



# Climate vs Weather

Question 1: How can we predict Climate (50 yrs)  
if we can't predict Weather (10 days)?





# What is Climate?

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if we can't predict Weather (10 days)?

Question 2: What is Climate?

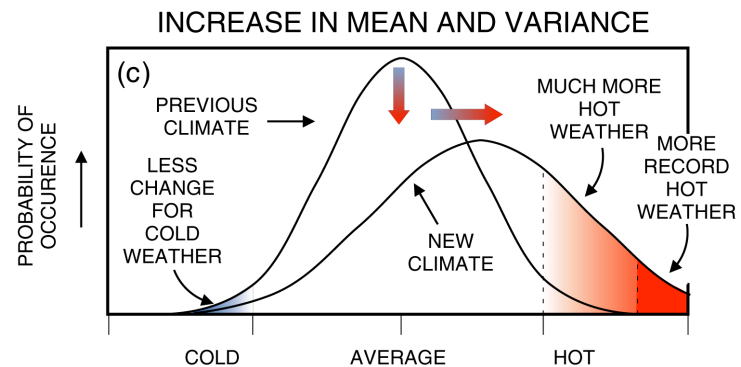
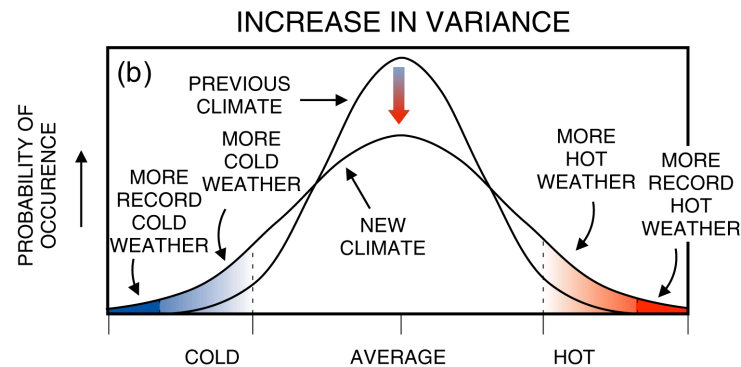
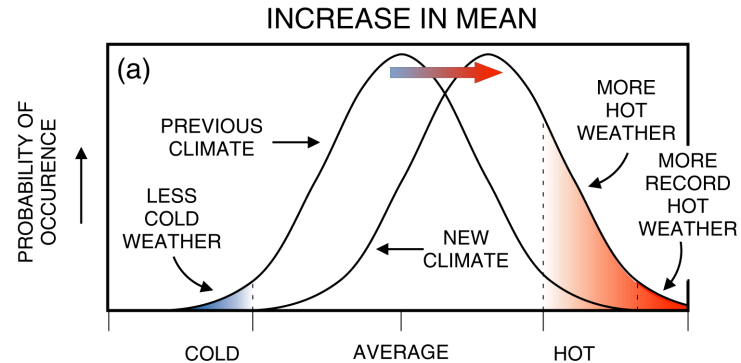
- A. Average Weather
- B. Record high and low temperatures
- C. The temperature range
- D. Distribution of possible weather
- E. Extreme events





# Characterizing Climate

***Climate change  
and its manifestation  
in terms of weather  
(climate extremes)***





# Climate vs Weather

Question 1: How can we predict Climate (50 yrs)  
if we can't predict Weather (10 days)?

Question 3: Why can't we predict weather?  
Weather is an inherently chaotic system  
Classic example: the Lorenz system



# Characterizing Climate

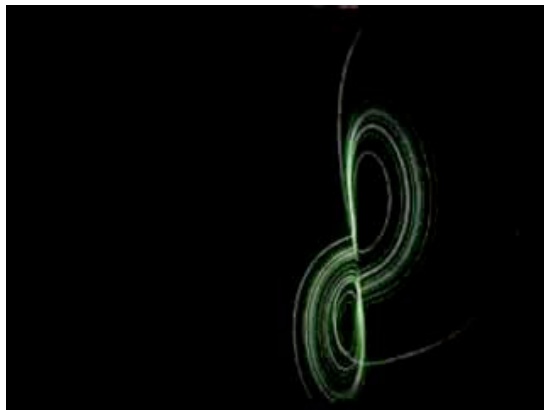
The chaos is the weather



Even in 3-dimensional systems, nearby initial conditions in a dynamical system can have VERY different destinies.

