Anhang A

Typentheorie: Syntax, Semantik, Inferenzregeln

A.1 Parametertypen

variable : Variablennamen (ML Datentyp `var`)
Zulässige Elemente sind Zeichenketten der Form $[\texttt{a}-\texttt{z}\,\texttt{A}-\texttt{Z}\,\texttt{0}-\texttt{9}\,\texttt{\_}\,\texttt{-}\,\texttt{\%}]^+$

natural : Natürliche Zahlen einschließlich der Null (ML Datentyp `int`).
Zulässige Elemente sind Zeichenketten der Form $\texttt{0} + [\texttt{1}-\texttt{9}][\texttt{0}-\texttt{9}]^*$.

token : Zeichenketten für Namen (ML Datentyp `tok`).
Zulässige Elemente bestehen aus allen Symbolen des NuPRL Zeichensatzes mit Ausnahme der Kontrollsymbole.

string : Zeichenketten für Texte (ML Datentyp `string`).
Zulässige Elemente bestehen aus allen Symbolen des NuPRL Zeichensatzes mit Ausnahme der Kontrollsymbole.

level-expression : Ausdrücke für das Level eines Typuniversums (ML Datentyp `level_exp`)
Die genaue Syntax wird in Abschnitt 3.2.3.1 bei der Diskussion der Universenhierarchie besprochen.

Die Namen für Parametertypen dürfen durch ihre ersten Buchstaben abgekürzt werden.

A.2 Operatorentabelle

kanonisch (Typen)	kanonisch (Elemente)	nichtkanonisch
	var$\{x:\mathtt{v}\}()$ x	
int$\{\}()$ $\mathbb{Z}$	**natnum**$\{n:\mathtt{n}\}()$ **minus**$\{\}($**natnum**$\{n:\mathtt{n}\}())$ n $-n$	**ind**$\{\}(\boxed{u};\,x,f_x.s;\,base;\,y,f_y.t)$ **minus**$\{\}(\boxed{u})$, **add**$\{\}(\boxed{u};\boxed{v})$, **sub**$\{\}(\boxed{u};\boxed{v})$ **mul**$\{\}(\boxed{u};\boxed{v})$, **div**$\{\}(\boxed{u};\boxed{v})$, **rem**$\{\}(\boxed{u};\boxed{v})$ **int_eq**$(\boxed{u};\,\boxed{v};\,s;\,t)$, **less**$(\boxed{u};\,\boxed{v};\,s;\,t)$ ind$(\boxed{u};\,x,f_x.s;\,base;\,y,f_y.t)$ $-\boxed{u}$, $\boxed{u}+\boxed{v}$, $\boxed{u}-\boxed{v}$ $\boxed{u}*\boxed{v}$, $\boxed{u}\div\boxed{v}$, $\boxed{u}$ rem $\boxed{v}$ if $\boxed{u}=\boxed{v}$ then s else t, if $\boxed{u}<\boxed{v}$ then s else t
lt$\{\}(u;v)$ $u<v$	**Axiom**$\{\}()$ Axiom	
void$\{\}()$ void		**any**$\{\}(e)$ any(e)
Atom$\{\}()$ Atom	**token**$\{string:\mathtt{t}\}()$ ¨*string*¨	**atom_eq**$(\boxed{u};\,\boxed{v};\,s;\,t)$ if $\boxed{u}=\boxed{v}$ then s else t
U$\{j:\mathtt{l}\}()$ U_j	(alle kanonischen Typen)	
fun$\{\}(S;\,x.T)$ $x{:}S\rightarrow T$	**lam**$\{\}(x.t)$ $\lambda x.t$	**apply**$\{\}(\boxed{f};t)$ $\boxed{f}\ t$
prod$\{\}(S;\,x.T)$ $x{:}S\times T$	**pair**$\{\}(s;t)$ $\langle s,t\rangle$	**spread**$\{\}(\boxed{e};\,x,y.u)$ let $\langle x,y\rangle=\boxed{e}$ in u
union$\{\}(S;T)$ $S+T$	**inl**$\{\}(s)$, **inr**$\{\}(t)$ inl(s), inr(t)	**decide**$\{\}(\boxed{e};\,x.u;\,y.v)$ case $\boxed{e}$ of inl$(x)\mapsto u \mid$ inr$(y)\mapsto v$
equal$\{\}(s;t;T)$ $s=t\in T$	**Axiom**$\{\}()$ Axiom	
list$\{\}(T)$ T list	**nil**$\{\}()$, **cons**$\{\}(t;l)$ [], $t.l$	**list_ind**$\{\}(\boxed{s};\,base;\,x,l,f_{xl}.t)$ list_ind$(\boxed{s};\,base;\,x,l,f_{xl}.t)$
set$\{\}(S;\,x.T)$ $\{x{:}S \mid T\}$	(Elemente von S)	
quotient$\{\}(T;\,x,y.E)$ $x,y:T/\!/E$	(Elemente von T)	
rec$\{\}(X.T_X)$ rectype $X=T_X$	(Elemente gemäß T_X)	**rec_ind**$\{\}(\boxed{e};\,f,x.t)$ let* $f(x)=t$ in $f(\boxed{e})$
pfun$\{\}(S;\,T)$ $S\not\rightarrow T$	**fix**$\{\}(f,x.t)$ letrec $f(x)=t$	**dom**$\{\}(\boxed{f})$, **apply_p**$\{\}(\boxed{f};t)$ dom$(\boxed{f})$, $\boxed{f}\,(t)$

Man beachte, daß in der Implementierung von NuPRL 4.0 die konkreten Namen und Displayformen zum Teil etwas anders sind als hier angegeben. An der Anpassung wird gearbeitet.

A.3 Redizes und Kontrakta

Redex		Kontraktum
$(\lambda x.u)\ t$	$\xrightarrow{\beta}$	$u[t/x]$
$\mathsf{let}\ \langle x,y\rangle = \langle s,t\rangle\ \mathsf{in}\ u$	$\xrightarrow{\beta}$	$u[s,t\,/\,x,y]$
$\mathsf{case}\ \mathsf{inl}(s)\ \mathsf{of}\ \mathsf{inl}(x) \mapsto u\ \mid\ \mathsf{inr}(y) \mapsto v$	$\xrightarrow{\beta}$	$u[s/x]$
$\mathsf{case}\ \mathsf{inr}(t)\ \mathsf{of}\ \mathsf{inl}(x) \mapsto u\ \mid\ \mathsf{inr}(y) \mapsto v$	$\xrightarrow{\beta}$	$v[t/y]$
$\mathsf{ind}(0;\, x,f_x.s;\, base;\, y,f_y.t)$	$\xrightarrow{\beta}$	$base$
$\mathsf{ind}(n;\, x,f_x.s;\, base;\, y,f_y.t)$	$\xrightarrow{\beta}$	$t[n,\mathsf{ind}(n\text{-}1;\, x,f_x.s;\, base;\, y,f_y.t)\ /x, f_x]\quad,\quad (n>0)$
$\mathsf{ind}(\text{-}n;\, x,f_x.s;\, base;\, y,f_y.t)$	$\xrightarrow{\beta}$	$s[\text{-}n,\mathsf{ind}(\text{-}n\text{+}1;\, x,f_x.s;\, base;\, y,f_y.t)\ /y, f_y],\quad (n>0)$
$\text{-}i$	$\xrightarrow{\beta}$	*Die Negation von i (als Zahl)*
$i\text{+}j$	$\xrightarrow{\beta}$	*Die Summe von i und j*
$i\text{-}j$	$\xrightarrow{\beta}$	*Die Differenz von i und j*
$i*j$	$\xrightarrow{\beta}$	*Das Produkt von i und j*
$i\div j$	$\xrightarrow{\beta}$	0*, falls j=0; ansonsten die Integer-Division von i und j*
$i\ \mathsf{rem}\ j$	$\xrightarrow{\beta}$	0*, falls j=0; ansonsten der Rest der Division von i und j*
$\mathsf{if}\ i\texttt{=}j\ \mathsf{then}\ s\ \mathsf{else}\ t$	$\xrightarrow{\beta}$	*s, falls $i = j$; ansonsten t*
$\mathsf{if}\ i\texttt{<}j\ \mathsf{then}\ s\ \mathsf{else}\ t$	$\xrightarrow{\beta}$	*s, falls $i < j$; ansonsten t*
$\mathsf{list\_ind}([];\, base;\, x,l,f_{xl}.t)$	$\xrightarrow{\beta}$	$base$
$\mathsf{list\_ind}(s.u;\, base;\, x,l,f_{xl}.t)$	$\xrightarrow{\beta}$	$t[s,u,\mathsf{list\_ind}(u;\, base;\, x,l,f_{xl}.t)\ /x, l, f_{xl}]$
$\mathsf{if}\ u\texttt{=}v\ \mathsf{then}\ s\ \mathsf{else}\ t$	$\xrightarrow{\beta}$	*s, falls $u = v$; ansonsten t*
$\mathsf{let}^*\ f(x) = t\ \mathsf{in}\ f(e)$	$\xrightarrow{\beta}$	$t[\lambda y.\mathsf{let}^*\ f(x) = t\ \mathsf{in}\ f(y),\ e\ /\ f, x]$
$(\mathsf{letrec}\ f(x) = t)\ (u)$	$\xrightarrow{\beta}$	$t[\mathsf{letrec}\ f(x) = t,\ u\ /\ f, x]$
$\mathsf{dom}(\mathsf{letrec}\ f(x) = t)$	$\xrightarrow{\beta}$	$\lambda x.\mathsf{rectype}\ F = \mathcal{E}[[t]]$

