CS 684: Algorithmic Game Theory	**Scribe: Jignesh Parmar**
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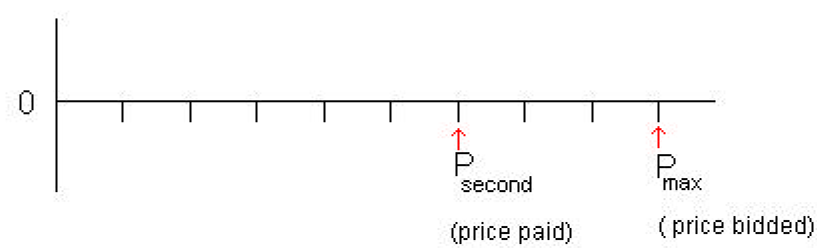
Auctions and Cost Sharing

There are several types of single item auctions:

- Simple auctions
- Increasing/Decreasing price auctions
- First price auctions
- Second price auctions

Today, we consider second price auctions (also called Vickrey auctions named for the Nobel prize economist William Vickrey). In second price auctions, the winner is the person with the highest bid, but the amount that he pays for the item is the second highest price bidded.



Second price auctions are truthful. That is, no matter what others do, one cannot gain profit by announcing incorrect value for the items. In second price auctions, if user **i** has a utility $\mathbf{u_i}$ for an item, and is asked to pay price **p,** the resulting benefit = $\mathbf{u_i}$ - **p**

(N.B. Second price auctions are not group-strategy proof. A group of conspirators can fool an auctioneer)

Cost sharing in VCG auctions is represented as follows:

N represents the users in the second price auction.

For each $S \subseteq N$, $C(S) \geq 0$ (cost is monotone increasing)

Each user $i \in N$ has benefit u_i for item being bidded on.

In the previous two lectures, c(s) was submodular and we discussed the minimum cost spanning tree. The goals were:

Select $S \subseteq N$ and give $x_i(s)$ cost share

1) $\sum_{i \in s} x_i - c(s)$

2) Cross monotone $x_i(A) \leq x_i(B)$, for $A \geq B$

In these cases, there was a group-strategy proof way to select users s and cost shares $x_i(s) \leq u_i$.

Another Goal is to maximize social welfare. Social welfare is defined as

$$\max_{s} \sum_{i \in s} u_i - c(s)$$

Social welfare is not maximized using the previous Shapley value or the Primal-Dual mechanisms. To maximize social welfare, we need to use an algorithm proposed by Vickrey, Clark and Groves (VCG) to select a subset S and to assign cost shares. (N.B. VCG is not budget-balanced)

In VCG, the overall goal is to maximize

$$\max_{s} \sum_{i \in s} u_i - c(s)$$

but the individual goals can be characterized as follows:

$$\text{Max} \quad u_i - x_i(u_1, u_2, \dots, u_k)$$

$$\max_{\hat{u}_i} \quad u_i d'_{s,i} - x_i(u_1, u_2, \dots \hat{u}_i \dots, u_k)$$

$$d'_{s,i} = \begin{cases} 1 & \text{if } i \in s \\ 0 & \text{otherwise} \end{cases}$$

In other words, for each item i that is included in the set s, the individual wants to maximize his utility minus his cost share.

VCG Prices for the items can be set as follows

$$i \in s,\ x_i = \quad c(s) - \sum_{\substack{j \in s \\ j \neq i}} u_j$$

$$i \notin s,\ x_i = -\left[\sum_{j \in s} u_j - c(s) \right]$$

Theorem: We state that the VCG mechanism is truthful, and that no individual can gain an advantage by claiming an incorrect utility.

(*N.B.* remember, when truthful, all players want to maximize: $\max_{s} \sum_{i \in s} u_i - c(s)$

Proof:

Assume that player i announces utility $\hat{u}_i$ which is not the same as u_i
He then gets $\sum_{j \in s} u_j - c(s)$ if he is not included in the set S.

If he is included in S , then we select $i \in s$, and he still gets $\sum_{j \in s} u_j - c(s)$

Either way his true benefit does not change, regardless of whether he announces his real utility or not.