Lorenz Attractor

The attractor is the climate

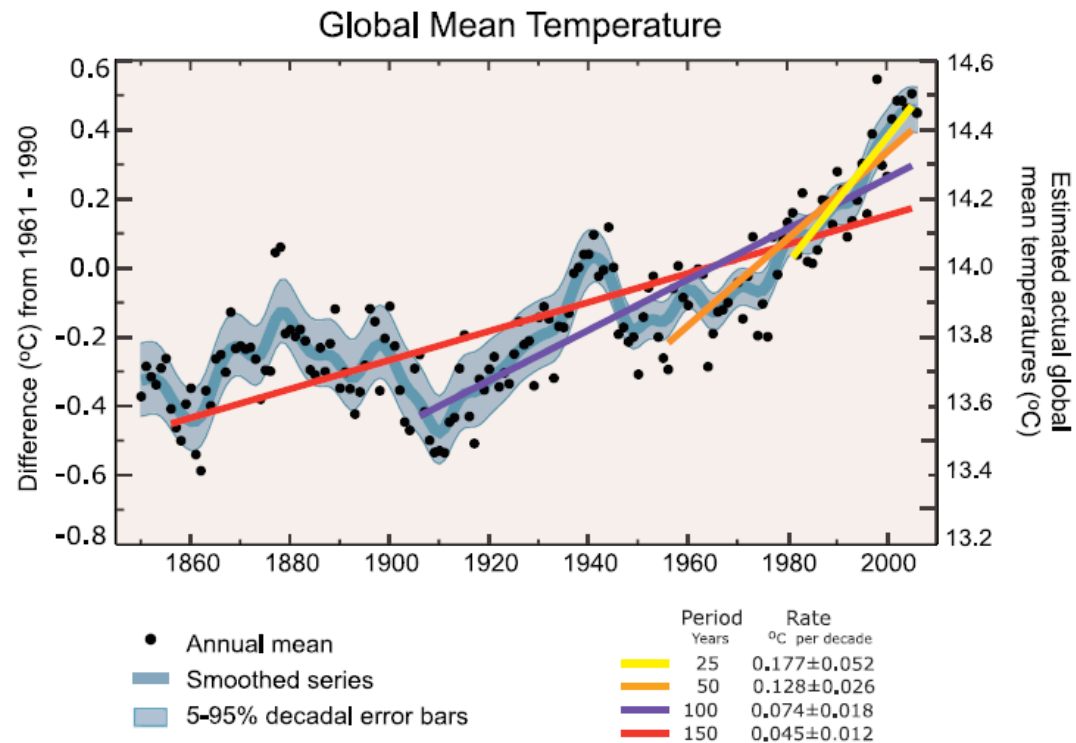


The attractor itself enjoys structure and it is the parameters of the attractor we are after in predicting the climate



# Evidence of a Changing Climate

Intergovernmental Panel on Climate Change (IPCC):  
Warming is “unequivocal”