A.4 Urteile

A.4.1 Typsemantik

$\mathbb{Z} = \mathbb{Z}$		
$i_1\texttt{<}j_1 = i_2\texttt{<}j_2$	falls	$i_1 = i_2 \in \mathbb{Z}$ und $j_1 = j_2 \in \mathbb{Z}$
$\mathsf{void} = \mathsf{void}$		
$\mathsf{Atom} = \mathsf{Atom}$		
$\mathsf{U}_{j_1} = \mathsf{U}_{j_2}$	falls	$j_1{=}j_2$ *(als natürliche Zahl)*
$x_1{:}S_1{\to}T_1 = x_2{:}S_2{\to}T_2$	falls	$S_1{=}S_2$ und $T_1[s_1/x_1]{=}T_2[s_2/x_2]$ für alle Terme s_1, s_2 mit $s_1{=}s_2 \in S_1$.
$T = S_2{\to}T_2$	falls	$T = x_2{:}S_2{\to}T_2$ für ein beliebiges $x_2 \in \mathcal{V}$.
$S_1{\to}T_1 = T$	falls	$x_1{:}S_1{\to}T_1 = T$ für ein beliebiges $x_1 \in \mathcal{V}$.
$x_1{:}S_1{\times}T_1 = x_2{:}S_2{\times}T_2$	falls	$S_1{=}S_2$ und $T_1[s_1/x_1]{=}T_2[s_2/x_2]$ für alle Terme s_1, s_2 mit $s_1{=}s_2 \in S_1$.
$T = S_2{\times}T_2$	falls	$T = x_2{:}S_2{\times}T_2$ für ein beliebiges $x_2 \in \mathcal{V}$.
$S_1{\times}T_1 = T$	falls	$x_1{:}S_1{\times}T_1 = T$ für ein beliebiges $x_1 \in \mathcal{V}$.
$S_1{+}T_1 = S_2{+}T_2$	falls	$S_1{=}S_2$ und $T_1{=}T_2$.
$s_1\,\texttt{=}\,t_1 \in T_1 = s_2\,\texttt{=}\,t_2 \in T_2$	falls	$T_1{=}T_2$ und $s_1{=}s_2 \in T_1$ und $t_1{=}t_2 \in T_1$.
$T_1\ \mathsf{list} = T_2\ \mathsf{list}$	falls	$T_1 = T_2$
$\{x_1 : S_1 \mid T_1\} = \{x_2 : S_2 \mid T_2\}$	falls	$S_1{=}S_2$ und es gibt eine Variable x, die weder in T_1 noch in T_2 vorkommt, und zwei Terme p_1 und p_2 mit der Eigenschaft $p_1 \in \forall x{:}S_1 .\ T_1[x/x_1] \Rightarrow T_2[x/x_2]$ und $p_2 \in \forall x{:}S_1 .\ T_2[x/x_2] \Rightarrow T_1[x/x_1]$.
$T = \{S_2 \mid T_2\}$	falls	$T = \{x_2 : S_2 \mid T_2\}$ für ein beliebiges $x_2 \in \mathcal{V}$.
$\{S_1 \mid T_1\} = T$	falls	$\{x_1 : S_1 \mid T_1\} = T$ für ein beliebiges $x_1 \in \mathcal{V}$.
$x_1,y_1 : T_1/\!/E_1 = x_2,y_2 : T_2/\!/E_2$	falls	$T_1 = T_2$ und es gibt (verschiedene) Variablen x, y, z, die weder in E_1 noch in E_2 vorkommen, und Terme p_1, p_2, r,s und t mit der Eigenschaft $p_1 \in \forall x{:}T_1 .\forall y{:}T_1 .\ E_1[x,y/x_1,y_1] \Rightarrow E_2[x,y/x_2,y_2]$ und $p_2 \in \forall x{:}T_1 .\forall y{:}T_1 .\ E_2[x,y/x_2,y_2] \Rightarrow E_1[x,y/x_1,y_1]$ und $r \in \forall x{:}T_1 .\ E_1[x,x/x_1,y_1]$ und $s \in \forall x{:}T_1 .\forall y{:}T_1 .\ E_1[x,y/x_1,y_1] \Rightarrow E_1[y,x/x_1,y_1]$ und $t \in \forall x{:}T_1 .\forall y{:}T_1 .\forall z{:}T_1 .$ $E_1[x,y/x_1,y_1] \Rightarrow E_1[y,z/x_1,y_1] \Rightarrow E_1[x,z/x_1,y_1]$
$\mathsf{rectype}\ X_1 = T_{X1} = \mathsf{rectype}\ X_2 = T_{X2}$	falls	$T_{X1}[X/X_1] = T_{X2}[X/X_2]$ für alle Typen X
$S_1{\not\to}T_1 = S_2{\not\to}T_2$	falls	$S_1 = S_2$ und $T_1 = T_2$

A.4.2 Elementsemantik – Universen

Elementsemantik – Universen		
$\mathbb{Z} = \mathbb{Z} \in \mathsf{U}_j$		
i_1<$j_1 = i_2$<$j_2 \in \mathsf{U}_j$	falls	$i_1 = i_2 \in \mathbb{Z}$ und $j_1 = j_2 \in \mathbb{Z}$
$\mathsf{void} = \mathsf{void} \in \mathsf{U}_j$		
$\mathsf{Atom} = \mathsf{Atom} \in \mathsf{U}_j$		
$\mathsf{U}_{j_1} = \mathsf{U}_{j_2} \in \mathsf{U}_j$	falls	$j_1 = j_2 < j$ *(als natürliche Zahl)*
$x_1{:}S_1 \to T_1 = x_2{:}S_2 \to T_2 \in \mathsf{U}_j$	falls	$S_1 = S_2 \in \mathsf{U}_j$ und $T_1[s_1/x_1] = T_2[s_2/x_2] \in \mathsf{U}_j$ für alle Terme s_1, s_2 mit $s_1 = s_2 \in S_1$.
$T = S_2 \to T_2 \in \mathsf{U}_j$	falls	$T = x_2{:}S_2 \to T_2 \in \mathsf{U}_j$ für ein beliebiges $x_2 \in \mathcal{V}$.
$S_1 \to T_1 = T \in \mathsf{U}_j$	falls	$x_1{:}S_1 \to T_1 = T \in \mathsf{U}_j$ für ein beliebiges $x_1 \in \mathcal{V}$.
$x_1{:}S_1 \times T_1 = x_2{:}S_2 \times T_2 \in \mathsf{U}_j$	falls	$S_1 = S_2 \in \mathsf{U}_j$ und $T_1[s_1/x_1] = T_2[s_2/x_2] \in \mathsf{U}_j$ für alle Terme s_1, s_2 mit $s_1 = s_2 \in S_1$.
$T = S_2 \times T_2 \in \mathsf{U}_j$	falls	$T = x_2{:}S_2 \times T_2 \in \mathsf{U}_j$ für ein beliebiges $x_2 \in \mathcal{V}$.
$S_1 \times T_1 = T \in \mathsf{U}_j$	falls	$x_1{:}S_1 \times T_1 = T \in \mathsf{U}_j$ für ein beliebiges $x_1 \in \mathcal{V}$.
$S_1 + T_1 = S_2 + T_2 \in \mathsf{U}_j$	falls	$S_1 = S_2 \in \mathsf{U}_j$ und $T_1 = T_2 \in \mathsf{U}_j$.
$s_1 = t_1 \in T_1 \ = \ s_2 = t_2 \in T_2 \in \mathsf{U}_j$	falls	$T_1 = T_2 \in \mathsf{U}_j$ und $s_1 = s_2 \in T_1$ und $t_1 = t_2 \in T_1$.
$T_1\ \mathsf{list} = T_2\ \mathsf{list} \in \mathsf{U}_j$	falls	$T_1 = T_2 \in \mathsf{U}_j$
$\{x_1{:}S_1 \mid T_1\} = \{x_2{:}S_2 \mid T_2\} \in \mathsf{U}_j$	falls	$S_1 = S_2 \in \mathsf{U}_j$ und $T_1[s/x_1] \in \mathsf{U}_j$ sowie $T_2[s/x_2] \in \mathsf{U}_j$ gilt für alle Terme s mit $s \in S_1$ und es gibt eine Variable x, die weder in T_1 noch in T_2 vorkommt, und zwei Terme p_1 und p_2 mit der Eigenschaft $p_1 \in \forall x{:}S_1 .\, T_1[x/x_1] \Rightarrow T_2[x/x_2]$ und $p_2 \in \forall x{:}S_1 .\, T_2[x/x_2] \Rightarrow T_1[x/x_1]$
$T = \{S_2 \mid T_2\} \in \mathsf{U}_j$	falls	$T = \{x_2{:}S_2 \mid T_2\} \in \mathsf{U}_j$ für ein beliebiges $x_2 \in \mathcal{V}$.
$\{S_1 \mid T_1\} = T \in \mathsf{U}_j$	falls	$\{x_1{:}S_1 \mid T_1\} = T \in \mathsf{U}_j$ für ein beliebiges $x_1 \in \mathcal{V}$.
$x_1, y_1 : T_1//E_1 = x_2, y_2 : T_2//E_2 \in \mathsf{U}_j$	falls	$T_1 = T_2 \in \mathsf{U}_j$ und für alle Terme s, t mit $s \in T_1$ und $t \in T_1$ gilt $E_1[s,t/x_1,y_1] \in \mathsf{U}_j$ sowie $E_2[s,t/x_2,y_2] \in \mathsf{U}_j$ und es gibt (verschiedene) Variablen x, y, z, die weder in E_1 noch in E_2 vorkommen, und Terme p_1, p_2, r, s und t mit der Eigenschaft $p_1 \in \forall x{:}T_1 .\forall y{:}T_1 .\, E_1[x,y/x_1,y_1] \Rightarrow E_2[x,y/x_2,y_2]$ und $p_2 \in \forall x{:}T_1 .\forall y{:}T_1 .\, E_2[x,y/x_2,y_2] \Rightarrow E_1[x,y/x_1,y_1]$ und $r \in \forall x{:}T_1 .E_1[x,x/x_1,y_1]$ und $s \in \forall x{:}T_1 .\forall y{:}T_1 .\, E_1[x,y/x_1,y_1] \Rightarrow E_1[y,x/x_1,y_1]$ und $t \in \forall x{:}T_1 .\forall y{:}T_1 .\forall z{:}T_1 .$ $E_1[x,y/x_1,y_1] \Rightarrow E_1[y,z/x_1,y_1] \Rightarrow E_1[x,z/x_1,y_1]$
$\mathsf{rectype}\ X_1 = T_{X1} = \mathsf{rectype}\ X_2 = T_{X2} \in \mathsf{U}_j$	falls	$T_{X1}[X/X_1] = T_{X2}[X/X_2] \in \mathsf{U}_j$ für alle Terme X mit $X \in \mathsf{U}_j$
$S_1 \not\to T_1 = S_2 \not\to T_2 \in \mathsf{U}_j$	falls	$S_1 = S_2 \in \mathsf{U}_j$ und $T_1 = T_2 \in \mathsf{U}_j$

