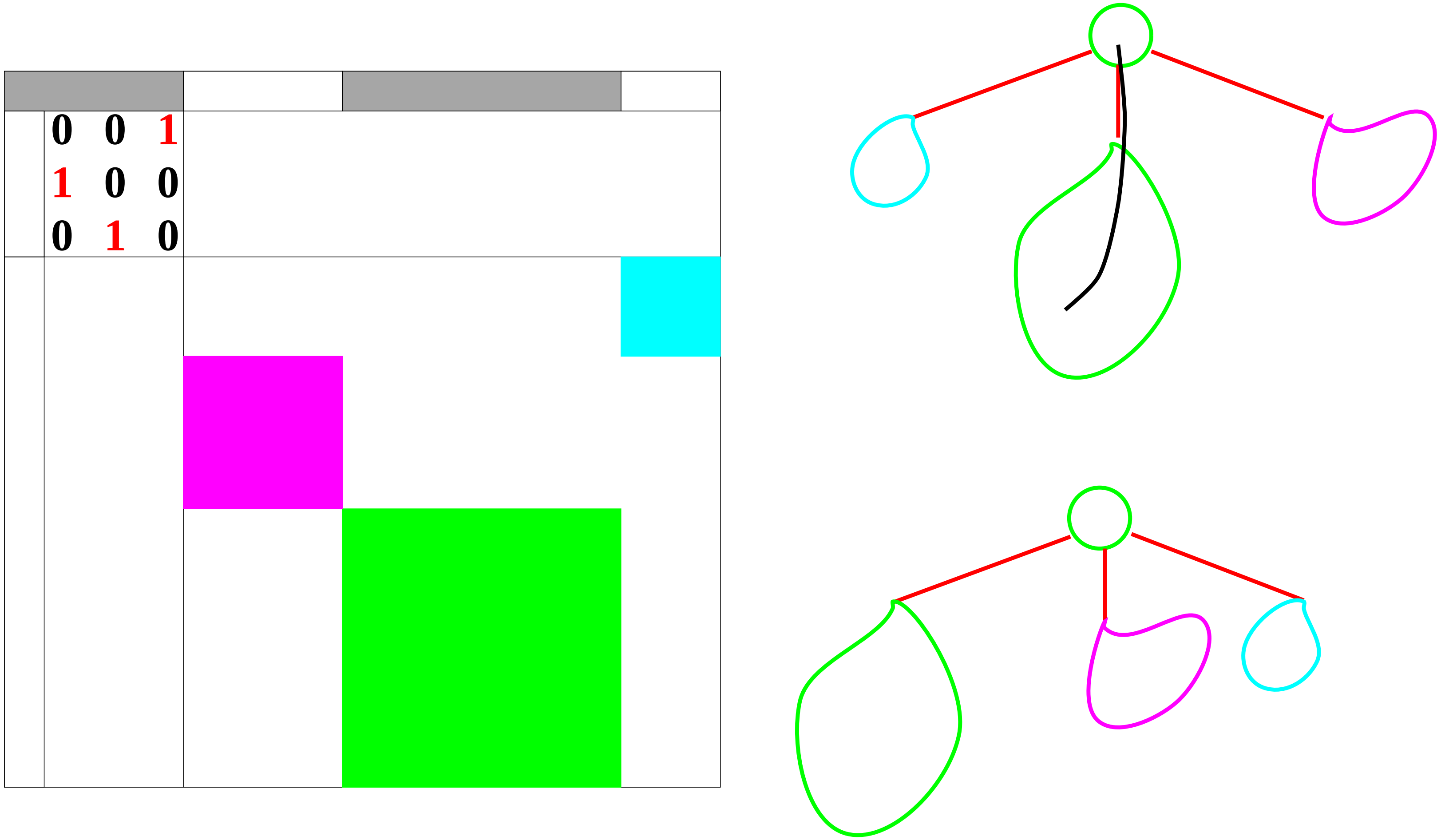
Output:

- Transformed program

How?

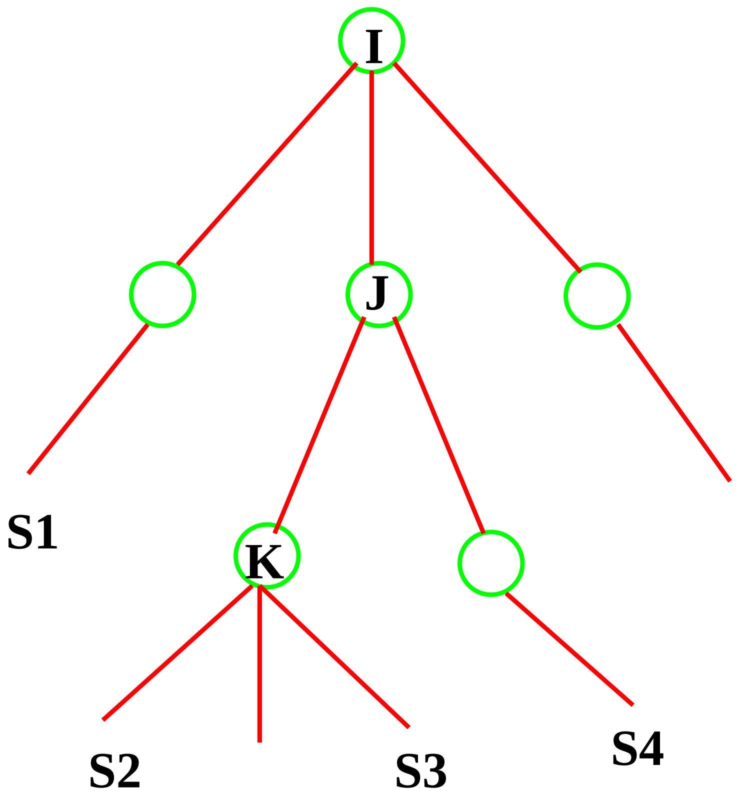
- Build transformed syntax tree
- For each statement, obtain bounds for surrounding loops
- Merge bounds for loops containing multiple statements and insert guards as necessary

Shape of a transformation



Discovering AST structure is a recursive problem

Completion - containment of dependences



Definition:

- A dependence between S and D is *contained* in the deepest common ancestor of S and D

Linear algebra formulation

Given two sets of dependence vectors $\mathfrak{D}_1$ and $\mathfrak{D}_2$

Find a vector v such that

$$\begin{aligned} \overrightarrow{v} * \overrightarrow{d_1}^T &\geq 0, d_1 \in \mathfrak{D}_1 \\ \overrightarrow{v} * \overrightarrow{d_2}^T &> 0, d_2 \in \mathfrak{D}_2 \end{aligned} \tag{1}$$

Completion Procedure

Input:

- Legal partial transformation & unsatisfied dependences

Output:

- Legal complete transformation

How?

- Generate a block at a time
- Ensure permutation for immediate children
- Must satisfy contained dependences
- Must not violate all other dependences

Related Work

- Started with Wolfe’s work - direct interchange
- Statement Alignment (Callahan, Ayuguade)
- Schedules (Feautrier) and Mappings (Kelly, Pugh)
  - Very general
  - Per statement, general affine remapping
  - Expressive, expensive

Conclusions

- Extension of linear framework for perfectly nested loops
- More restricted than *mappings, schedules*
- Standard dependence abstractions usable
- Quick search for good transformations
- Automatic completion procedure