# Pandora's Box

## Observations:

FACTS

carbon in atmosphere

EVIDENCE

rising temperatures

## Theory:

PHYSICS

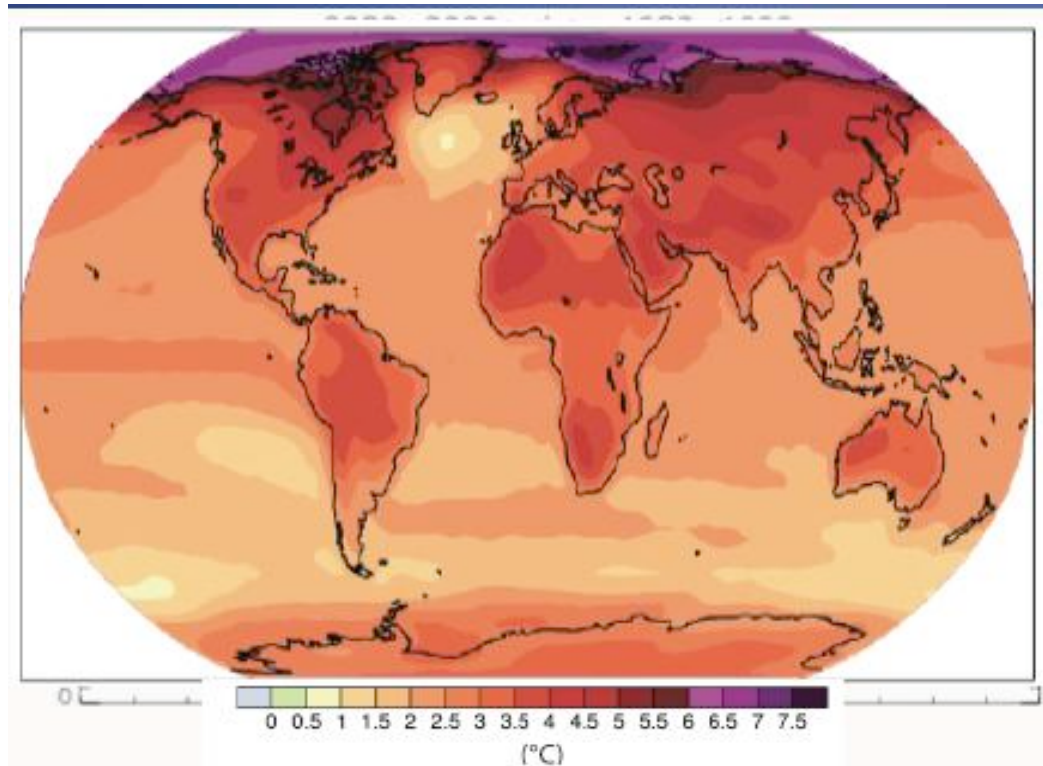
greenhouse effect

PREDICTION

mathematical models



# Why do we try to predict?



Warming is NOT uniform



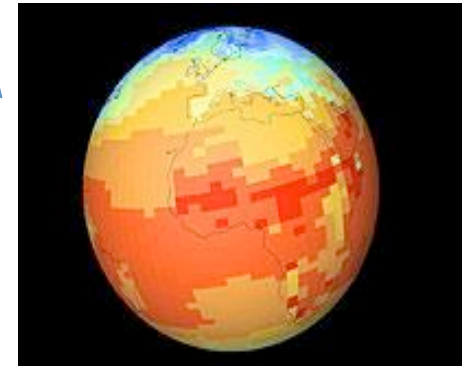


# Why use computational models?



1

many

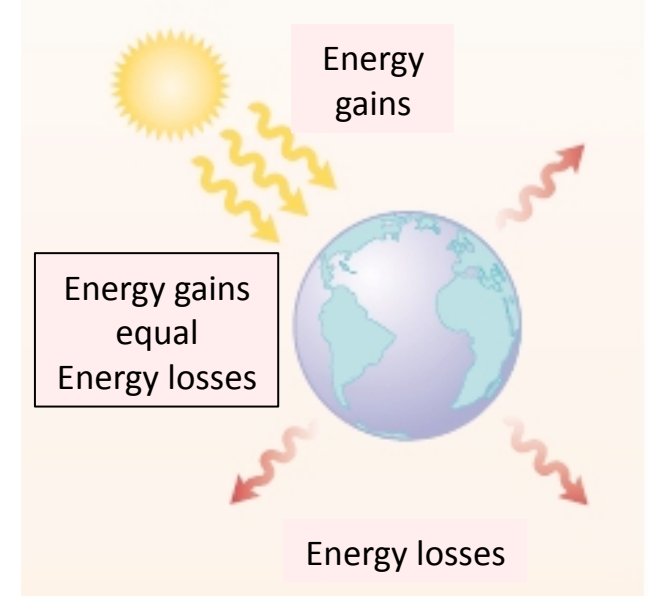


Friday: Natalie Mahowald will talk about full climate models

Today: Intro to most basic concepts of energy balance models



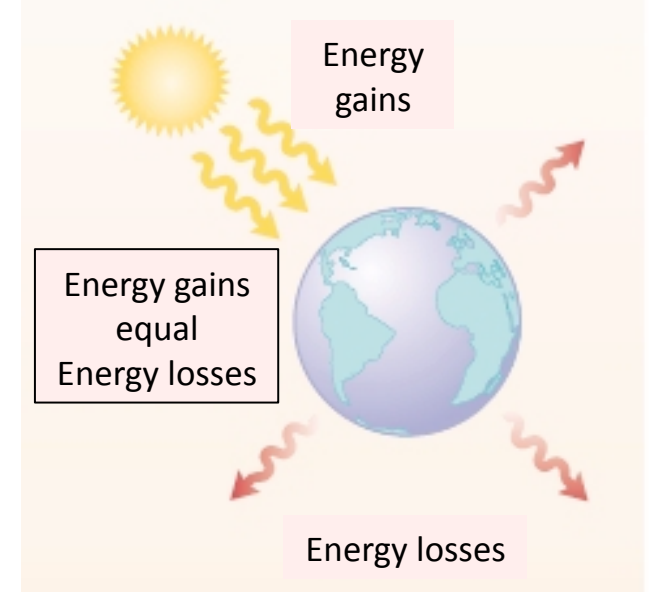
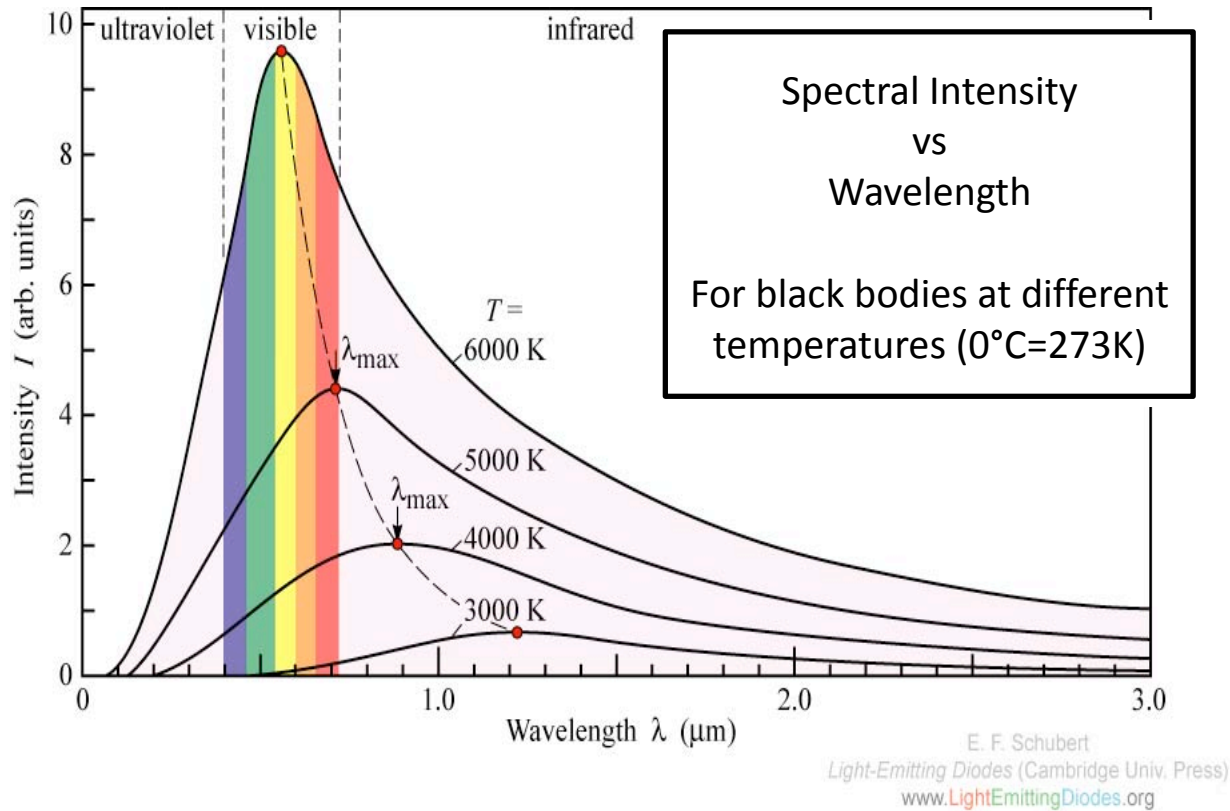
# Simplest Energy Balance Models