A.4.3 Elementsemantik

$i = i \in \mathbb{Z}$		
$\mathsf{Axiom} = \mathsf{Axiom} \in s$<$t$	falls	es ganze Zahlen i und j gibt, wobei i kleiner als j ist, für die gilt $s \xrightarrow{l} i$ und $t \xrightarrow{l} j$
$s = t \in \mathsf{void}$		*gilt niemals !*
$\ddot{}string\ddot{} = \ddot{}string\ddot{} \in \mathsf{Atom}$		
$\lambda x_1.t_1 = \lambda x_2.t_2 \in x{:}S \to T$	falls	$x{:}S \to T$ Typ und $t_1[s_1/x_1] = t_2[s_2/x_2] \in T[s_1/x]$ für alle Terme s_1, s_2 mit $s_1 = s_2 \in S$.
$\langle s_1, t_1 \rangle = \langle s_2, t_2 \rangle \in x{:}S \times T$	falls	$x{:}S \times T$ Typ und $s_1 = s_2 \in S$ und $t_1 = t_2 \in T[s_1/x]$.
$\mathsf{inl}(s_1) = \mathsf{inl}(s_2) \in S+T$	falls	$S+T$ Typ und $s_1 = s_2 \in S$.
$\mathsf{inr}(t_1) = \mathsf{inr}(t_2) \in S+T$	falls	$S+T$ Typ und $t_1 = t_2 \in T$.
$\mathsf{Axiom} = \mathsf{Axiom} \in s = t \in T$	falls	$s = t \in T$
$[] = [] \in T\ \mathsf{list}$	falls	T Typ
$t_1 .l_1 = t_2 .l_2 \in T\ \mathsf{list}$	falls	T Typ und $t_1 = t_2 \in T$ und $l_1 = l_2 \in T\ \mathsf{list}$
$s = t \in \{x{:}S \mid T\}$	falls	$\{x{:}S \mid T\}$ Typ und $s = t \in S$ und es gibt einen Term p mit der Eigenschaft $p \in T[s/x]$
$s = t \in x, y : T//E$	falls	$x, y : T//E$ Typ und $s \in T$ und $t \in T$ und es gibt einen Term p mit der Eigenschaft $p \in E[s,t/x,y]$
$s = t \in \mathsf{rectype}\ X = T_X$	falls	$\mathsf{rectype}\ X = T_X$ Typ und $s = t \in T_X[\mathsf{rectype}\ X = T_X \,/\, X]$
$\mathsf{letrec}\ f_1(x_1) = t_1 = \mathsf{letrec}\ f_2(x_2) = t_2 \in S \not\to T$	falls	$S \not\to T$ Typ und für alle Terme s_1 und s_2 mit $s_1 = s_2 \in S$ $\{x{:}S \mid \mathsf{dom}(\mathsf{letrec}\ f_1(x_1) = t_1)(x)\}$ $= \{x{:}S \mid \mathsf{dom}(\mathsf{letrec}\ f_2(x_2) = t_2)(x)\}$ und $t_1[\mathsf{letrec}\ f_1(x_1) = t_1,\ s_1 \,/\, f_1, x_1]$ $= t_2[\mathsf{letrec}\ f_2(x_2) = t_2,\ s_2 \,/\, f_2, x_2] \in T$

A.5 Inferenzregeln

A.5.1 Ganze Zahlen

$\Gamma \vdash \mathbb{Z} \;\in\; \mathsf{U}_j$ ⌊Ax⌋
by <u>intEq</u>

$\Gamma \vdash n \;\in\; \mathbb{Z}$ ⌊Ax⌋
by <u>natnumEq</u>

$\Gamma \vdash \mathbb{Z}$ ⌊**ext** n⌋
by <u>natnumI n</u>

$\Gamma \vdash \mathsf{ind}(u_1;\, x_1, f_{x1}.s_1;\, base_1;\, y_1, f_{y_1}.t_1)$
$\quad = \mathsf{ind}(u_2;\, x_2, f_{x2}.s_2;\, base_2;\, y_2, f_{y_2}.t_2)$
$\quad \in T[u_1/z]$ ⌊Ax⌋
by <u>indEq z T</u>
$\Gamma \vdash u_1 = u_2$ ⌊Ax⌋
$\Gamma,\ x{:}\mathbb{Z},\ v{:}\,x<0,\ f_x{:}T[(x+1)/z]$
$\quad \vdash s_1[x, f_x / x_1, f_{x1}] = s_2[x, f_x / x_2, f_{x2}] \;\in\; T[x/z]$ ⌊Ax⌋
$\Gamma \vdash base_1 = base_2 \;\in\; T[0/z]$ ⌊Ax⌋
$\Gamma,\ x{:}\mathbb{Z},\ v{:}\,0<x,\ f_x{:}T[(x-1)/z]$
$\quad \vdash t_1[x, f_x / y_1, f_{y_1}] = t_2[x, f_x / y_2, f_{y_2}] \;\in\; T[x/z]$ ⌊Ax⌋

$\Gamma,\ z{:}\mathbb{Z},\ \Delta \vdash C$
⌊**ext** $\mathsf{ind}(z;\, x, f_x.s[\mathsf{Axiom}/v];\, base;\, x, f_x.t[\mathsf{Axiom}/v])$⌋
by <u>intE i</u>
$\Gamma,\ z{:}\mathbb{Z},\ \Delta,\ x{:}\mathbb{Z},\ v{:}\,x<0,\ f_x{:}C[(x+1)/z]$
$\quad \vdash C[x/z]$ ⌊**ext** s⌋
$\Gamma,\ z{:}\mathbb{Z},\ \Delta \vdash C[0/z]$ ⌊**ext** $base$⌋
$\Gamma,\ z{:}\mathbb{Z},\ \Delta,\ x{:}\mathbb{Z},\ v{:}\,0<x,\ f_x{:}C[(x-1)/z]$
$\quad \vdash C[x/z]$ ⌊**ext** t⌋

$\Gamma \vdash -s_1 = -s_2$ ⌊Ax⌋
by <u>minusEq</u>
$\Gamma \vdash s_1 = s_2$ ⌊Ax⌋

$\Gamma \vdash \mathbb{Z}$ ⌊**ext** $-s$⌋
by <u>minusI</u>
$\Gamma \vdash \mathbb{Z}$ ⌊**ext** s⌋

$\Gamma \vdash s_1{+}t_1 = s_2{+}t_2$ ⌊Ax⌋
by <u>addEq</u>
$\Gamma \vdash s_1 = s_2$ ⌊Ax⌋
$\Gamma \vdash t_1 = t_2$ ⌊Ax⌋

$\Gamma \vdash \mathbb{Z}$ ⌊**ext** $s{+}t$⌋
by <u>addI</u>
$\Gamma \vdash \mathbb{Z}$ ⌊**ext** s⌋
$\Gamma \vdash \mathbb{Z}$ ⌊**ext** t⌋

$\Gamma \vdash s_1{-}t_1 = s_2{-}t_2$ ⌊Ax⌋
by <u>subEq</u>
$\Gamma \vdash s_1 = s_2$ ⌊Ax⌋
$\Gamma \vdash t_1 = t_2$ ⌊Ax⌋

$\Gamma \vdash \mathbb{Z}$ ⌊**ext** $s{-}t$⌋
by <u>subI</u>
$\Gamma \vdash \mathbb{Z}$ ⌊**ext** s⌋
$\Gamma \vdash \mathbb{Z}$ ⌊**ext** t⌋

$\Gamma \vdash s_1{*}t_1 = s_2{*}t_2$ ⌊Ax⌋
by <u>mulEq</u>
$\Gamma \vdash s_1 = s_2$ ⌊Ax⌋
$\Gamma \vdash t_1 = t_2$ ⌊Ax⌋

$\Gamma \vdash \mathbb{Z}$ ⌊**ext** $s{*}t$⌋
by <u>mulI</u>
$\Gamma \vdash \mathbb{Z}$ ⌊**ext** s⌋
$\Gamma \vdash \mathbb{Z}$ ⌊**ext** t⌋

$\Gamma \vdash s_1{\div}t_1 = s_2{\div}t_2$ ⌊Ax⌋
by <u>divEq</u>
$\Gamma \vdash s_1 = s_2$ ⌊Ax⌋
$\Gamma \vdash t_1 = t_2$ ⌊Ax⌋
$\Gamma \vdash t_1{\neq}0$ ⌊Ax⌋

$\Gamma \vdash \mathbb{Z}$ ⌊**ext** $s{\div}t$⌋
by <u>divI</u>
$\Gamma \vdash \mathbb{Z}$ ⌊**ext** s⌋
$\Gamma \vdash \mathbb{Z}$ ⌊**ext** t⌋

$\Gamma \vdash s_1 \;\mathsf{rem}\; t_1 = s_2 \;\mathsf{rem}\; t_2$ ⌊Ax⌋
by <u>remEq</u>
$\Gamma \vdash s_1 = s_2$ ⌊Ax⌋
$\Gamma \vdash t_1 = t_2$ ⌊Ax⌋
$\Gamma \vdash t_1{\neq}0$ ⌊Ax⌋

$\Gamma \vdash \mathbb{Z}$ ⌊**ext** $s \;\mathsf{rem}\; t$⌋
by <u>remI</u>
$\Gamma \vdash \mathbb{Z}$ ⌊**ext** s⌋
$\Gamma \vdash \mathbb{Z}$ ⌊**ext** t⌋

