Machine Learning Theory (CS 6783)

Tu-Th 1:25 to 2:40 PM
Phillips Hall, 407

Instructor : Karthik Sridharan

About the course

- No exams !
- 5 assignments that count towards your grades (55%)
- One term project (40%)
- 5% for class participation

Pre-requisites

- Basic probability theory
- Basics of algorithms and analysis
- Introductory level machine learning course
- *Mathematical maturity, comfortable reading/writing formal mathematical proofs.*

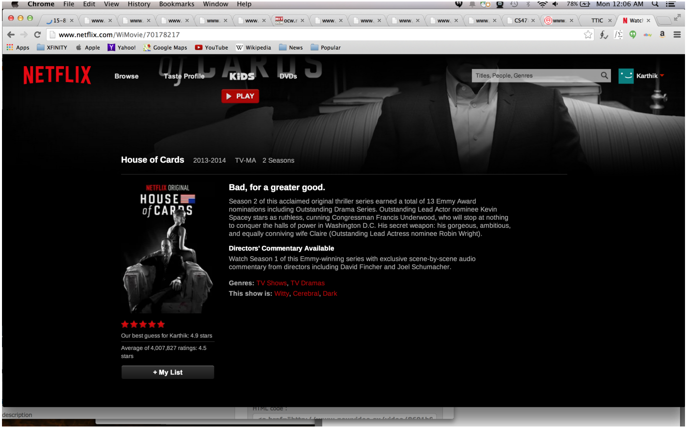
Lets get started …

What is Machine Learning

Use past observations to automatically learn to make better predictions/decisions in the future.

Where is it used ?

Recommendation Systems



WHERE IS IT USED ?

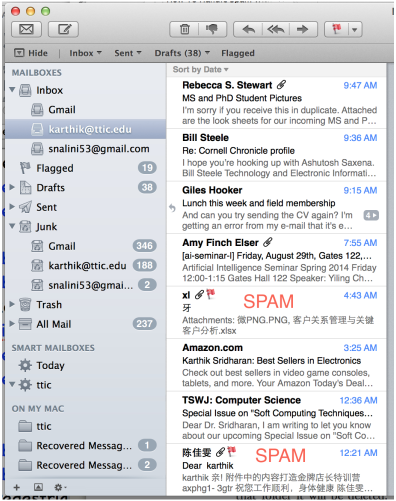
Pedestrian Detection

WHERE IS IT USED ?

Market Predictions

Where is it used ?

Spam Classification



Where is it used ?

- Online advertising (improving click through rates)
- Climate/weather prediction
- Text categorization
- Unsupervised clustering (of articles …)
- …

What is Learning Theory

What is Learning Theory

Oops …

Cognitive **theories** look beyond behavior to explain brain-based **learning**. And constructivism views **learning** as a process in which the learner actively constructs or builds new ideas or concepts. Behaviorism. Behaviorism as a **theory** was primarily developed by B. F. Skinner.

Learning theory (education) - Princeton University
www.princeton.edu/…/**Learning_theory**_(education)… Princeton University

Feedback

What is Machine Learning Theory

- How do we formalize machine learning problems
- Right framework for right problems (Eg. online , statistical)
- How do we pick the right model to use and what are the tradeoffs between various models
- How many instances do we need to see to learn to given accuracy
- How do we design learning algorithms with provable guarantees on performance
- *Computational learning theory : which problems are efficiently learnable*

Outline of Topics

- Learning problem and frameworks, settings, minimax rates
- Statistical learning theory
  - Probably Approximately Correct (PAC) and Agnostic PAC frameworks
  - Empirical Risk Minimization, Uniform convergence, Empirical process theory
  - Bound on learning rates: MDL bounds, PAC Bayes theorem, Rademacher complexity, VC dimension, covering numbers, fat-shattering dimension
  - Supervised learning : necessary and sufficient conditions for learnability
- Online learning theory
  - Sequential minimax and value of online learning game
  - Regret bounds: Sequential Rademacher complexity, Littlestone dimension, sequential covering numbers, sequential fat-shattering dimension
  - Online supervised learning : necessary & sufficient conditions for learnability
- Algorithms for online convex optimization: Exponential weights algorithm, strong convexity, exp-concavity and rates, Online mirror descent
- Deriving generic learning algorithms : relaxations, random play-outs
- If time permits, uses of learning theory results in optimization, approximation algorithms, perhaps a bit of bandits, . . .

