

1. (a) It will be helpful to solve the following special case first. Given a coin with bias p (that is, $\Pr(\text{heads}) = p$ and $\Pr(\text{tails}) = 1 - p$), what is the expected number of flips before seeing the first head?
2. Think of the von Neumann trick we discussed in class for simulating a fair coin ($\Pr(\text{heads}) = \Pr(\text{tails}) = 1/2$) with a bias- p coin.
3. (a) Define a new random variable Y taking value $a \cdot \mathcal{E}X$ with probability $\Pr(X \geq a \cdot \mathcal{E}X)$ and 0 with probability $\Pr(X < a \cdot \mathcal{E}X) = 1 - \Pr(X \geq a \cdot \mathcal{E}X)$.
(c) For the application of the Chebyshev inequality, first give a general formula for the standard deviation of the number of heads in N independent flips of a bias- p coin.