$\Gamma \vdash \mathsf{if}\ u_1{=}v_1\ \mathsf{then}\ s_1\ \mathsf{else}\ t_1$
$\quad = \mathsf{if}\ u_2{=}v_2\ \mathsf{then}\ s_2\ \mathsf{else}\ t_2 \;\in\; T$ ⌊Ax⌋
by <u>int_eqEq</u>
$\Gamma \vdash u_1 = u_2$ ⌊Ax⌋
$\Gamma \vdash v_1 = v_2$ ⌊Ax⌋
$\Gamma,\ v{:}\,u_1{=}v_1 \vdash s_1 = s_2 \;\in\; T$ ⌊Ax⌋
$\Gamma,\ v{:}\,u_1{\neq}v_1 \vdash t_1 = t_2 \;\in\; T$ ⌊Ax⌋

$\Gamma \vdash \mathsf{if}\ u_1{<}v_1\ \mathsf{then}\ s\ \mathsf{else}\ t$
$\quad = \mathsf{if}\ u_2{<}v_2\ \mathsf{then}\ s_2\ \mathsf{else}\ t_2 \;\in\; T$ ⌊Ax⌋
by <u>lessEq</u>
$\Gamma \vdash u_1 = u_2$ ⌊Ax⌋
$\Gamma \vdash v_1 = v_2$ ⌊Ax⌋
$\Gamma,\ v{:}\,u_1{<}v_1 \vdash s_1 = s_2 \;\in\; T$ ⌊Ax⌋
$\Gamma,\ v{:}\,u_1{\geq}v_1 \vdash t_1 = t_2 \;\in\; T$ ⌊Ax⌋

$\Gamma \vdash \mathsf{ind}(i;\, x, f_x.s;\, base;\, y, f_y.t) = t_2 \;\in\; T$ ⌊Ax⌋
by <u>indRedDown</u>
$\Gamma \vdash t[i, \mathsf{ind}(i{+}1;\, x, f_x.s;\, base;\, y, f_y.t) \,/ x, f_x]$
$\quad = t_2 \;\in\; T$ ⌊Ax⌋
$\Gamma \vdash i{<}0$ ⌊Ax⌋

$\Gamma \vdash \mathsf{ind}(i;\, x, f_x.s;\, base;\, y, f_y.t) = t_2 \;\in\; T$ ⌊Ax⌋
by <u>indRedUp</u>
$\Gamma \vdash t[i, \mathsf{ind}(i{-}1;\, x, f_x.s;\, base;\, y, f_y.t) \,/ y, f_y]$
$\quad = t_2 \;\in\; T$ ⌊Ax⌋
$\Gamma \vdash 0{<}i$ ⌊Ax⌋

$\Gamma \vdash \mathsf{ind}(i;\, x, f_x.s;\, base;\, y, f_y.t) = t_2 \;\in\; T$ ⌊Ax⌋
by <u>indRedBase</u>
$\Gamma \vdash base = t_2 \;\in\; T$ ⌊Ax⌋
$\Gamma \vdash i{=}0$ ⌊Ax⌋

$\Gamma \vdash \mathsf{if}\ u{=}v\ \mathsf{then}\ s\ \mathsf{else}\ t = t_2 \;\in\; T$ ⌊Ax⌋
by <u>int_eqRedT</u>
$\Gamma \vdash s = t_2 \;\in\; T$ ⌊Ax⌋
$\Gamma \vdash u{=}v$ ⌊Ax⌋

$\Gamma \vdash \mathsf{if}\ u{=}v\ \mathsf{then}\ s\ \mathsf{else}\ t = t_2 \;\in\; T$ ⌊Ax⌋
by <u>int_eqRedF</u>
$\Gamma \vdash t = t_2 \;\in\; T$ ⌊Ax⌋
$\Gamma \vdash u{\neq}v$ ⌊Ax⌋

$\Gamma \vdash \mathsf{if}\ u{<}v\ \mathsf{then}\ s\ \mathsf{else}\ t = t_2 \;\in\; T$ ⌊Ax⌋
by <u>lessRedT</u>
$\Gamma \vdash s = t_2 \;\in\; T$ ⌊Ax⌋
$\Gamma \vdash u{<}v$ ⌊Ax⌋

$\Gamma \vdash \mathsf{if}\ u{<}v\ \mathsf{then}\ s\ \mathsf{else}\ t = t_2 \;\in\; T$ ⌊Ax⌋
by <u>lessRedF</u>
$\Gamma \vdash t = t_2 \;\in\; T$ ⌊Ax⌋
$\Gamma \vdash u{\geq}v$ ⌊Ax⌋

$\Gamma \vdash 0 \leq s \;\mathsf{rem}\; t \;\wedge\; s \;\mathsf{rem}\; t < t$ ⌊Ax⌋
by <u>remBounds1</u>
$\Gamma \vdash 0 \leq s$ ⌊Ax⌋
$\Gamma \vdash 0 < t$ ⌊Ax⌋

$\Gamma \vdash 0 \leq s \;\mathsf{rem}\; t \;\wedge\; s \;\mathsf{rem}\; t < -t$ ⌊Ax⌋
by <u>remBounds2</u>
$\Gamma \vdash 0 \leq s$ ⌊Ax⌋
$\Gamma \vdash t < 0$ ⌊Ax⌋

$\Gamma \vdash s \;\mathsf{rem}\; t \leq 0 \;\wedge\; s \;\mathsf{rem}\; t > -t$ ⌊Ax⌋
by <u>remBounds3</u>
$\Gamma \vdash s \leq 0$ ⌊Ax⌋
$\Gamma \vdash 0 < t$ ⌊Ax⌋

$\Gamma \vdash s \;\mathsf{rem}\; t \leq 0 \;\wedge\; s \;\mathsf{rem}\; t > t$ ⌊Ax⌋
by <u>remBounds4</u>
$\Gamma \vdash s \leq 0$ ⌊Ax⌋
$\Gamma \vdash t < 0$ ⌊Ax⌋

$\Gamma \vdash s = (s{\div}t){*}t + (s \;\mathsf{rem}\; t)$ ⌊Ax⌋
by <u>divremSum</u>
$\Gamma \vdash s{\in}\mathbb{Z}$ ⌊Ax⌋
$\Gamma \vdash t{\neq}0$ ⌊Ax⌋

$\Gamma \vdash s_1{<}t_1 = s_2{<}t_2 \;\in\; \mathsf{U}_j$ ⌊Ax⌋
by <u>ltEq</u>
$\Gamma \vdash s_1 = s_2$ ⌊Ax⌋
$\Gamma \vdash t_1 = t_2$ ⌊Ax⌋

$\Gamma \vdash \mathsf{Axiom} \;\in\; s{<}t$ ⌊Ax⌋
by <u>axiomEq_lt</u>
$\Gamma \vdash s{<}t$ ⌊Ax⌋

A.5.2 Void

$\Gamma \vdash \mathsf{void} \;\in\; \mathsf{U}_j$ ⌊Ax⌋
by <u>voidEq</u>

$\Gamma \vdash \mathsf{any}(s) = \mathsf{any}(t) \;\in\; T$ ⌊Ax⌋
by <u>anyEq</u>
$\Gamma \vdash s = t \;\in\; \mathsf{void}$ ⌊Ax⌋

$\Gamma,\ z{:}\,\mathsf{void},\ \Delta \vdash C$ ⌊**ext** $\mathsf{any}(z)$⌋
by <u>voidE i</u>

A.5.3 Universen

$\Gamma \vdash \mathsf{U}_j \;\in\; \mathsf{U}_k$ ⌊Ax⌋
by <u>univEq *</u>

$\Gamma \vdash T \;\in\; \mathsf{U}_k$ ⌊Ax⌋
by <u>cumulativity j *</u>
$\Gamma \vdash T \;\in\; \mathsf{U}_j$ ⌊Ax⌋

*: *Die Regel ist nur anwendbar, wenn $j < k$ ist.*

A.5.4 Atom

$\Gamma \vdash \mathsf{Atom} = \mathsf{Atom} \in \mathsf{U}_j \;\lfloor\mathsf{Ax}\rfloor$
by $\underline{\texttt{atomEq}}$

$\Gamma \vdash \text{"}string\text{"} \in \mathsf{Atom} \;\lfloor\mathsf{Ax}\rfloor$
by $\underline{\texttt{tokEq}}$

$\Gamma \vdash \mathsf{Atom} \;\lfloor\mathbf{ext}\ \text{"}string\text{"}\rfloor$
by $\underline{\texttt{tokI}\ \text{"}string\text{"}}$

$\Gamma \vdash \mathsf{if}\ u_1{=}v_1\ \mathsf{then}\ s_1\ \mathsf{else}\ t_1$
$\quad = \mathsf{if}\ u_2{=}v_2\ \mathsf{then}\ s_2\ \mathsf{else}\ t_2 \in T \;\lfloor\mathsf{Ax}\rfloor$
by $\underline{\texttt{atom\_eqEq}}$
$\Gamma \vdash u_1{=}u_2 \in \mathsf{Atom} \;\lfloor\mathsf{Ax}\rfloor$
$\Gamma \vdash v_1{=}v_2 \in \mathsf{Atom} \;\lfloor\mathsf{Ax}\rfloor$
$\Gamma,\ v{:}\ u_1{=}v_1 \in \mathsf{Atom} \vdash s_1 = s_2 \in T \;\lfloor\mathsf{Ax}\rfloor$
$\Gamma,\ v{:}\ \neg(u_1{=}v_1 \in \mathsf{Atom}) \vdash t_1 = t_2 \in T \;\lfloor\mathsf{Ax}\rfloor$

$\Gamma \vdash \mathsf{if}\ u{=}v\ \mathsf{then}\ s\ \mathsf{else}\ t = t_2 \in T \;\lfloor\mathsf{Ax}\rfloor$
by $\underline{\texttt{atom\_eqRedT}}$
$\Gamma \vdash s = t_2 \in T \;\lfloor\mathsf{Ax}\rfloor$
$\Gamma \vdash u{=}v \in \mathsf{Atom} \;\lfloor\mathsf{Ax}\rfloor$