# Simplest Energy Balance Models

## Black Body Radiation - Planck



Sun:  $\sim 6000\text{K}$ , emits mainly in visible spectrum (shortwave)

Earth:  $\sim 300\text{K}$ , emits mainly in IR (longwave)

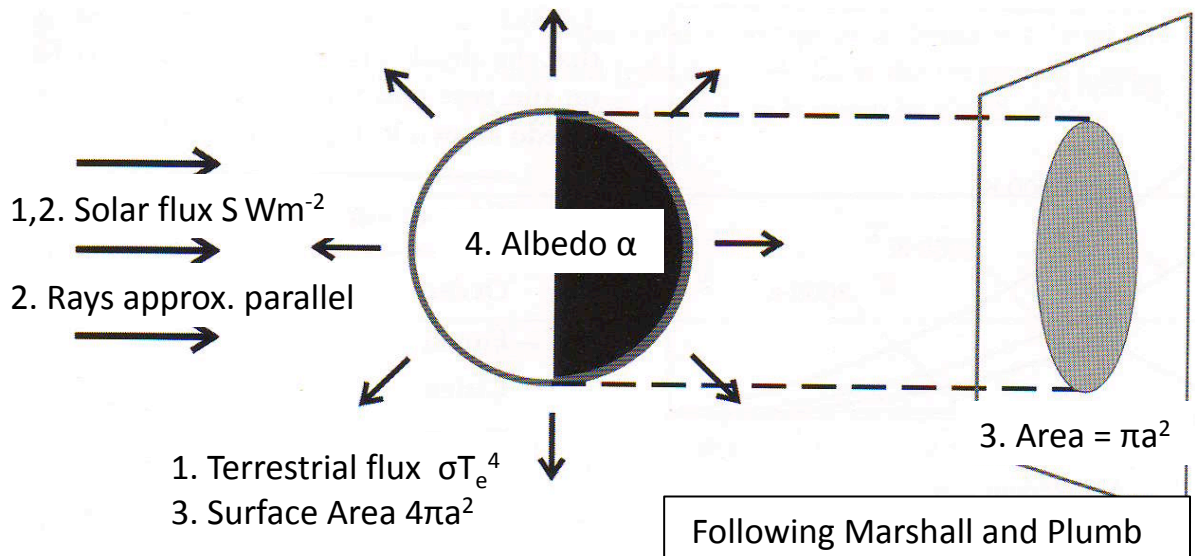
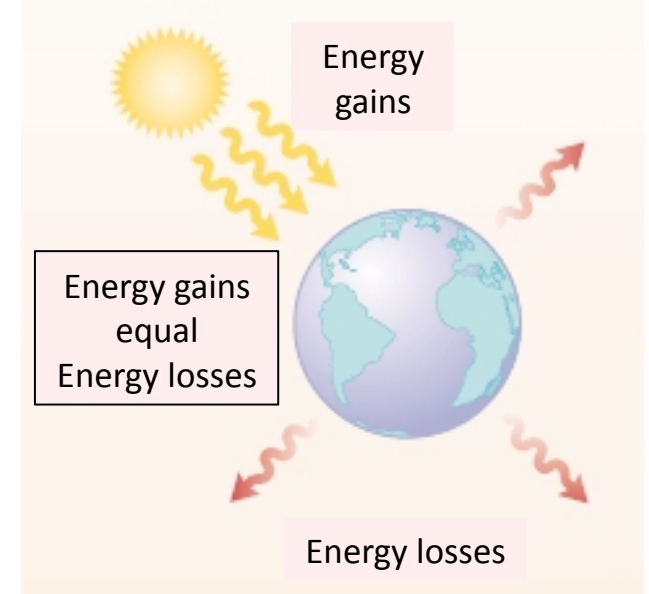
Integrate over wavelength for temp  $T$  to get total emission flux  $\sigma T^4$  -- Stefan-Boltzmann



