COM S 783 / OR&IE 634 Approximation Algorithms **Fall Semester, 2003**

Assignment 4: 11/10/03

Lecturer: Yuval Rabani *Due date: 12/03/03*

1. What's the integrality gap of the MAX CUT semidefinite relaxation for the 5 node cycle C_5? What happens when you add the triangle inequalities?

2. Show an infinite sequence of increasingly large examples where the primal-dual algorithm for Steiner tree gives poor guarantees when applied to solving the Steiner arborescence problem in directed graphs.

3. Consider a local search algorithm for FACILITY LOCATION that has just two local moves: adding or deleting a facility. (So moving a facility to a different location cannot be done in one move.) Prove that this algorithm does not guarantee a good approximation ratio.

4. Show a lower bound on the integrality gap of the LP relaxation for FACILITY LOCATION. (It does not have to match the upper bound shown in class.)

5. **(bonus question)** Let $G = (V, E)$ be an undirected graph. A set $\mathcal{T}$ of subtrees of G with non-negative weights $\lambda : \mathcal{T} \to \mathbb{R}^+$ is called a *tree packing* iff for every $e \in E$, the total weight of trees using e is at most 1. The weight of the packing is $\sum_{T \in \mathcal{T}} \lambda(T)$. The Nash-Williams theorem states that for every G there is a packing of spanning trees of G with *integer* coefficients and weight at least half the minimum cut in G. Prove a somewhat weaker theorem for Steiner trees: For every set $U \subset V$ there is a packing of Steiner trees in G spanning U (not necessarily with integer coefficients) with weight at least half the minimum cut in G that separates U into two non-empty parts.

   Hint: Use the primal-dual algorithm for Steiner tree.