$\Gamma \vdash \mathsf{if}\ u{=}v\ \mathsf{then}\ s\ \mathsf{else}\ t = t_2 \in T \;\lfloor\mathsf{Ax}\rfloor$
by $\underline{\texttt{atom\_eqRedF}}$
$\Gamma \vdash t = t_2 \in T \;\lfloor\mathsf{Ax}\rfloor$
$\Gamma \vdash \neg(u{=}v \in \mathsf{Atom}) \;\lfloor\mathsf{Ax}\rfloor$

A.5.5 Funktionenraum

$\Gamma \vdash x_1{:}S_1{\to}T_1 = x_2{:}S_2{\to}T_2 \in \mathsf{U}_j \;\lfloor\mathsf{Ax}\rfloor$
by $\underline{\texttt{functionEq}}$
$\Gamma \vdash S_1 = S_2 \in \mathsf{U}_j \;\lfloor\mathsf{Ax}\rfloor$
$\Gamma,\ x{:}S_1 \vdash T_1[x/x_1] = T_2[x/x_2] \in \mathsf{U}_j \;\lfloor\mathsf{Ax}\rfloor$

$\Gamma \vdash \lambda x_1.t_1 = \lambda x_2.t_2 \in x{:}S{\to}T \;\lfloor\mathsf{Ax}\rfloor$
by $\underline{\texttt{lambdaEq}\ j}$
$\Gamma,\ x'{:}S \vdash t_1[x'/x_1] = t_2[x'/x_2] \in T[x'/x] \;\lfloor\mathsf{Ax}\rfloor$
$\Gamma \vdash S \in \mathsf{U}_j \;\lfloor\mathsf{Ax}\rfloor$

$\Gamma \vdash x{:}S{\to}T \;\lfloor\mathbf{ext}\ \lambda x'.t\rfloor$
by $\underline{\texttt{lambdaI}\ j}$
$\Gamma,\ x'{:}S \vdash T[x'/x] \;\lfloor\mathbf{ext}\ t\rfloor$
$\Gamma \vdash S \in \mathsf{U}_j \;\lfloor\mathsf{Ax}\rfloor$

$\Gamma \vdash f_1\,t_1 = f_2\,t_2 \in T[t_1/x] \;\lfloor\mathsf{Ax}\rfloor$
by $\underline{\texttt{applyEq}\ x{:}S{\to}T}$
$\Gamma \vdash f_1 = f_2 \in x{:}S{\to}T \;\lfloor\mathsf{Ax}\rfloor$
$\Gamma \vdash t_1 = t_2 \in S \;\lfloor\mathsf{Ax}\rfloor$

$\Gamma,\ f{:}\ x{:}S{\to}T,\ \Delta \vdash C \;\lfloor\mathbf{ext}\ t[f\,s, \mathsf{Axiom}\,/\,y,v]\rfloor$
by $\underline{\texttt{functionE}\ i\ s}$
$\Gamma,\ f{:}\ x{:}S{\to}T,\ \Delta \vdash s \in S \;\lfloor\mathsf{Ax}\rfloor$
$\Gamma,\ f{:}\ x{:}S{\to}T,\ y{:}T[s/x],$
$\quad v{:}\ y = f\,s \in T[s/x],\ \Delta \vdash C \;\lfloor\mathbf{ext}\ t\rfloor$

$\Gamma \vdash (\lambda x.t)\,s = t_2 \in T \;\lfloor\mathsf{Ax}\rfloor$
by $\underline{\texttt{applyRed}}$
$\Gamma \vdash t[s/x] = t_2 \in T \;\lfloor\mathsf{Ax}\rfloor$

$\Gamma \vdash f_1 = f_2 \in x{:}S{\to}T \;\lfloor\mathbf{ext}\ t\rfloor$
by $\underline{\texttt{functionExt}\ j\ x_1{:}S_1{\to}T_1\ x_2{:}S_2{\to}T_2}$
$\Gamma,\ x'{:}S \vdash f_1\,x' = f_2\,x' \in T[x'/x] \;\lfloor\mathbf{ext}\ t\rfloor$
$\Gamma \vdash S \in \mathsf{U}_j \;\lfloor\mathsf{Ax}\rfloor$
$\Gamma \vdash f_1 \in x_1{:}S_1{\to}T_1 \;\lfloor\mathsf{Ax}\rfloor$
$\Gamma \vdash f_2 \in x_2{:}S_2{\to}T_2 \;\lfloor\mathsf{Ax}\rfloor$

$\Gamma,\ f{:}\ S{\to}T,\ \Delta \vdash C \;\lfloor\mathbf{ext}\ t[f\,s,/y]\rfloor$
by $\underline{\texttt{functionE\_indep}\ i}$
$\Gamma,\ f{:}\ S{\to}T,\ \Delta \vdash S \;\lfloor\mathbf{ext}\ s\rfloor$
$\Gamma,\ f{:}\ S{\to}T,\ y{:}T,\ \Delta \vdash C \;\lfloor\mathbf{ext}\ t\rfloor$

A.5.6 Produktraum

$\Gamma \vdash x_1{:}S_1{\times}T_1 = x_2{:}S_2{\times}T_2 \in \mathsf{U}_j \;\lfloor\mathsf{Ax}\rfloor$
by $\underline{\texttt{productEq}}$
$\Gamma \vdash S_1 = S_2 \in \mathsf{U}_j \;\lfloor\mathsf{Ax}\rfloor$
$\Gamma,\ x'{:}S_1 \vdash T_1[x'/x_1] = T_2[x'/x_2] \in \mathsf{U}_j \;\lfloor\mathsf{Ax}\rfloor$

$\Gamma \vdash \langle s_1,t_1\rangle = \langle s_2,t_2\rangle \in x{:}S{\times}T \;\lfloor\mathsf{Ax}\rfloor$
by $\underline{\texttt{pairEq}\ j}$
$\Gamma \vdash s_1 = s_2 \in S \;\lfloor\mathsf{Ax}\rfloor$
$\Gamma \vdash t_1 = t_2 \in T[s_1/x] \;\lfloor\mathsf{Ax}\rfloor$
$\Gamma,\ x'{:}S \vdash T[x'/x] \in \mathsf{U}_j \;\lfloor\mathsf{Ax}\rfloor$

$\Gamma \vdash x{:}S{\times}T \;\lfloor\mathbf{ext}\ \langle s,t\rangle\rfloor$
by $\underline{\texttt{pairI}\ j\ s}$
$\Gamma \vdash s \in S \;\lfloor\mathsf{Ax}\rfloor$
$\Gamma \vdash T[s/x] \;\lfloor\mathbf{ext}\ t\rfloor$
$\Gamma,\ x'{:}S \vdash T[x'/x] \in \mathsf{U}_j \;\lfloor\mathsf{Ax}\rfloor$

$\Gamma \vdash \mathsf{let}\ \langle x_1,y_1\rangle = e_1\ \mathsf{in}\ t_1$
$\quad = \mathsf{let}\ \langle x_2,y_2\rangle = e_2\ \mathsf{in}\ t_2 \in C[e_1/z] \;\lfloor\mathsf{Ax}\rfloor$
by $\underline{\texttt{spreadEq}\ z\ C\ x{:}S{\times}T}$
$\Gamma \vdash e_1 = e_2 \in x{:}S{\times}T \;\lfloor\mathsf{Ax}\rfloor$
$\Gamma,\ s{:}S,\ t{:}T[s/x],\ y{:}\ e_1{=}\langle s,t\rangle \in x{:}S{\times}T$
$\quad \vdash t_1[s,t/x_1,y_1] = t_2[s,t/x_2,y_2] \in C[\langle s,t\rangle/z] \;\lfloor\mathsf{Ax}\rfloor$

$\Gamma,\ z{:}\ x{:}S{\times}T,\ \Delta \vdash C \;\lfloor\mathbf{ext}\ \mathsf{let}\ \langle s,t\rangle = z\ \mathsf{in}\ u\rfloor$
by $\underline{\texttt{productE}\ i}$
$\Gamma,\ z{:}\ x{:}S{\times}T,\ s{:}S,\ t{:}T[s/x],\ \Delta[\langle s,t\rangle/z]$
$\quad \vdash C[\langle s,t\rangle/z] \;\lfloor\mathbf{ext}\ u\rfloor$

$\Gamma \vdash \mathsf{let}\ \langle x,y\rangle = \langle s,t\rangle\ \mathsf{in}\ u = t_2 \in T \;\lfloor\mathsf{Ax}\rfloor$
by $\underline{\texttt{spreadRed}}$
$\Gamma \vdash u[s,t/x,y] = t_2 \in T \;\lfloor\mathsf{Ax}\rfloor$

$\Gamma \vdash \langle s_1,t_1\rangle = \langle s_2,t_2\rangle \in S{\times}T \;\lfloor\mathsf{Ax}\rfloor$
by $\underline{\texttt{pairEq\_indep}}$
$\Gamma \vdash s_1 = s_2 \in S \;\lfloor\mathsf{Ax}\rfloor$
$\Gamma \vdash t_1 = t_2 \in T \;\lfloor\mathsf{Ax}\rfloor$

$\Gamma \vdash S{\times}T \;\lfloor\mathbf{ext}\ \langle s,t\rangle\rfloor$
by $\underline{\texttt{pairI\_indep}}$
$\Gamma \vdash S \;\lfloor\mathbf{ext}\ s\rfloor$
$\Gamma \vdash T \;\lfloor\mathbf{ext}\ t\rfloor$

A.5.7 Disjunkte Vereinigung (Summe)

