Errors to be Corrected
in the Third Printing of

A Logical Approach to Discrete Math

by

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A number of errors will be corrected in the third printing of "A Logical Approach to Discrete Math". If you have a copy of an earlier printing, you may want to copy these changes into it. The lines of a page are labeled +0, +1, +2, ... from the top and -0, -1, -2, ... from the bottom.

Inside front cover, table "Types used in this Text". In the fourth type, change "negative integers: $1, 2, 3, \ldots$" to "negative integers: $-1, -2, -3, \ldots$".

Page xi, last paragraph. Add Konrad Gries, Gil Neiger, and Stan Warford to the acknowledgements.

Page xii, line -4 (just before the footnote). Add email addresses gries@cs.cornell.edu and fbs@cs.cornell.edu.

Page 6, historical note, lines +6 and +12. Change "counting" to "numbering".

Page 6, historical note, line -3. Delete the comma after "Arabic".

Page 10, first paragraph of subsection INFERENCE RULE SUBSTITUTION. Change this paragraph to the following.

> Our first use of textual substitution comes in the form of an *inference rule*, which provides a syntactic mechanism for deriving "truths", or *theorems* as we call them. Later, we see that theorems correspond to expressions that are true in all states. An inference rule consists of a list of expressions, called its *premises* or *hypotheses*, above a line and an expression, called its *conclusion*, below the line. It asserts that if the premises are theorems, then the conclusion is a theorem.
>
> Inference rule Substitution uses an expression E, a list of variables v, and a corresponding list of expressions F:

Page 11, line +4. Change "holds in all states, then so does" to "is a theorem, then so is".

Page 11, line +7. Change "is true in every state" to "is a theorem".

Page 11, line +8. Change "is true in every state" to "is also a theorem".

Page 11, line +10. Change "is true in every state" to "is a theorem".

Page 11, line +11. Change "holds as well" to "is also a theorem".

Page 11, last paragraph. Change the second sentence to "The first two are expressions that we postulate are theorems (and they are true in every state)."

Page 12, line -6. Change "holds in all states" to "is a theorem"

Page 12, line -6. Change "holds, by" to "is a theorem, by".

Page 13, historical note, line +8. Change "coinsidentia" to "coincidentia".

Page 15, line +1. Change "holds in every state" to "is a theorem".

Page 15, lines +8–+10. Change three occurrences of "holds in all states" to "is a theorem".

Page 28, line -2. Fix the word "Also" at the beginning of the line.

Page 28, line -0. Change "$(x + y \cdot z)$" to "$x + (y \cdot z)$".

Page 32, Metatheorem (2.3). The Metatheorem is misstated. The metatheorem and the three lines below it should read,

> (2.3) Metatheorem Duality.
> (a) P is valid iff $\neg P_D$ is valid.
> (b) $P \equiv Q$ is valid iff $P_D \equiv Q_D$ is valid.
>
> Table 2.2 illustrates Metatheorem (2.3). In the table, all expressions on the left are valid; hence, so are the

Page 33, line +1 of Table 2.3. Change "and becomes $\wedge$" to "and, but becomes $\wedge$".

Page 36, lines +5–+6. Change "Bob can see with j eyes" to "Bob has sight in j of his eyes".

Page 36, line -8. Change "$(x \Rightarrow y)$." to $(y \Rightarrow x)$.".

Page 42. Change the first line to

> A *theorem* of our propositional calculus is either (i) an axiom, (ii) the conclusion of an inference rule whose premises are theorems, or (iii) a".

The next edition will see a bigger change here. We will have have four inference rules instead of three. The additional rule will be a form of modus ponens for equivalence:

$$\textbf{Equanimity}: \quad \frac{E,\ E \equiv F}{F} \quad .$$

We will then be able to say that a theorem is either an axiom or the conclusion of an inference rule whose premises are theorems. That is simpler and more conventional.