# Energy Balance, No Atmosphere

**Hypothesis:** Earth's temperature is a consequence of

1. Blackbody radiation
2. Distance from Sun
3. Size
4. Albedo (reflectivity)





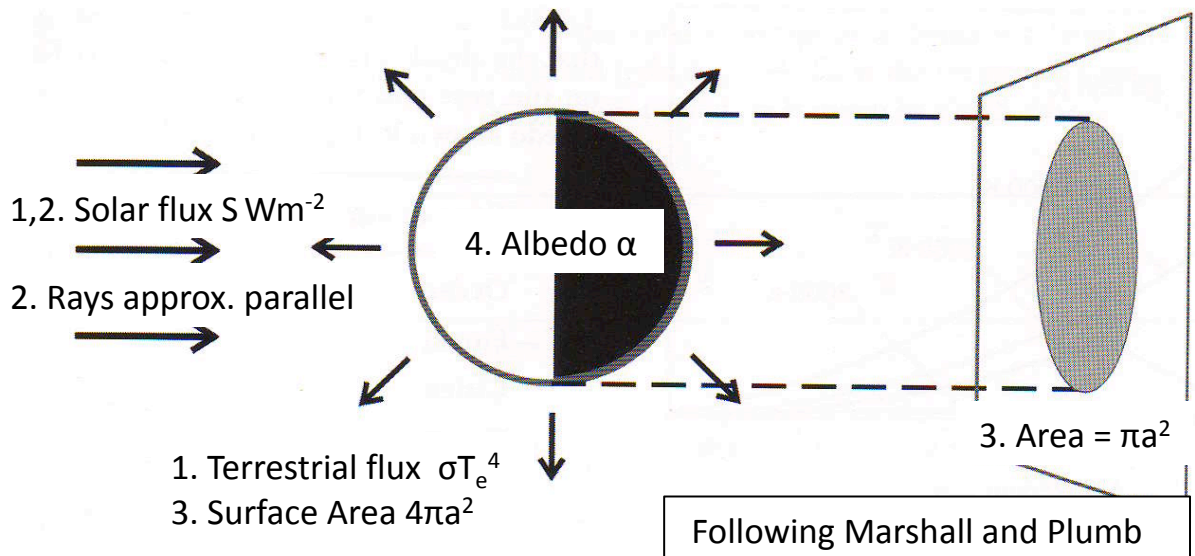
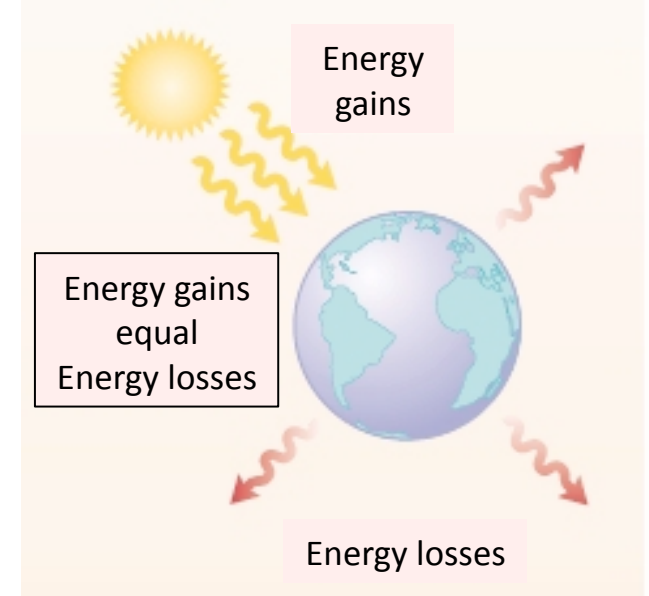
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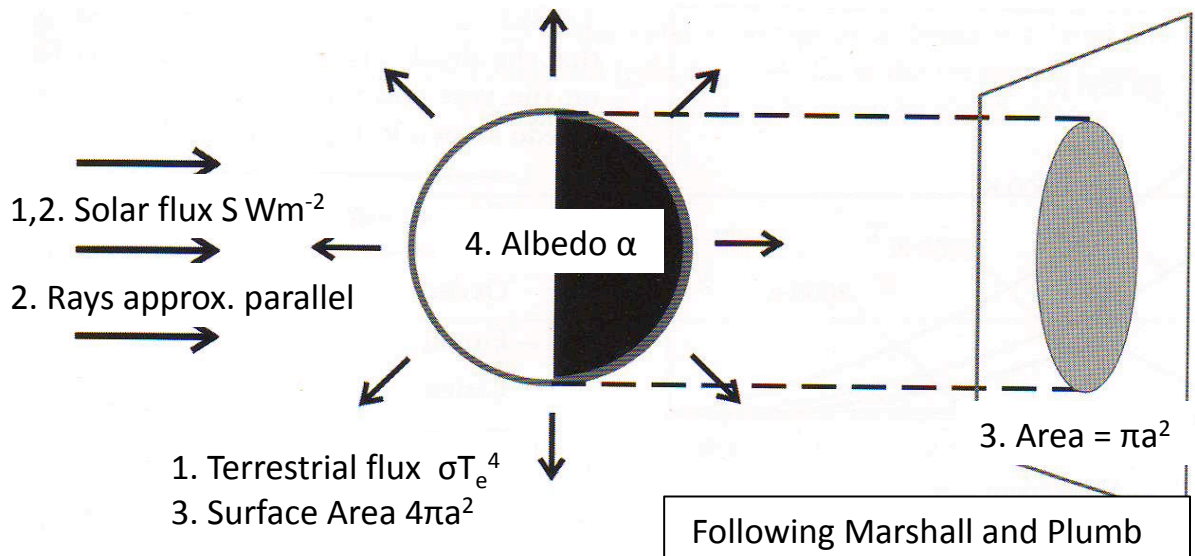