$\Gamma \vdash S_1{+}T_1 \;=\; S_2{+}T_2 \;\in\; \mathsf{U}_j \;\lfloor\mathrm{Ax}\rfloor$
by $\underline{\texttt{unionEq}}$
$\quad\Gamma \vdash S_1 \;=\; S_2 \;\in\; \mathsf{U}_j \;\lfloor\mathrm{Ax}\rfloor$
$\quad\Gamma \vdash T_1 \;=\; T_2 \;\in\; \mathsf{U}_j \;\lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash \mathsf{inl}(s_1) \;=\; \mathsf{inl}(s_2) \;\in\; S{+}T \;\lfloor\mathrm{Ax}\rfloor$
by $\underline{\texttt{inlEq}\ j}$
$\quad\Gamma \vdash s_1 \;=\; s_2 \;\in\; S \;\lfloor\mathrm{Ax}\rfloor$
$\quad\Gamma \vdash T \;\in\; \mathsf{U}_j \;\lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash S{+}T \;\lfloor\textbf{ext}\ \mathsf{inl}(s)\rfloor$
by $\underline{\texttt{inlI}\ j}$
$\quad\Gamma \vdash S \;\lfloor\textbf{ext}\ s\rfloor$
$\quad\Gamma \vdash T \;\in\; \mathsf{U}_j \;\lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash \mathsf{inr}(t_1) \;=\; \mathsf{inr}(t_2) \;\in\; S{+}T \;\lfloor\mathrm{Ax}\rfloor$
by $\underline{\texttt{inrEq}\ j}$
$\quad\Gamma \vdash t_1 \;=\; t_2 \;\in\; T \;\lfloor\mathrm{Ax}\rfloor$
$\quad\Gamma \vdash S \;\in\; \mathsf{U}_j \;\lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash S{+}T \;\lfloor\textbf{ext}\ \mathsf{inr}(t)\rfloor$
by $\underline{\texttt{inrI}\ j}$
$\quad\Gamma \vdash T \;\lfloor\textbf{ext}\ t\rfloor$
$\quad\Gamma \vdash S \;\in\; \mathsf{U}_j \;\lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash \mathsf{case}\ e_1\ \mathsf{of}\ \mathsf{inl}(x_1) \mapsto u_1 \mid \mathsf{inr}(y_1) \mapsto v_1$
$\qquad = \mathsf{case}\ e_2\ \mathsf{of}\ \mathsf{inl}(x_2) \mapsto u_2 \mid \mathsf{inr}(y_2) \mapsto v_2$
$\qquad \in\; C[e_1/z] \;\lfloor\mathrm{Ax}\rfloor$
by $\underline{\texttt{decideEq}\ z\ C\ S{+}T}$
$\quad\Gamma \vdash e_1 \;=\; e_2 \;\in\; S{+}T \;\lfloor\mathrm{Ax}\rfloor$
$\quad\Gamma,\ s{:}S,\ y{:}\ e_1{=}\mathsf{inl}(s) \in S{+}T$
$\qquad \vdash u_1[s/x_1] = u_2[s/x_2] \;\in\; C[\mathsf{inl}(s)/z] \;\lfloor\mathrm{Ax}\rfloor$
$\quad\Gamma,\ t{:}T,\ y{:}\ e_1{=}\mathsf{inr}(t) \in S{+}T$
$\qquad \vdash v_1[t/y_1] = v_2[t/y_2] \;\in\; C[\mathsf{inr}(t)/z] \;\lfloor\mathrm{Ax}\rfloor$

$\Gamma,\ z{:}S{+}T,\ \Delta \vdash C$
$\qquad \lfloor\textbf{ext}\ \mathsf{case}\ z\ \mathsf{of}\ \mathsf{inl}(x) \mapsto u \mid \mathsf{inr}(y) \mapsto v\rfloor$
by $\underline{\texttt{unionE}\ i}$
$\quad\Gamma,\ z{:}S{+}T,\ x{:}S,\ \Delta[\mathsf{inl}(x)/z]$
$\qquad \vdash C[\mathsf{inl}(x)/z] \;\lfloor\textbf{ext}\ u\rfloor$
$\quad\Gamma,\ z{:}S{+}T,\ y{:}T,\ \Delta[\mathsf{inr}(y)/z]$
$\qquad \vdash C[\mathsf{inr}(y)/z] \;\lfloor\textbf{ext}\ v\rfloor$

$\Gamma \vdash \mathsf{case}\ \mathsf{inl}(s)\ \mathsf{of}\ \mathsf{inl}(x) \mapsto u \mid \mathsf{inr}(y) \mapsto v$
$\qquad = t_2 \;\in\; T \;\lfloor\mathrm{Ax}\rfloor$
by $\underline{\texttt{decideRedL}}$
$\quad\Gamma \vdash u[s/x] \;=\; t_2 \;\in\; T \;\lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash \mathsf{case}\ \mathsf{inr}(t)\ \mathsf{of}\ \mathsf{inl}(x) \mapsto u \mid \mathsf{inr}(y) \mapsto v$
$\qquad = t_2 \;\in\; T \;\lfloor\mathrm{Ax}\rfloor$
by $\underline{\texttt{decideRedR}}$
$\quad\Gamma \vdash v[t/y] \;=\; t_2 \;\in\; T \;\lfloor\mathrm{Ax}\rfloor$

A.5.8 Gleichheit

$\Gamma \vdash s_1{=}t_1 \in T_1 \;=\; s_2{=}t_2 \in T_2 \;\in\; \mathsf{U}_j \;\lfloor\mathrm{Ax}\rfloor$
by $\underline{\texttt{equalityEq}}$
$\quad\Gamma \vdash T_1 \;=\; T_2 \;\in\; \mathsf{U}_j \;\lfloor\mathrm{Ax}\rfloor$
$\quad\Gamma \vdash s_1 \;=\; s_2 \;\in\; T_1 \;\lfloor\mathrm{Ax}\rfloor$
$\quad\Gamma \vdash t_1 \;=\; t_2 \;\in\; T_1 \;\lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash \mathsf{Axiom} \;\in\; s{=}t \in T \;\lfloor\mathrm{Ax}\rfloor$
by $\underline{\texttt{axiomEq}}$
$\quad\Gamma \vdash s \;=\; t \;\in\; T \;\lfloor\mathrm{Ax}\rfloor$

$\Gamma,\ z{:}\ s{=}t \in T,\ \Delta \vdash C \;\lfloor\textbf{ext}\ u\rfloor$
by $\underline{\texttt{equalityE}\ i}$
$\quad\Gamma,\ z{:}\ s{=}t \in T,\ \Delta[\mathsf{Axiom}/z]$
$\qquad \vdash C[\mathsf{Axiom}/z] \;\lfloor\textbf{ext}\ u\rfloor$

$\Gamma,\ x{:}T,\ \Delta \vdash x \;\in\; T \;\lfloor\mathrm{Ax}\rfloor$
by $\underline{\texttt{hypEq}\ i}$

$\Gamma \vdash s{=}t \in T \;\lfloor\mathrm{Ax}\rfloor$
by $\underline{\texttt{symmetry}}$
$\quad\Gamma \vdash t{=}s \in T \;\lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash s{=}t \in T \;\lfloor\mathrm{Ax}\rfloor$
by $\underline{\texttt{transitivity}\ t'}$
$\quad\Gamma \vdash s{=}t' \in T \;\lfloor\mathrm{Ax}\rfloor$
$\quad\Gamma \vdash t'{=}t \in T \;\lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash C[s/x] \;\lfloor\textbf{ext}\ u\rfloor$
by $\underline{\texttt{subst}\ j\ \ s{=}t \in T\ \ x\ \ C}$
$\quad\Gamma \vdash s{=}t \in T \;\lfloor\mathrm{Ax}\rfloor$
$\quad\Gamma \vdash C[t/x] \;\lfloor\textbf{ext}\ u\rfloor$
$\quad\Gamma,\ x{:}T \vdash C \;\in\; \mathsf{U}_j \;\lfloor\mathrm{Ax}\rfloor$

A.5.9 Listen

$\Gamma \vdash T_1\ \mathsf{list} \;=\; T_2\ \mathsf{list} \;\in\; \mathsf{U}_j \;\lfloor\mathrm{Ax}\rfloor$
by $\underline{\texttt{listEq}}$
$\quad\Gamma \vdash T_1 \;=\; T_2 \;\in\; \mathsf{U}_j \;\lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash [\,] \;\in\; T\ \mathsf{list} \;\lfloor\mathrm{Ax}\rfloor$
by $\underline{\texttt{nilEq}\ j}$
$\quad\Gamma \vdash T \;\in\; \mathsf{U}_j \;\lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash T\ \mathsf{list} \;\lfloor\textbf{ext}\ [\,]\rfloor$
by $\underline{\texttt{nilI}\ j}$
$\quad\Gamma \vdash T \;\in\; \mathsf{U}_j \;\lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash t_1.l_1 \;=\; t_2.l_2 \;\in\; T\ \mathsf{list} \;\lfloor\mathrm{Ax}\rfloor$
by $\underline{\texttt{consEq}}$
$\quad\Gamma \vdash t_1 \;=\; t_2 \;\in\; T \;\lfloor\mathrm{Ax}\rfloor$
$\quad\Gamma \vdash l_1 \;=\; l_2 \;\in\; T\ \mathsf{list} \;\lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash T\ \mathsf{list} \;\lfloor\textbf{ext}\ t.l\rfloor$
by $\underline{\texttt{consI}}$
$\quad\Gamma \vdash T \;\lfloor\textbf{ext}\ t\rfloor$
$\quad\Gamma \vdash T\ \mathsf{list} \;\lfloor\textbf{ext}\ l\rfloor$

$\Gamma \vdash \mathsf{list\_ind}(s_1;\ base_1;\ x_1,l_1,f_{xl1}.t_1)$
$\quad = \mathsf{list\_ind}(s_2;\ base_2;\ x_2,l_2,f_{xl2}.t_2) \;\in\; T[s_1/z] \;\lfloor\mathrm{Ax}\rfloor$
by $\underline{\texttt{list\_indEq}\ z\ T\ S\ \mathsf{list}}$
$\quad\Gamma \vdash s_1 \;=\; s_2 \;\in\; S\ \mathsf{list} \;\lfloor\mathrm{Ax}\rfloor$
$\quad\Gamma \vdash base_1 \;=\; base_2 \;\in T[[\,]/z] \;\lfloor\mathrm{Ax}\rfloor$
$\quad\Gamma,\ x{:}S,\ l{:}S\ \mathsf{list},\ f_{xl}{:}T[l/z] \vdash$
$\qquad t_1[x,l,f_{xl}\,/\,x_1,l_1,f_{xl1}] \;=\; t_1[x,l,f_{xl}\,/\,x_2,l_2,f_{xl2}]$
$\qquad \in\; T[x.l/z] \;\lfloor\mathrm{Ax}\rfloor$