Further, this extra inference rule will eliminate the need for calling (3.6) a *new* proof method. Consider the following proof, for boolean expressions $E0, E1, E2$ and $E3$.

$$\begin{array}{ll} & E0 \\ = & \quad \langle \ldots \rangle \\ & E1 \\ = & \quad \langle \ldots \rangle \\ & E2 \\ = & \quad \langle \ldots \rangle \\ & E3 \end{array}$$

Inference rule Leibniz is still used for each step, with inference rule Substitution perhaps being used in the hint to generate the premise of a Leibniz step. Transitivity is used implicitly, twice, to conclude that $E0 = E3$ is a theorem. Further, if $E3$ is a theorem, the new inference rule allows us to conclude that $E0$ is a theorem.

Page 46, lines +3–+4 after the box. Change "By the reflexivity of $\equiv$" to "By Identity of $\equiv$ (3.3)"

Page 47, line -12. Change "they obvious" to "they are obvious".

Page 85, line with first hint of the proof in the middle of the page. Change "$-$" in the lefthand column to "$=$".

Page 90, paragraph 1 of Section 5.2. Change "low signal voltage" to "low signal voltages".

Page 90, paragraph 3 of Section 5.2. Change "affect" to "effect".

Page 122, fifth line before the exercises. Change "in which your are working" to "in which you are working".

Page 139, Table 8.1. In the fourth type, change "negative integers: $1, 2, 3, \ldots$" to "negative integers: $-1, -2, -3, \ldots$".

Page 141, line +6. Change "expressed a follows" to "expressed as follows".

Page 148, footnote, line +2. Add a comma after "(8.14) does not hold".

Page 148, footnote, line +3. Change "Empty range (8.13)" to "Range split (8.16)".

Page 149, Axiom (8.15). Change "$P, Q{:}B$ or R is finite" to "each quantification is defined".

Page 149, Axiom (8.16). Change "$P{:}B$ or R and S are finite" to "each quantification is defined"

Page 149, footnote 7. Rewrite the footnote as follows.

The sum $(+i \mid 0 < i : i) = 1+2+3+\ldots$ is not defined. Using $0 = i+(-i)$, we have the following instance of Axiom (8.15). Its LHS is 0 but its RHS is undefined.

$$(+i \mid 0 < i : 0) = (+i \mid 0 < i : i) + (+i \mid 0 < i : -i)$$

This is the reason for the caveat on some of the axioms.

The sum $(+i \mid 0 < i : 1/i^2) = 1/1 + 1/4 + 1/8 + 1/16 + \ldots$ is defined to equal $\pi^2/6$, even though it is an infinite sum, because $(+i \mid 0 < i \leq n : 1/i^2)$ "converges" to $\pi^2/6$ as n gets larger. Similarly, $(\equiv i \mid 0 < i : \mathit{true})$ is defined to equal true, since $(\equiv i \mid 0 < i \leq n : \mathit{true}) \equiv \mathit{true}$ for all n. But $(\equiv i \mid 0 < i : \mathit{false})$ is undefined (why?).

A complete discussion of when a quantification is defined is outside the scope of this text. Entire books are written on the subject of convergence of summations. However, quantifications with finite ranges are always defined, and quantifications using operator $\wedge$ and $\vee$ are always defined.

Page 150, Axiom (8.17). Change "$P{:}B$ or R and S are finite" to "each quantification is defined".

Page 150, Axiom (8.18). Add the caveat "Provided each quantification is defined,".

Page 150, Axiom (8.19). Change "$\star$ is idempotent or $P{:}B$ or R and S are finite" to "each quantification is defined".

Page 150, line -8. Replace "The restrictions on these laws" by "The "occurs" restrictions on these laws".

Page 152, Theorem 8.23 (Split off term). Replace "For $n{:}\mathbb{N}$," by "For $n{:}\mathbb{N}$ and dummies $i{:}\mathbb{N}$,".

Page 152, line -6. Change "5^2+" to "$5^2\cdot$".

Page 155, Exercise 8.2. Change "is type" to "has type".

