Pricing Fixed-Income Securities and Sensitivity Analysis

27 March 1998

CS522 Lab Note

The Financial Toolbox includes many functions to compute accrued interest, determine prices and yields, and calculate convexity and duration of fixed-income securities.

Terminology

The **settlement date** of a bond is the date when money first changes hands; i.e., when a buyer pays for a bond. It need not coincide with the issue date. The **issue date** is the date a bond is first offered for sale. That date usually determines when interest payments, known as **coupons**, are made. The **maturity date** of a bond is the date when the issuer returns the final face value, also known as the **redemption value** or **par value**, to the buyer. Typically the **purchase price**, the price actually paid for a bond, is not the same as the redemption value. The **yield** of a bond is determined by the ration of redemption value to purchase price over the life of the bond. It is the nominal annual interest rate that gives a future value of the purchase price equal to the redemption value of the bond. The coupon payments determine part of that yield.

Handling and Converting Dates

Date strings (27-Mar-1998) are what we work with when we handle dates in real life. The Financial Toolbox works internally with serial date numbers. A serial date represents a calendar date as the number of days that have passed since a fixed base date. In Matlab, serial date number 1 is January 1,0000. Matlab also uses serial time to represent fractions of days beginning at midnight; for example, 6 p.m. equals 0.75 serial days.

Example

`today` returns the current date as a serial date number

```
>> today
ans =
    729840
```

`datenum` returns a serial date number given a date string. The date string can be any of the following forms :

’27-mar-1998’	’mar 27, 1998’	’27-mar-98’
’27-mar’ (current year assumed)	’03/27/98’	’03/27’ (current year assumed)
’27-mar-1998, 15:35’	’27-mar-1998, 3:35 pm’	’03/27/98/15:35’
’03/27/98/3:35 pm’	’15:35’	

```
>> datenum(’26-mar-98’)
ans =
      729840
```

`datestr` converts a date number or a date string to a date string.

```
>> datestr(datenum(’15:35’), 0)
ans =
      31-Dec-1997 15:34:60
```

Example 1 : Sensitivity of Bond Prices to Changes in Interest Rates

The modified duration and convexity of a portfolio are calculated using Matlab functions bonddur adn bondconv.

```
>> B1 = [datenum('04/10/1998') datenum('06/17/2010')  100 .07  2 0
         datenum('04/10/1998') datenum('06/09/2015')  100 .06  2 0
         datenum('04/10/1998') datenum('05/14/2025') 1000 .045 2 0];

>> yields1 = [.05 .06 .065]';

>> temp = zeros(3,2); prices1 = zeros(3,1); durations1 = zeros(3,1);
>> convexities1 = zeros(3,1);

>> [temp(:,1) temp(:,2)] = prbond(B1(:,1), B1(:,2), B1(:,3), B1(:,4),...
   yields1, B1(:,5), B1(:,6));

>> prices1 = temp(:,1) + temp(:,2)
prices1 =
        120.27
        102.00
        764.91

>> [temp(:,1) temp(:,2)] = bonddur(B1(:,1), B1(:,2), B1(:,3), B1(:,4),...
    yields1, B1(:,5), B1(:,6));

>> durations1 = temp(:, 2)
durations1 =
         8.36
        10.42
        13.33

>> [temp(:,1) temp(:,2)] = bondconv(B1(:,1), B1(:,2), B1(:,3), B1(:,4),...
    yields1, B1(:,5), B1(:,6));

>> convexities1 = temp(:, 2)
convexities1 =

        91.04
       149.81
       273.42

>> dY = .002; portf_price = 100000; portf_weights = ones(1,3)/3;
>> portf_amnts = portf_price * portf_weights ./ prices1'
portf_amnts =
        277.15        326.79         43.58

>> portf_duration = dot(portf_weights, durations1')
portf_duration =
        10.70

>> portf_convexity = portf_weights * convexities1
portf_convexity =
        171.42
```

We can calculate the first- and second-order approximations of the portfolio price using the duration and convexity of the portfolio when there is a change in the yield. The approximations are checked with the true portfolio price obtained by `prbond`.

```
>> perc_approx1 = -portf_duration * dY * 100
perc_approx1 =
         -2.14

>> perc_approx2 = perc_approx1 + portf_convexity * dY^2 * 100 / 2
perc_approx2 =
         -2.11

>> price_approx1 = portf_price + perc_approx1 * portf_price / 100
price_approx1 =
      97859.23

>> price_approx2 = portf_price + perc_approx2 * portf_price / 100
price_approx2 =
      97893.51

>> new_prices = zeros(3,1);
>> [temp(:,1) temp(:,2)] = prbond(B1(:,1), B1(:,2), B1(:,3), B1(:,4),...
    yields1+dY, B1(:,5), B1(:,6));
>> new_prices = temp(:,1) + temp(:,2)
new_prices =
        118.28
         99.91
        744.93

>> new_price = portf_amnts * new_prices
new_price =
      97893.08
```

Example 2 : Constructing a Bond Portfolio to Hedge Against Duration and Convexity

This example constructs a bond portfolio to hedge the portfolio of Example 1 againts duration and convexity.

```
>> B2 = [datenum('04/10/1998') datenum('06/15/2005')  500 .07  2 0
         datenum('04/10/1998') datenum('10/02/2010') 1000 .066 2 0
         datenum('04/10/1998') datenum('03/01/2025')  250 .08  2 0];

>> yields2 = [.06 .07 .075]';

>> temp = zeros(3,2); prices2 = zeros(3,1); durations2 = zeros(3,1);
>> convexities2 = zeros(3,1);

>> [temp(:,1) temp(:,2)] = prbond(B2(:,1), B2(:,2), B2(:,3), B2(:,4),...
    yields2, B2(:,5), B2(:,6));
>> prices2 = (temp(:,1) + temp(:,2))'
prices2 =
        539.92        968.49        266.51

>> [temp(:,1) temp(:,2)] = bonddur(B2(:,1), B2(:,2), B2(:,3), B2(:,4),...
    yields2, B2(:,5), B2(:,6));
```

```
>> durations2 = temp(:,2)’
durations2 =
          5.53          8.32         11.28

>> [temp(:,1) temp(:,2)] = bondconv(B2(:,1), B2(:,2), B2(:,3), B2(:,4),...
    yields2, B2(:,5), B2(:,6));

>> convexities2 = temp(:,2)’
convexities2 =
         38.16         90.63        205.24

>> A = [durations2; convexities2; 1 1 1]
A =
          5.53          8.32         11.28
         38.16         90.63        205.24
          1.00          1.00          1.00

>> b = [portf_duration; portf_convexity; 1]
b =
         10.70
        171.42
          1.00

>> weights = A \ b
weights =
         -0.21
          0.60
          0.61

>> new_portf_duration =  durations2 * weights
new_portf_duration =
         10.70

>> portf_duration
portf_duration =
         10.70

>> new_portf_convexity = convexities2 * weights
new_portf_convexity =
        171.42

>> portf_convexity
portf_convexity =
        171.42

>> hedge_amounts = portf_price * weights’ ./ prices2
hedge_amounts =
        -38.89         62.08        228.42
```