$\Gamma,\ z{:}T\ \mathsf{list},\ \Delta \vdash C$
$\quad \lfloor\textbf{ext}\ \mathsf{list\_ind}(z;\ base;\ x,l,f_{xl}.t)\rfloor$
by $\underline{\texttt{listE}\ i}$
$\quad\Gamma,\ z{:}T\ \mathsf{list},\ \Delta \vdash C[[\,]/z] \;\lfloor\textbf{ext}\ base\rfloor$
$\quad\Gamma,\ z{:}T\ \mathsf{list},\ \Delta,\ x{:}T,\ l{:}T\ \mathsf{list},\ f_{xl}{:}C[l/z]$
$\qquad \vdash C[x.l/z] \;\lfloor\textbf{ext}\ t\rfloor$

$\Gamma \vdash \mathsf{list\_ind}([\,];\ base;\ x,l,f_{xl}.t) \;=\; t_2 \;\in\; T \;\lfloor\mathrm{Ax}\rfloor$
by $\underline{\texttt{list\_indRedBase}}$
$\quad\Gamma \vdash base \;=\; t_2 \;\in\; T \;\lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash \mathsf{list\_ind}(s.u;\ base;\ x,l,f_{xl}.t) \;=\; t_2 \;\in\; T \;\lfloor\mathrm{Ax}\rfloor$
by $\underline{\texttt{list\_indRedUp}}$
$\quad\Gamma \vdash t[s,u,\mathsf{list\_ind}(u;\ base;\ x,l,f_{xl}.t)\ /x,l,f_{xl}]$
$\qquad = t_2 \;\in\; T \;\lfloor\mathrm{Ax}\rfloor$

A.5.10 Teilmengen

$\Gamma \vdash \{x_1{:}S_1 \mid T_1\} = \{x_2{:}S_2 \mid T_2\} \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
by <u>`setEq`</u>
$\Gamma \vdash S_1 = S_2 \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma,\ x{:}S_1 \vdash T_1[x/x_1] = T_2[x/x_2] \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash s = t \in \{x{:}S \mid T\} \ \lfloor\mathrm{Ax}\rfloor$
by <u>`elementEq` j</u>
$\Gamma \vdash s = t \in S \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma \vdash T[s/x] \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma,\ x'{:}S \vdash T[x'/x] \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash \{x{:}S \mid T\} \ \lfloor\textbf{ext } s\rfloor$
by <u>`elementI` j s</u>
$\Gamma \vdash s \in S \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma \vdash T[s/x] \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma,\ x'{:}S \vdash T[x'/x] \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$

$\Gamma,\ z{:}\{x{:}S \mid T\},\ \Delta \vdash C \ \lfloor\textbf{ext } (\lambda y.t)\ z\rfloor$
by <u>`setE` i</u>
$\Gamma,\ z{:}\{x{:}S \mid T\},\ y{:}S,\ [\![v]\!]{:}T[y/x],\ \Delta[y/z]$
$\vdash C[y/z] \ \lfloor\textbf{ext } t\rfloor$

$\Gamma \vdash s = t \in \{S \mid T\} \ \lfloor\mathrm{Ax}\rfloor$
by <u>`elementEq_indep`</u>
$\Gamma \vdash s = t \in S \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma \vdash T \ \lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash \{S \mid T\} \ \lfloor\textbf{ext } s\rfloor$
by <u>`elementI_indep`</u>
$\Gamma \vdash S \ \lfloor\textbf{ext } s\rfloor$
$\Gamma \vdash T \ \lfloor\mathrm{Ax}\rfloor$

A.5.11 Quotienten

$\Gamma \vdash x_1,y_1 : T_1/\!/E_1 = x_2,y_2 : T_2/\!/E_2 \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
by <u>`quotientEq_weak`</u>
$\Gamma \vdash T_1 = T_2 \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma,\ x{:}T_1,\ y{:}T_1$
$\vdash E_1[x,y/x_1,y_1] = E_2[x,y/x_2,y_2] \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma,\ x{:}T_1,\ y{:}T_1 \vdash E_1[x,x/x_1,y_1] \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma,\ x{:}T_1,\ y{:}T_1,\ v{:}E_1[x,y/x_1,y_1]$
$\vdash E_1[y,x/x_1,y_1] \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma,\ x{:}T_1,\ y{:}T_1,\ z{:}T_1,\ v{:}E_1[x,y/x_1,y_1],$
$v'{:}E_1[y,z/x_1,y_1] \vdash E_1[x,z/x_1,y_1] \ \lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash x_1,y_1 : T_1/\!/E_1 = x_2,y_2 : T_2/\!/E_2 \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
by <u>`quotientEq`</u>
$\Gamma \vdash x_1,y_1 : T_1/\!/E_1 \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma \vdash x_2,y_2 : T_2/\!/E_2 \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma \vdash T_1 = T_2 \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma,\ v{:}\ T_1{=}T_2 \in \mathsf{U}_j,\ x{:}T_1,\ y{:}T_1$
$\vdash E_1[x,y/x_1,y_1] \Rightarrow E_2[x,y/x_2,y_2] \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma,\ v{:}\ T_1{=}T_2 \in \mathsf{U}_j,\ x{:}T_1,\ y{:}T_1$
$\vdash E_2[x,y/x_2,y_2] \Rightarrow E_1[x,y/x_1,y_1] \ \lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash s = t \in x,y : T/\!/E \ \lfloor\mathrm{Ax}\rfloor$
by <u>`memberEq_weak` j</u>
$\Gamma \vdash x,y : T/\!/E \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma \vdash s = t \in T \ \lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash x,y : T/\!/E \ \lfloor\textbf{ext } t\rfloor$
by <u>`memberI` j</u>
$\Gamma \vdash x,y : T/\!/E \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma \vdash T \ \lfloor\textbf{ext } t\rfloor$

$\Gamma \vdash s = t \in x,y : T/\!/E \ \lfloor\mathrm{Ax}\rfloor$
by <u>`memberEq` j</u>
$\Gamma \vdash x,y : T/\!/E \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma \vdash s \in T \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma \vdash t \in T \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma \vdash E[s,t/x,y] \ \lfloor\mathrm{Ax}\rfloor$

$\Gamma,\ v{:}\ s{=}t \in x,y : T/\!/E,\ \Delta \vdash C \ \lfloor\textbf{ext } u\rfloor$
by <u>`quotient_eqE` i j</u>
$\Gamma,\ v{:}\ s{=}t \in x,y : T/\!/E,\ [\![v']\!]{:}E[s,t/x,y],\ \Delta$
$\vdash C \ \lfloor\textbf{ext } u\rfloor$
$\Gamma,\ v{:}\ s{=}t \in x,y : T/\!/E,\ \Delta,\ x'{:}T,\ y'{:}T$
$\vdash E[x',y'/x,y] \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$

$\Gamma,\ z{:}\ x,y : T/\!/E,\ \Delta \vdash s{=}t \in S \ \lfloor\mathrm{Ax}\rfloor$
by <u>`quotientE` i j</u>
$\Gamma,\ z{:}\ x,y : T/\!/E,\ \Delta,\ x'{:}T,\ y'{:}T$
$\vdash E[x',y'/x,y] \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma,\ z{:}\ x,y : T/\!/E,\ \Delta \vdash S \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma,\ z{:}\ x,y : T/\!/E,\ \Delta,\ x'{:}T,\ y'{:}T,$
$v{:}\ E[x',y'/x,y] \vdash s[x'/z] = t[y'/z] \in S[x'/z] \ \lfloor\mathrm{Ax}\rfloor$

A.5.12 Induktive Typen

$\Gamma \vdash \mathsf{rectype}\ X_1 = T_{X1} = \mathsf{rectype}\ X_2 = T_{X2} \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
by <u>`recEq`</u>
$\Gamma,\ X{:}\mathsf{U}_j \vdash T_{X1}[X/X_1] = T_{X2}[X/X_2] \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash s = t \in \mathsf{rectype}\ X = T_X \ \lfloor\mathrm{Ax}\rfloor$
by <u>`rec_memEq` j</u>
$\Gamma \vdash s = t \in T_X[\mathsf{rectype}\ X = T_X\ /\ X] \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma \vdash \mathsf{rectype}\ X = T_X \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash \mathsf{rectype}\ X = T_X \ \lfloor\textbf{ext } t\rfloor$
by <u>`rec_memI` j</u>
$\Gamma \vdash T_X[\mathsf{rectype}\ X = T_X\ /\ X] \ \lfloor\textbf{ext } t\rfloor$
$\Gamma \vdash \mathsf{rectype}\ X = T_X \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash \mathsf{let}^*\ f_1(x_1) = t_1\ \mathsf{in}\ f_1(e_1)$
$= \mathsf{let}^*\ f_2(x_2) = t_2\ \mathsf{in}\ f_2(e_2) \in T[e_1/z] \ \lfloor\mathrm{Ax}\rfloor$
by <u>`rec_indEq` z T $\mathsf{rectype}\ X = T_X$ j</u>
$\Gamma \vdash e_1 = e_2 \in \mathsf{rectype}\ X = T_X \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma \vdash \mathsf{rectype}\ X = T_X \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma,\ P{:}\ (\mathsf{rectype}\ X = T_X) \to \mathbb{P}_j,$
$f{:}\ (y{:}\{x{:}\mathsf{rectype}\ X = T_X \mid P(x)\} \to T[y/z]),$
$x{:}\ T_X[\{x{:}\mathsf{rectype}\ X = T_X \mid P(x)\}\ /\ X]$
$\vdash t_1[f,x\ /\ f_1,x_1] = t_2[f,x\ /\ f_2,x_2] \in T[x/z] \ \lfloor\mathrm{Ax}\rfloor$