Page 158, line +7. Change "that $+$ is symmetric and" to "that $\cdot$ is symmetric and".

Page 166, heading of box on top. Change "Instantiation" to "Introduction".

Page 166, line -5. Change the hint "Range split for idempotent $\vee$ (8.18)" to "Distributivity (8.15)"

Page 167, Metatheorem (9.30). Change "Suppose $\neg occurs(\text{`}\hat{x}\text{'}, \text{`}Q\text{'})$." to "Suppose $\neg occurs(\text{`}\hat{x}\text{'}, \text{`}P, Q, R\text{'})$."

Page 167, sixth line in the proof. Change "Dummy renaming (8.21), where $\neg occurs(\text{`}\hat{x}, \text{'}, \text{`}Q\text{'})$". "Dummy renaming (8.21), choosing $\hat{x}$ that is not in Q".

Page 167, seventh line in the proof. Change "$(\forall \hat{x} \mid \neg(R \wedge P)[x := \hat{x}]) \Rightarrow Q$" to "$(\forall \hat{x} \mid \neg(R \wedge P)[x := \hat{x}]) \vee Q$".

Page 167, eighth line in the proof. Change "Distributivity of $\vee$ over $\forall$ (9.5)" to "Distributivity of $\vee$ over $\forall$ (9.5) — $\neg occurs(\text{`}\hat{x}\text{'}, \text{`}Q\text{'})$".

Page 167, formula (9.31). Change the formula from $(\exists x{:}\mathbb{N} \mid : x + a = 0)$ to $(\exists x{:}\mathbb{Z} \mid : x + a = 0)$.

Page 168, line +3. Change the formula $(\exists x{:}\mathbb{N} \mid : x + a = 0)$ to $(\exists x{:}\mathbb{Z} \mid : x + a = 0)$.

Page 168, line -7 (last display on page). Change the formula $(\exists x{:}\mathbb{N} \mid : x + a + 5 = 0)$ to $(\exists x{:}\mathbb{Z} \mid : x + a + 5 = 0)$.

Page 174, Exercise 9.11. Change the formula $(\forall x \mid : P) \equiv P[x := E]$ to $(\forall x \mid : P) \Rightarrow P[x := E]$.

Page 196, line +1. Change "t type" to "t a type".

Page 202, formula (11.19). Change (11.19) from $\sim\sim S \equiv S$ to $\sim\sim S = S$.

Page 206, formula (11.49). Change $S - T \equiv S \cap \sim T$ to $S - T = S \cap \sim T$.

Page 214, Exercise (11.13a). Change $S - T \equiv S \cap \sim T$ to $S - T = S \cap \sim T$.

Page 225, line +6. Change "Phidean" to "Phideas".

Page 317, theorem (15.98). Change "**Idempotency**: b **gcd** $b = b$" to "b **gcd** $b = abs.b$".

Page 317, theorem (15.100). Change "**Identity**: 0 **gcd** $b = b$" to "0 **gcd** $b = abs.b$".

Page 318, theorem (15.104). Add the antecedent $0 \leq d$.

Page 327, line -4 (in Table 15.1). Change "VI" to "VII".

Page 335, Exercise 15.71. Change "Idempotency (15.98), b **gcd** $b = b$" to "(15.98), b **gcd** $b = abs.b$".

Page 335, Exercise 15.73. Change "Identity (15.100), 0 **gcd** $b = b$" to "(15.100), 0 **gcd** $b = abs.b$".

Page 335, Exercise 15.77. Add the antecedent $0 \leq d$ to the formula.

Page 338, line -5. Change "(Remember that $0! = 0$.)" to "(Remember that $0! = 1$.)".

Page 344, line -15. Change "using Symmetry (16.14) and Absorption (16.15)." to "using Symmetry (16.14) and Absorption (16.16).".

Page 344, line -12. Change "Absorption theorems (16.16) and (16.17) together (see exercise" to "theorems (16.16) and (16.17) together (see Exercise".

Page 345, line +8. Change "Definition (16.9), twice" to "Definition of $C(s, r, S)$, twice".