Learning Problem : Basic Notation

- Input space/ feature space : $\mathcal{X}$

  (Eg. bag-of-words, n-grams, vector of grey-scale values, user-movie pair to rate)

  Feature extraction is an art, . . . an art we won't cover in this course

- Output space/ label space $\mathcal{Y}$

  (Eg. $\{\pm 1\}$, $[K]$, $\mathbb{R}$-valued output, structured output)

- Loss function : $\ell : \mathcal{Y} \times \mathcal{Y} \mapsto \mathbb{R}$

  (Eg. $0-1$ loss $\ell(y', y) = \mathbf{1}\{y' \neq y\}$, sq-loss $\ell(y', y) = (y - y')^2$), absolute loss $\ell(y', y) = |y - y'|$

  Measures performance/cost per instance (inaccuracy of prediction/ cost of decision).

- Model class/Hypothesis class $\mathcal{F} \subset \mathcal{Y}^{\mathcal{X}}$

  (Eg. $\mathcal{F} = \{x \mapsto f^\top x : \|f\|_2 \leq 1\}$, $\mathcal{F} = \{x \mapsto \text{sign}(f^\top x)\}$)

Formalizing Learning Problems

- How is data generated ?
- How do we measure performance or success ?
- Where do we place our prior assumption or model assumptions ?

Formalizing Learning Problems

- How is data generated ?
- How do we measure performance or success ?
- Where do we place our prior assumption or model assumptions ?
- *What we observe ?*

Probably Approximately Correct Learning

$$\mathcal{Y} = \{\pm 1\}\,,\quad \ell(y', y) = \mathbf{1}\{y' \neq y\}\,,\quad \mathcal{F} \subset \mathcal{Y}^{\mathcal{X}}$$

- Learner only observes training sample $S = \{(x_1, y_1), \ldots, (x_n, y_n)\}$
  - $x_1, \ldots, x_n \sim \mathbf{D}_X$
  - $\forall t \in [n], y_t = f^*(x_t)$ where $f^* \in \mathcal{F}$
- Goal : find $\hat{y} \in \mathcal{Y}^{\mathcal{X}}$ to minimize

$$\mathbb{P}_{x \sim D_X}\left(\hat{y}(x) \neq f^*(x)\right)$$

(Either in expectation or with high probability)

Probably Approximately Correct Learning

Definition

Given $\delta > 0$, $\epsilon > 0$, sample complexity $n(\epsilon, \delta)$ is the smallest n such that we can always find forecaster $\hat{y}$ s.t. with probability at least $1 - \delta$,

$$\mathbb{P}_{x \sim D_X} (\hat{y}(x) \neq f^*(x)) \leq \epsilon$$

(efficiently PAC learnable if we can learn efficiently in $1/\delta$ and $1/\epsilon$)

Eg. : learning output for deterministic systems

Non-parametric Regression

$$\mathcal{Y} \subset \mathbb{R}\ , \quad \ell(y', y) = (y - y')^2\ , \quad \mathcal{F} \subset \mathcal{Y}^{\mathcal{X}}$$

- Learner only observes training sample $S = \{(x_1, y_1), \ldots, (x_n, y_n)\}$
  - $x_1, \ldots, x_n \sim \mathbf{D}_X$
  - $\forall t \in [n], y_t = f^*(x_t) + \varepsilon_t$ where $f^* \in \mathcal{F}$ and $\varepsilon_t \sim N(0, \sigma)$

- Goal : find $\hat{y} \in \mathbb{R}^{\mathcal{X}}$ to minimize

$$\|\hat{y} - f^*\|^2_{L_2(D_X)} = \mathbb{E}_{x \sim D_X}\left[(\hat{y}(x) - f^*(x))^2\right]$$

(Either in expectation or in high probability)

Eg. : clinical trials (inference problems) model class known.