$\Gamma,\ z{:}\ \mathsf{rectype}\ X = T_X,\ \Delta \vdash C$
$\lfloor\textbf{ext } \mathsf{let}^*\ f(x) = t[\lambda y.\Lambda\ /\ P]\ \mathsf{in}\ f(z)\rfloor$
by <u>`recE` i j</u>
$\Gamma,\ z{:}\ \mathsf{rectype}\ X = T_X,\ \Delta$
$\vdash \mathsf{rectype}\ X = T_X \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma,\ z{:}\ \mathsf{rectype}\ X = T_X,\ \Delta,$
$P{:}\ (\mathsf{rectype}\ X = T_X) \to \mathbb{P}_j,$
$f{:}\ (y{:}\{x{:}\mathsf{rectype}\ X = T_X \mid P(x)\} \to C[y/z]),$
$x{:}\ T_X[\{x{:}\mathsf{rectype}\ X = T_X \mid P(x)\}\ /\ X]$
$\vdash C[x/z] \ \lfloor\textbf{ext } t\rfloor$

$\Gamma,\ z{:}\ \mathsf{rectype}\ X = T_X,\ \Delta \vdash C \ \lfloor\textbf{ext } t[z/x]\rfloor$
by <u>`recE_unroll` i</u>
$\Gamma,\ z{:}\ \mathsf{rectype}\ X = T_X,\ \Delta,$
$x{:}\ T_X[\mathsf{rectype}\ X = T_X\ /\ X],$
$v{:}\ z{=}x \in T_X[\mathsf{rectype}\ X = T_X\ /\ X] \vdash C[x/z] \ \lfloor\textbf{ext } t\rfloor$

A.5.13 Rekursive Funktionen

$\Gamma \vdash S_1 \not\to T_1 = S_2 \not\to T_2 \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
by <u>`pfunEq`</u>
$\Gamma \vdash S_1 = S_2 \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma \vdash T_1 = T_2 \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash (\mathsf{letrec}\ f_1(x_1) = t_1)$
$= (\mathsf{letrec}\ f_2(x_2) = t_2) \in S \not\to T \ \lfloor\mathrm{Ax}\rfloor$
by <u>`fixEq` j</u>
$\Gamma \vdash \mathsf{letrec}\ f_1(x_1) = t_1 \in S \not\to T \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma \vdash \mathsf{letrec}\ f_2(x_2) = t_2 \in S \not\to T \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma \vdash \{x{:}S \mid \mathsf{dom}(\mathsf{letrec}\ f_1(x_1) = t_1)(x)\}$
$= \{x{:}S \mid \mathsf{dom}(\mathsf{letrec}\ f_2(x_2) = t_2)(x)\} \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma,\ y{:}\{x{:}S \mid \mathsf{dom}(\mathsf{letrec}\ f_1(x_1) = t_1)(x)\}$
$\vdash (\mathsf{letrec}\ f_1(x_1) = t_1)\ (y)$
$= (\mathsf{letrec}\ f_2(x_2) = t_2)\ (y) \in T \ \lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash \mathsf{letrec}\ f(x) = t \in S \not\to T \ \lfloor\mathrm{Ax}\rfloor$
by <u>`fixMem` j</u>
$\Gamma \vdash S \not\to T \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma,\ f'{:}S \not\to T,\ x'{:}S \vdash \mathcal{E}[\![\,t[f',x'\ /\ f,x]\,]\!] \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma,\ f'{:}S \not\to T,\ x'{:}S,\ \mathcal{E}[\![\,t[f',x'\ /\ f,x]\,]\!]$
$\vdash t[f',x'\ /\ f,x] \in T \ \lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash f_1\,(t_1) = f_2\,(t_2) \in T \ \lfloor\mathrm{Ax}\rfloor$
by <u>`apply_pEq` $S \not\to T$</u>
$\Gamma \vdash f_1 = f_2 \in S \not\to T \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma \vdash t_1 = t_2 \in \{x{:}S \mid \mathsf{dom}(f_1)(x)\} \ \lfloor\mathrm{Ax}\rfloor$

$\Gamma,\ f{:}S \not\to T,\ \Delta \vdash C \ \lfloor\textbf{ext } t[f\,(s),\mathsf{Axiom}\ /\ y,v]\rfloor$
by <u>`pfunE` i s</u>
$\Gamma,\ f{:}S \not\to T,\ \Delta \vdash s \in \{x{:}S \mid \mathsf{dom}(f_1)(x)\} \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma,\ f{:}S \not\to T,\ \Delta,\ y{:}T,\ v{:}\ y{=}f\,(s) \in T \vdash C \ \lfloor\textbf{ext } t\rfloor$

$\Gamma \vdash (\mathsf{letrec}\ f(x) = t)\ (u) = t_2 \in T \ \lfloor\mathrm{Ax}\rfloor$
by <u>`apply_pRed`</u>
$\Gamma \vdash t[\mathsf{letrec}\ f(x) = t,\ u\ /\ f,x] = t_2 \in T \ \lfloor\mathrm{Ax}\rfloor$

$\Gamma \vdash \mathsf{dom}(f_1) = \mathsf{dom}(f_2) \in S \to \mathbb{P}_j \ \lfloor\mathrm{Ax}\rfloor$
by <u>`domEq` $S \not\to T$</u>
$\Gamma \vdash f_1 = f_2 \in S \not\to T \ \lfloor\mathrm{Ax}\rfloor$
$\Gamma \vdash S \not\to T \in \mathsf{U}_j \ \lfloor\mathrm{Ax}\rfloor$

A.5.14 Strukturelle Regeln und sonstige Regeln

$\Gamma,\ x{:}T,\ \Delta \vdash T\ \ \lfloor\textsf{ext}\ x\rfloor$
by $\underline{\texttt{hypothesis}\ i}$

$\Gamma \vdash T\ \ \lfloor\textsf{ext}\ t\rfloor$
by $\underline{\texttt{intro}\ t}$
$\Gamma \vdash t \in T\ \ \lfloor\textsf{Ax}\rfloor$

$\Gamma, \Delta \vdash C\ \ \lfloor\textsf{ext}\ (\lambda x.t)\ s\rfloor$
by $\underline{\texttt{cut}\ i\ T}$
$\Gamma, \Delta \vdash T\ \ \lfloor\textsf{ext}\ s\rfloor$
$\Gamma, x{:}T, \Delta \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$

$\Gamma,\ x{:}T,\ \Delta \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$
by $\underline{\texttt{thin}\ i}$
$\Gamma, \Delta \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$

$\Gamma \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$
by $\underline{\texttt{compute}\ tagC}$
$\Gamma \vdash C{\downarrow}_{tagC}\ \ \lfloor\textsf{ext}\ t\rfloor$

$\Gamma \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$
by $\underline{\texttt{rev\_compute}\ tagC}$
$\Gamma, \vdash C{\uparrow}_{tagC}\ \ \lfloor\textsf{ext}\ t\rfloor$

$\Gamma,\ z{:}T,\ \Delta \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$
by $\underline{\texttt{computeHyp}\ i\ tagT}$
$\Gamma,\ z{:}T{\downarrow}_{tagT},\ \Delta \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$

$\Gamma,\ z{:}T,\ \Delta \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$
by $\underline{\texttt{rev\_computeHyp}\ i\ tagT}$
$\Gamma,\ z{:}T{\uparrow}_{tagT},\ \Delta \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$

$\Gamma \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$
by $\underline{\texttt{unfold}\ \textit{def-name}}$
$\Gamma \vdash C_{\downarrow}\ \ \lfloor\textsf{ext}\ t\rfloor$

$\Gamma \vdash C_{\downarrow}\ \ \lfloor\textsf{ext}\ t\rfloor$
by $\underline{\texttt{fold}\ \textit{def-name}}$
$\Gamma \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$

$\Gamma,\ z{:}T,\ \Delta \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$
by $\underline{\texttt{unfoldHyp}\ i\ \textit{def-name}}$
$\Gamma,\ z{:}T_{\downarrow},\ \Delta \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$

$\Gamma,\ z{:}T_{\downarrow},\ \Delta \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$
by $\underline{\texttt{foldHyp}\ i\ \textit{def-name}}$
$\Gamma,\ z{:}T,\ \Delta \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$

$\Gamma,\ z{:}T,\ \Delta \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$
by $\underline{\texttt{replaceHyp}\ i\ S\ j}$
$\Gamma,\ z{:}S,\ \Delta \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$
$\Gamma,\ z{:}T,\ \Delta \vdash T = S \in \mathsf{U}_j\ \ \lfloor\textsf{Ax}\rfloor$

$\Gamma \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$
by $\underline{\texttt{lemma}\ \textit{theorem-name}}$

$\Gamma \vdash t \in T\ \ \lfloor\textsf{Ax}\rfloor$
by $\underline{\texttt{extract}\ \textit{theorem-name}}$

$\Gamma \vdash C\ \ \lfloor\textsf{ext}\ t[\sigma]\rfloor$
by $\underline{\texttt{instantiate}\ \Gamma'\ C'\ \sigma}$
$\Gamma' \vdash C'\ \ \lfloor\textsf{ext}\ t\rfloor$

$\Gamma \vdash C\ \ \lfloor\textsf{ext}\ t[y/x]\rfloor$
by $\underline{\texttt{rename}\ y\ x}$
$\Gamma[x/y] \vdash C[x/y]\ \ \lfloor\textsf{ext}\ t\rfloor$

$\Gamma \vdash s = t \in T\ \ \lfloor\textsf{Ax}\rfloor$
by $\underline{\texttt{equality}}$

$\Gamma \vdash C\ \ \lfloor\textsf{ext}\ t\rfloor$
by $\underline{\texttt{arith}\ j}$
$\Gamma \vdash s_1 \in \mathbb{Z}\ \ \lfloor\textsf{Ax}\rfloor$
$\vdots$