The Design and Analysis of Algorithms	Problem Set 5
CS 681 Fall 2003	Due Friday, November 21, 2003

Solve 3 of the following 4 problems. You may solve all 4 for extra credit. We will maintain a FAQ for the problem set on the course Web page. Each of the first three problem can be proved NP-complete using a problem that we discussed in class, either proved NP-complete (3-SAT, Circuit Satisfiability, 3D Matching), or discussed algorithm for it for the special case of bounded tree-width graphs (Vertex Cover or Independent Set, Dominating Set, Graph Coloring).

(1) (15 points) The *Multi-Cut problem* is given by a undirected graph $G = (V, E)$ with nonnegative capacities $c_e \geq 0$ on the edges $e \in E$ and k pairs of nodes $(s_1, t_1), (s_2, t_2), \ldots, (s_k, t_k)$, and a threshold value γ. The problem is to decide if there is a subset F of the edges E of total capacity at most γ, that is $\sum_{e \in F} c_e \leq \gamma$, such that each given pairs s_i and t_i is in separate components after deleting F. Note that the special case, when $k = 1$ is the traditional (s, t) min-cut problem.

Show that Multi-Cut Problem is NP-complete even when G is a tree. Hint: try very simple kinds of trees.

(2) (15 points) The *Directed Disjoint Paths* problem is defined as follows. We are given a directed graph G and k pairs of nodes $(s_1, t_1), (s_2, t_2), \ldots, (s_k, t_k)$. The problem is to decide whether there exist node-disjoint paths $P_1, P_2, \ldots, P_k$ so that P_i goes from s_i to t_i.

Show that *Directed Disjoint Paths* is NP-complete. Hint: for a reduction from SAT start with an (s_i, t_i) pair corresponding to each clause in the SAT formula.

(3) (15 points) Suppose you're consulting for one of the many companies in New Jersey that designs communication networks, and they come to you with the following problem. They're studying a specific n-node communication network, modeled as a directed graph $G = (V, E)$. For reasons of fault-tolerance they want to divide up G into as many virtual "domains" as possible: a *domain* in G is a set X of nodes, of size at least 2, so that for each pair of nodes $u, v \in X$ there are directed paths from u to v and v to u that are contained entirely in X.

Show that the following DOMAIN DECOMPOSITION problem is NP-complete. Given a directed graph $G = (V, E)$ and a number k, can V be *partitioned* into at least k sets, each of which is a domain? Hint: try 3D Matching.

(4) (15 Points) Consider an optimization version of the *Hitting Set* problem defined as follows. We are given a set $A = \{a_1, \ldots, a_n\}$ and a collection $B_1, B_2, \ldots, B_m$ of subsets of A. Also, each element $a_i \in A$ has a *weight* $w_i \geq 0$. The problem is to find a hitting set $H \subseteq A$ such that the total weight of the elements in H, $\sum_{a_i \in H} w_i$, is as small as possible. (H is a hitting set if $H \cap B_i$ is not empty for each i). Let $b = \max_i |B_i|$ denote the maximum size of any of the sets $B_1, B_2, \ldots, B_m$. Give a polynomial time approximation algorithm for this problem that finds a hitting set whose total weight is at most b times the minimum possible.