Page 390, Theorem (18.6). Change "with respect to an operator" to "with respect to an associative operator".

Page 397, line +19. Change "Signficant" to "Significant".

Page 403, line +9. Change "complete amateur" to "completely amateur".

Page 405, last line of proof of (d). Change "$h.m + h.n$" to "$h(b^m) + h(b^n)$".

Page 408, line +4. Change "Figure 18.2" to "Table 18.2".

Page 408, line +6. Change "making a 90-degree rotation followed by a 180-degee" to "making a 180-degree rotation followed by a 90-degee".

Page 408, Table 18.2. This table is wrong, since it was written for relation composition $\circ$ instead of function composition $\bullet$. Change the table to

$\bullet$	I	R	R'	R''	H	V	D	D'
I	I	R	R'	R''	H	V	D	D'
R	R	R'	R''	I	D'	D	H	V
R'	R'	R''	I	R	V	H	D'	D
R''	R''	I	R	R'	D	D'	V	H
H	H	D	V	D'	I	R'	R	R''
V	V	D'	H	D	R'	I	R''	R
D	D	V	D'	H	R''	R	I	R'
D'	D'	H	D	V	R	R''	R'	I

Page 412, line +4. Change "$(HD'VD)$" to $(HDVD')$.

Page 412, lines Change lines +7–+14. Change these to the following.

I: $(I)\,(R)\,(R')\,(R'')\,(H)\,(V)\,(D)\,(D')$
R: $(IRR'R'')\,(HDVD')$
R': $(IR')\,(RR'')\,(HV)\,(DD')$
R'': $(IR''R'R)\,(HD'VD)$
H: $(IH)\,(RD'R''D)\,(R'V)$
V: $(IV)\,(RDR''D')\,(R'H)$
D: $(ID)\,(RH)\,(R'D')\,(R''V)$
D': $(ID')\,(RV)\,(R'D)\,(R''H)$

Page 418, Exercise 18.16. Change "$\langle S, \circ \rangle$" to "$\langle S, \circ, 1 \rangle$".

Page 418, Exercise 18.18. Change "$\langle S, \circ \rangle$" to "$\langle S, \circ, 1 \rangle$".

Page 418, Exercise 18.26e. Change "$\langle \mathbb{N}, \cdot \rangle$" to "$\langle \mathbb{N}, \cdot, 1 \rangle$".

Page 419, Exercise 18.31. Change "$\langle S, \circ \rangle$" to "$\langle S, \circ, 1 \rangle$".

Page 419, Exercise 18.37. Change "function h" to "function "hn".

Page 421, Exercise 18.75. Change "$b = 0)$" to "$b = 0$".

Page 421, Exercise 18.77. Change "$b \cdot \sim y = 0$" to "$b \otimes \sim y = 0$".

Page 424, line +7. Change the second occurrence of "$\{b, c\}$" to "$\{b, d\}$".

Page 462, line +7. Change "Exercise (20.5)" to "Exercise 20.5".

Page 462, line -9. Change "infinite sets given in theorem (20.8)." to "infinite sets given in Theorem (20.8).".

Page 464, line -9. Change "a cardinal number In" to "a cardinal number. In".

Page 468, line +12. Delete the period at the end of the displayed formula.

Page 469, Top paragraph. Change the two occurrences of "$\circ$" to "$\bullet$".

Page 474, Reference [23]. Change "*The Enigma*" to "*Alan Turing: The Enigma*".

Page 475, Reference [45]. Change "Freeeman" to "Freeman".

List of theorems at the back (there are two copies), Axiom (8.15). Change "$P, Q{:}B$ or R is finite" to "each quantification is defined".

List of theorems at the back (there are two copies), Axiom (8.16). Change "$P{:}B$ or R and S are finite" to "each quantification is defined"

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List of theorems at the back (there are two copies), Axiom (8.19). Change "$\star$ is idempotent or $P{:}B$ or R and S are finite" to "each quantification is defined".