Algorithmic aspects of communication networks	Lecture 10
OR 739 Spring 1997	February 12, 1997

Admission control and routing on $n \times n$ mesh.

Divide into $r \times r$ submeshes, with $r \sim \log_2(2m)$. We will assume:

- $k = 5$, any edge can be used 5 times
- every s, t pair from different submeshes

aggregated graph: contract each submesh to a node $\sim \frac{n^2}{r^2}$ nodes.

Admission s, t path:

- find path from s to little square boundary
- find path from t to little square boundary
- connect s and t in aggregated graph

Last time:

(1) Admitted pairs can be routed using every edge at most 5 times.
$\mathcal{A}$ = admitted pairs
$\mathcal{O}$ = set of pairs in optimal solution
we are interested in $\mathcal{O} \setminus \mathcal{A} = \mathcal{O}_g \cup \mathcal{O}_l$
$\mathcal{O}_g$ = pairs in $\mathcal{O}$ rejected in aggregated graph
$\mathcal{O}_l$ = pairs in $\mathcal{O}$ rejected in local routing

(2) $|\mathcal{O}_g| \leq 5 \log_2(2m)|\mathcal{A}|$ comes from proof of admission control theorem.

Let's fix an enumeration of the little squares $i = 1, 2, \ldots$. Let
$\mathcal{O}_i = \{(s, t)$ pairs in $\mathcal{O}$ rejected by subsquare $i\}$
$\mathcal{O}_l = \bigcup_i \mathcal{O}_i$
$\mathcal{A}_i = \{(s, t)$ accepted and s or t in subsquare $i\}$
$\mathcal{A} = \bigcup_i \mathcal{A}_i$

Facts:

(1) $|\mathcal{A}| = \frac{1}{2} \sum_i |\mathcal{A}_i|$. This follows from the assumption that every s, t pair comes from two different submeshes.

(2) $|\mathcal{O}_i| \leq 25r$. To see this just observe that $25r$ is an upper bound on the number of edges leaving any little square, counting multiplicity $k = 5$.

(3) $\mathcal{A}_i = \emptyset \Rightarrow \mathcal{O}_i = \emptyset$. Because by the way the local routing is done: if $\mathcal{A}_i = \emptyset$ then routing in the subsquare i will succeed, and hence $\mathcal{O}_i$ must also be empty.

(1), (2), (3) $\Rightarrow |\mathcal{O}_i| \leq 25r|\mathcal{A}_i|$

Lemma 1 $|\mathcal{O}_l| \leq 50\log_2(2m)|\mathcal{A}|$

Proof. $\mathcal{O}_l \leq \sum_i |\mathcal{O}_i| \leq \sum_i 25r|\mathcal{A}_i| = 50r|\mathcal{A}|$ ■

So we get,

Theorem 1 $|\mathcal{O}| \leq (55\log_2(2m)+1)|\mathcal{A}|$

Good things in mesh routing:

- aggregation creates large capacity $\Rightarrow$ routing in aggregated graph "easy"
- useful if "local routing is easy"

Bad things:

- large constant
- special graph

<u>**Off-line routing and congestion control**</u> - rerouting is O.K.

Given a routing, let
$f(e) = \#$ paths through edge e
$\lambda = \max_e f(e) =$ congestion
$\lambda^* =$ min congestion of any routing

Lemma 2 *$c \geq 0$ cost on edges, then*

$$\sum_{s,t} dist_c(s,t) \leq \lambda^* \sum_e c(e)$$

Proof. Let $P_{s,t}$ be the s-t path in optimal routing

$$\sum_{s,t} dist_c(s,t) \leq \sum_{s,t}(\sum_{e\in P_{s,t}} c(e)) = \sum_e c(e)f^*(e) \leq \lambda^* \sum_e c(e)$$

where $f^*(e) = \#$ of paths through edge e in optimal solution ■

Given a routing f, λ and cost c we have

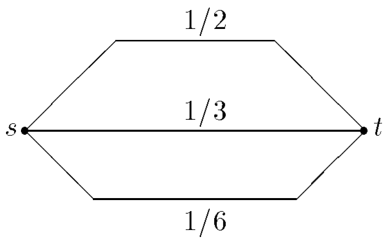
$$\frac{\sum_{s,t} dist_c(s,t)}{\sum_e c(e)} \leq \lambda^* \leq \lambda$$

Corollary 1 *If c, f, λ are such that*

$$\lambda \leq (1+\epsilon)\frac{\sum_{s,t} dist_c(s,t)}{\sum_e c(e)}$$

then $\lambda \leq (1+\epsilon)\lambda^*$

fractional version of λ^*:
Next we shortly discuss how this lower bound on the optimum λ^* relates to linear programming. Consider the fractional version of the routing problem, where instead of picking an s-t path, we assign a non-negative fraction to every possible s-t path in the graph in such a way that the fractions add up to 1.



In this fractional version the problem of minimizing the congestion corresponds to the following linear program:

$$\begin{array}{lll} \min & \lambda & \\ \text{s.t.} & \sum_{P \in \mathcal{P}_{s,t}} x_P = 1 & \text{for every } (s,t) \text{ pair, } \mathcal{P}_{s,t} = \{\text{paths from } s \text{ to } t\} \\ & \sum_{P: e \in P} x_P \leq \lambda & \text{for every edge } e \\ & x_P \geq 0 & \text{for every path } P \end{array}$$

The case in which we restrict each x_P to be 0 or 1 corresponds to the original routing problem. The bound given in lemma 2 could be obtained by applying weak duality to this LP; actually weak duality implies the stronger inequality:

$$\frac{\sum_{s,t} dist_c(s,t)}{\sum_e c(e)} \leq \lambda^*_{\text{frac}} \leq \lambda^* \leq \lambda$$

where λ^*_{frac} = optimal value of the LP. Strong LP duality implies that in fact

$$\frac{\sum_{s,t} dist_c(s,t)}{\sum_e c(e)} = \lambda^*_{\text{frac}}$$

Our goal will be to find f, λ and c such that the two extremes in the first inequality above are close to each other.