Non-parametric Regression

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- Goal : find $\hat{y} \in \mathbb{R}^{\mathcal{X}}$ to minimize

$$\begin{aligned}\|\hat{y} - f^*\|^2_{L_2(D_X)} &= \mathbb{E}_{x \sim D_X}\left[(\hat{y}(x) - f^*(x))^2\right] \\ &= \mathbb{E}_{x \sim D_X}\left[(\hat{y}(x) - y)^2\right] - \inf_{f \in \mathcal{F}} \mathbb{E}_{x \sim D_X}\left[(f(x) - y)^2\right]\end{aligned}$$

(Either in expectation or in high probability)

Eg. : clinical trials (inference problems) model class known.

Statistical Learning (Agnostic PAC)

- Learner only observes training sample $S = \{(x_1, y_1), \ldots, (x_n, y_n)\}$ drawn iid from joint distribution $\mathbf{D}$ on $\mathcal{X} \times \mathcal{Y}$

- Goal : find $\hat{y} \in \mathbb{R}^{\mathcal{X}}$ to minimize expected loss over future instances

$$\mathbb{E}_{(x,y)\sim\mathbf{D}}\left[\ell(\hat{y}(x), y)\right] - \inf_{f\in\mathcal{F}} \mathbb{E}_{(x,y)\sim\mathbf{D}}\left[\ell(f(x), y)\right] \le \epsilon$$

$$L_{\mathbf{D}}(\hat{y}) - \inf_{f\in\mathcal{F}} L_{\mathbf{D}}(f) \le \epsilon$$

Well suited for *Prediction* problems.

Statistical Learning (Agnostic PAC)

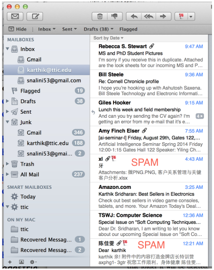
Definition

Given $\delta > 0$, $\epsilon > 0$, sample complexity $n(\epsilon, \delta)$ is the smallest n such that we can always find forecaster $\hat{y}$ s.t. with probability at least $1 - \delta$,

$$L_{\mathbf{D}}(\hat{y}) - \inf_{f \in \mathcal{F}} L_{\mathbf{D}}(f) \leq \epsilon$$

LEARNING PROBLEMS

Pedestrian Detection

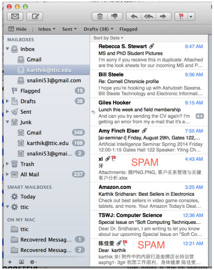


Spam Classification

Learning Problems

Pedestrian Detection

(Batch/Statistical setting)



Spam Classification

(Online/adversarial setting)

Online Learning (Sequential Prediction)

For $t = 1$ to n

Learner receives $x_t \in \mathcal{X}$

Learner predicts output $\hat{y}_t \in \mathcal{Y}$

True output $y_t \in \mathcal{Y}$ is revealed

End for

Goal : minimize regret

$$\mathbf{Reg}_n(\mathcal{F}) := \frac{1}{n}\sum_{t=1} \ell(\hat{y}_t, y_t) - \inf_{f\in\mathcal{F}} \frac{1}{n}\sum_{t=1} \ell(f(x_t), y_t)$$

Other Problems/Frameworks

- Unsupervised learning, clustering
- Semi-supervised learning
- Active learning and selective sampling
- Online convex optimization
- Bandit problems, partial monitoring, ...

Sneek Peek

- No Free Lunch Theorems
- Minimax rates for various setting/problems
- Comparing the various settings