Computer Graphics	**Solution to Homework 0**
CS417/CS418 Fall 1998	**Due Wednesday, January 27, 1999**

- This diagnostic homework will be weighted to count much less than other homeworks.
- For this homework, NO PARTNERS.
- Hand in each part (CS417, CS418, administrative) SEPARATELY.
- As always in this class, you are graded on correctness *and also clarity and conciseness.*

Administrative Part

0. Fill out the waiver AND e-mail the requested information AND write legibly & sign & date on your homework the following statement, filling in your own name and CUID# as appropriate: "I, ⟨*your name*⟩, CUID#⟨*your CUID#*⟩, wrote up this assignment."

   You must complete this question to receive credit; e-mail will be accepted until Friday midnight.

CS417 Part

1. Suppose r is a root of the polynomial $x^2 + bx + c$, i.e. $r^2 + br + c = 0$. Express the other root of the polynomial in terms of r, b, c using only subtraction or division — square root and exponentiation (powers) are not allowed.

   HINT: Suppose the polynomial were factored into two linear terms; how are the terms related to the roots and coefficients of the polynomial?

   Answer: If s is the other root, then the polynomial factors as

   $$(x - r)(x - s) = x^2 - (r + s)x + rs = x^2 + bx + c,$$

   so $b = -(r + s)$ and $c = rs$, so $s = -b - r$ and if $r \neq 0$ then also $s = c/r$.

2. Use the following formulas:

   $$\cos(\theta + \phi) = \cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi) \quad (1)$$
   $$\sin(\theta + \phi) = \sin(\theta)\cos(\phi) + \cos(\theta)\sin(\phi) \quad (2)$$
   $$(x, y) = (r\cos(\theta), r\sin(\theta)), \text{ for } r = \sqrt{x^2 + y^2} \text{ and some } \theta. \quad (3)$$

   to derive a formula for the result (x', y') of rotating (x, y) clockwise ρ radians around the origin. Show your work. Your final answer should involve only x, y, ρ — neither r nor θ should appear.

   HINT: Does the radius (distance r from the origin) change? What's the new angle?

   Answer: If the original radius and angle are r and θ from (3), then the new radius and angle are r and $\theta + \phi$. Substituting $\theta + \phi$ for θ into (3) yields

   $$\begin{aligned}(x', y') &= (r\cos(\theta + \phi), r\sin(\theta + \phi)) \\ &= (r[\cos(\theta)\cos(\phi) - \sin(\theta)\sin(\phi)], r[\sin(\theta)\cos(\phi) + \cos(\theta)\sin(\phi)]) && \text{by (1), (2)} \\ &= (r\cos(\theta)\cos(\phi) - r\sin(\theta)\sin(\phi), r\sin(\theta)\cos(\phi) + r\cos(\theta)\sin(\phi)) && \text{arithmetic} \\ &= (x\cos(\phi) - y\sin(\phi), y\cos(\phi) + x\sin(\phi)) && \text{by (3).}\end{aligned}$$

3. Let $a = (x_a, y_a)$ and $b = (x_b, y_b)$ be two vectors, i.e. two points in the plane $\mathbb{R}^2$.

   (a) Give a formula for the projection $c = (x_c, y_c)$ of vector a onto vector b, i.e. the point c obtained by projecting point a onto the line defined by the origin and point b.

   Answer: The answer from standard linear algebra is

$$c = \frac{a \cdot b}{b \cdot b} b, \text{ i.e. } (x_c, y_c) = (tx_b, ty_b) \text{ with } t = \frac{x_a x_b + y_a y_b}{x_b^2 + y_b^2}.$$

   (b) Give a formula for the reflection $d = (x_d, y_d)$ of vector a across vector b, i.e. the point d obtained by reflecting point a across the line defined by the origin and point b.

   HINT: You may use c in your answer even if you couldn't figure out its formula.

   Answer: Geometry tells us $d-c = c-a$, so $d = 2c-a$, i.e. $(x_d, y_d) = (2x_c - x_a, 2y_c - y_a)$.

CS418 Part

4. Enter `type sphere` at the Matlab prompt to see the code (including helpful comments) for `sphere`, which is useful for plotting a sphere with equally many latitude and longitude lines.

   Write a Matlab function `mysphere` so that `[x,y,z] = mysphere(m,n); surf(x,y,z)` plots a sphere with `m` latitude lines and `n` longitude lines. Demonstrate with $m = 11$ and $n = 40$. Turn in your code and a printout of the plot. (If you're uncomfortable writing a function, then instead write a script/sequence of commands. In this case, your code should start with `m=11; n=40` and never again reference constants 11,40).

   HINT: Figure out the definitions of `theta`, `phi`, `cosphi`, and `sintheta` and alter these four lines of code appropriately (no other computations need to be changed).

   Answer: Observe that the comments have been updated. Thus, `help mysphere` will actually BE helpful, and also now we know which variables are for latitude and which for longitude.

```
function [xx,yy,zz] = mysphere(m,n)
%MYSPHERE Generate sphere.
%   [X,Y,Z] = MYSPHERE(M,N) generates three (m+1)-by-(n+1)
%   matrices so that SURF(X,Y,Z) produces a unit sphere
%   with m latitude and n longitude lines.
%   Modified from code by Clay M. Thompson 4-24-91, CBM 8-21-92.

% -pi <= theta <= pi is a row vector -- longitude
% -pi/2 <= phi <= pi/2 is a column vector -- latitude
theta = (-n:2:n)/n*pi;
phi = (-m:2:m)'/m*pi/2;
cosphi = cos(phi); cosphi(1) = 0; cosphi(m+1) = 0;
sintheta = sin(theta); sintheta(1) = 0; sintheta(n+1) = 0;

xx = cosphi*cos(theta);
yy = cosphi*sintheta;
zz = sin(phi)*ones(1,n+1);
```

   Here's how we use it:

```
[x,y,z]=mysphere(11,40); surf(x,y,z);
% extra:  fix aspect ratio, hide axes
axis equal; axis off
```