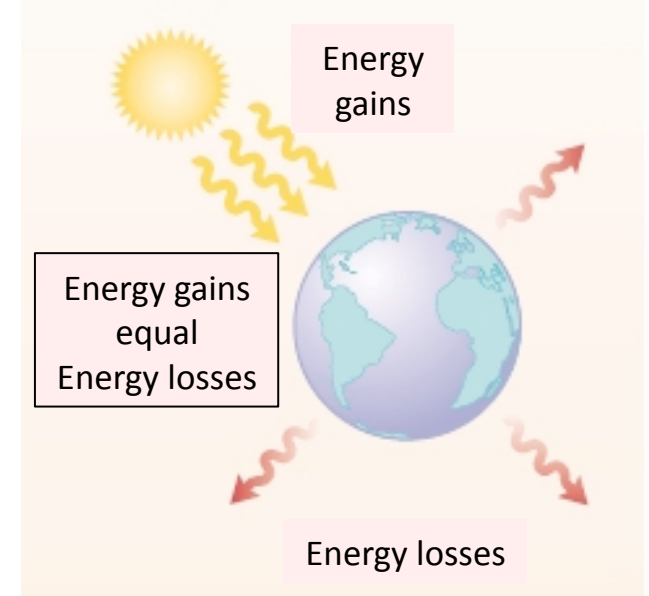
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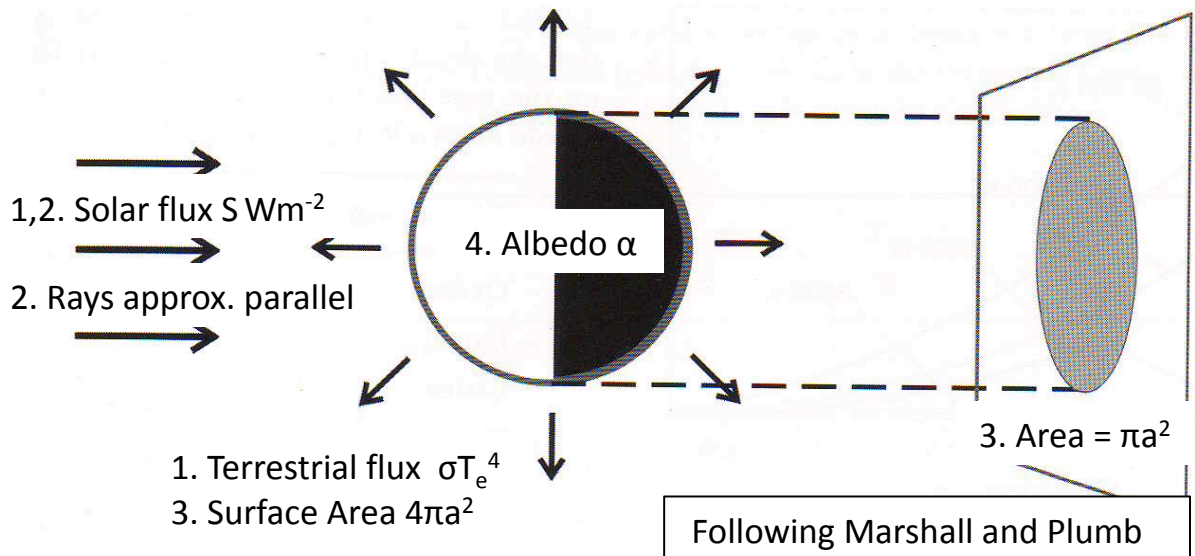
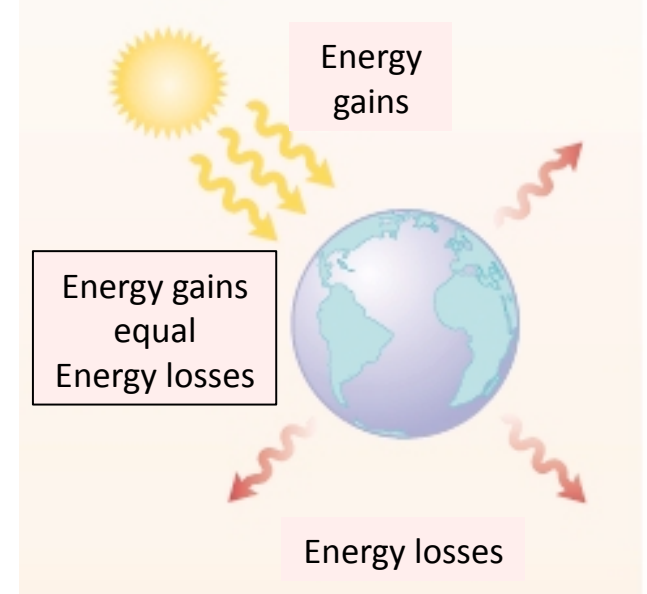
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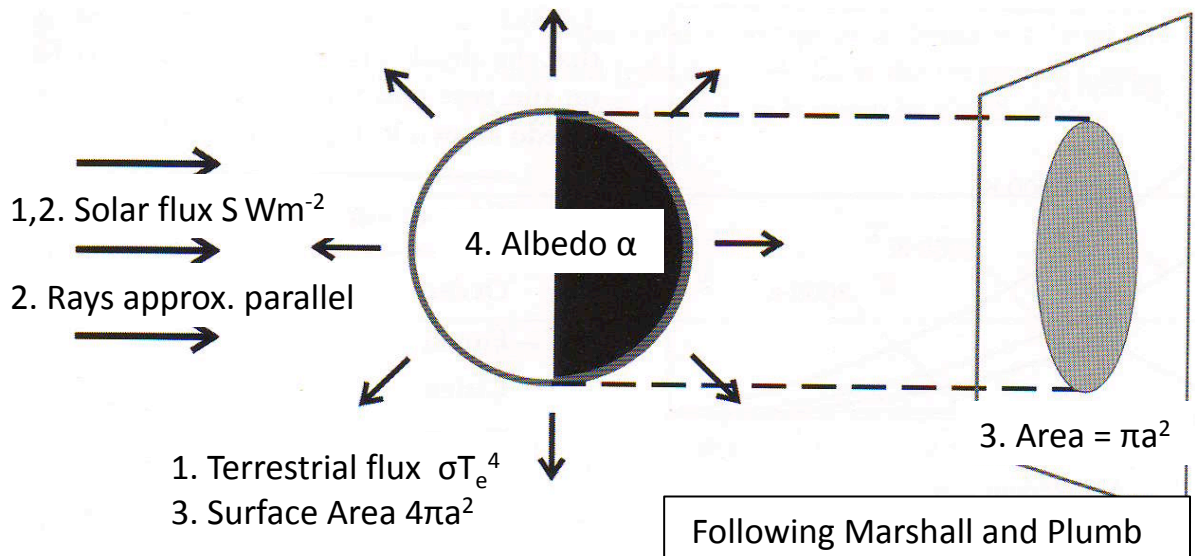
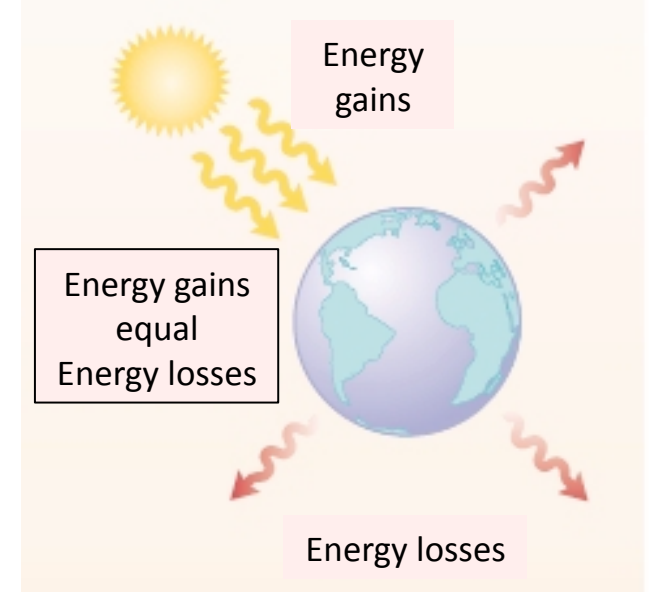
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## Balance

$$S\pi a^2 = \alpha S\pi a^2 + \sigma T_e^4 4\pi a^2$$

$$\rightarrow (1-\alpha)S = 4\sigma T_e^4$$

$$\rightarrow T_e = 255K = -18^\circ C$$







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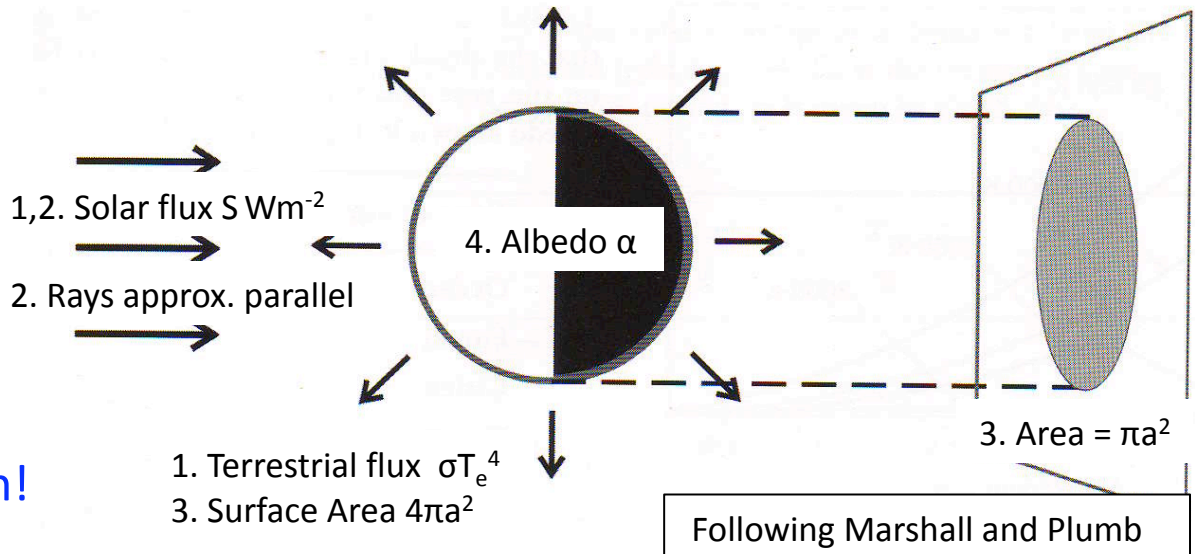
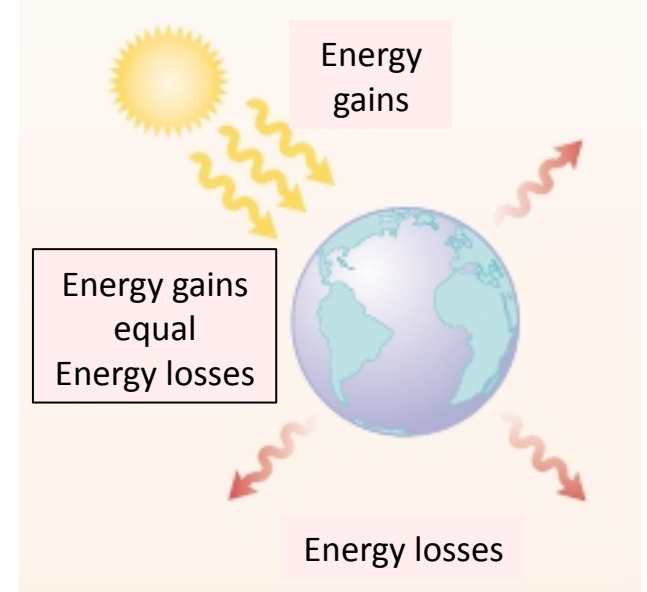
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**Too Cold – Snowball Earth!**





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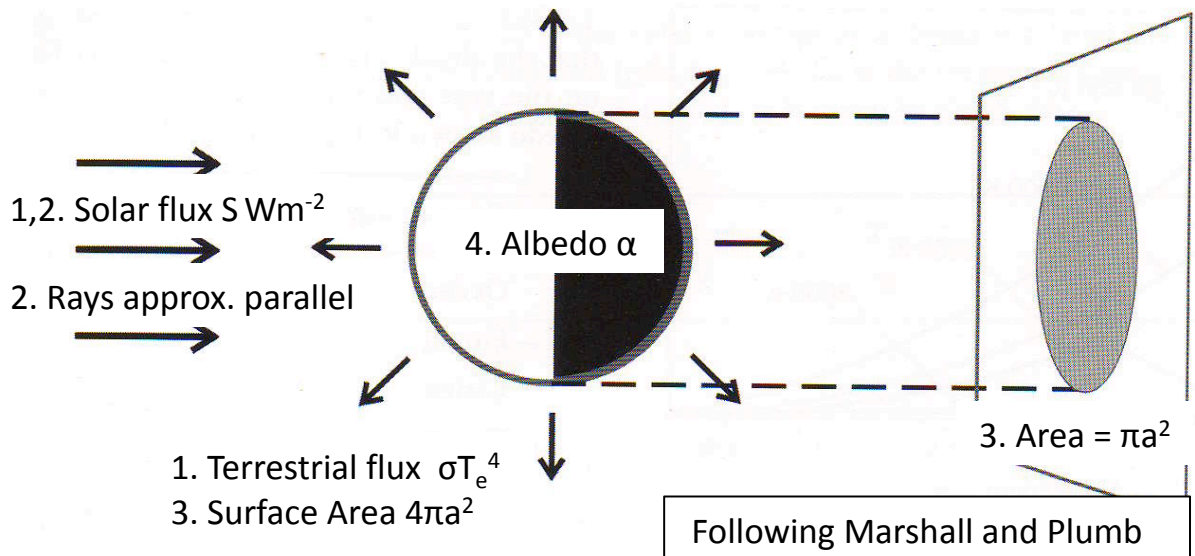
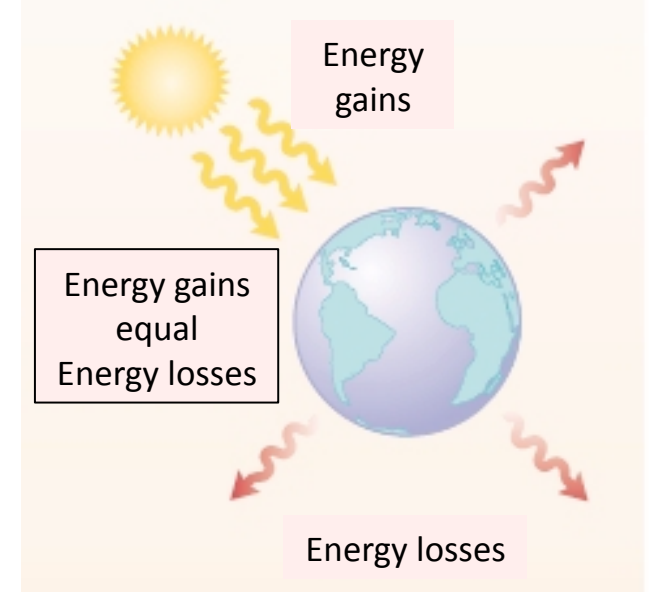
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Observed global average surface temp  $T_s = 288K = 15^\circ C$





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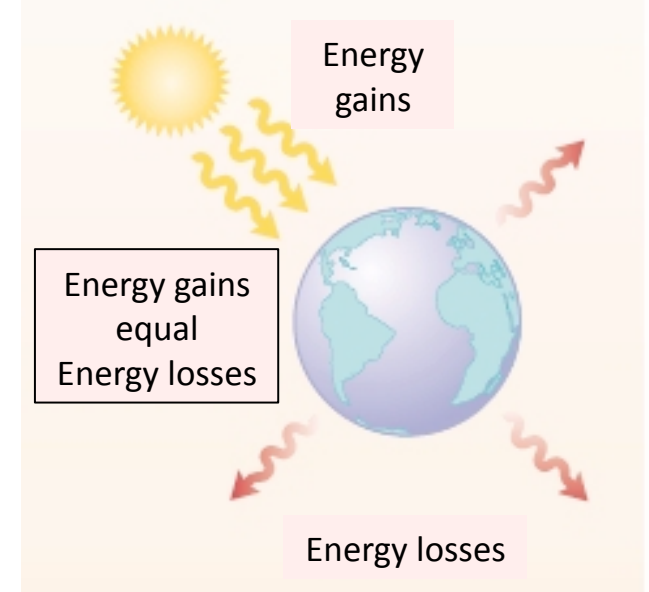
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**Conclusions:**

With these hypotheses, surface temp is 33°C too low  
We need to include atmosphere to correct this

255K is a good estimate for the top of the atmosphere.  
Which does satisfy the hypotheses more closely.



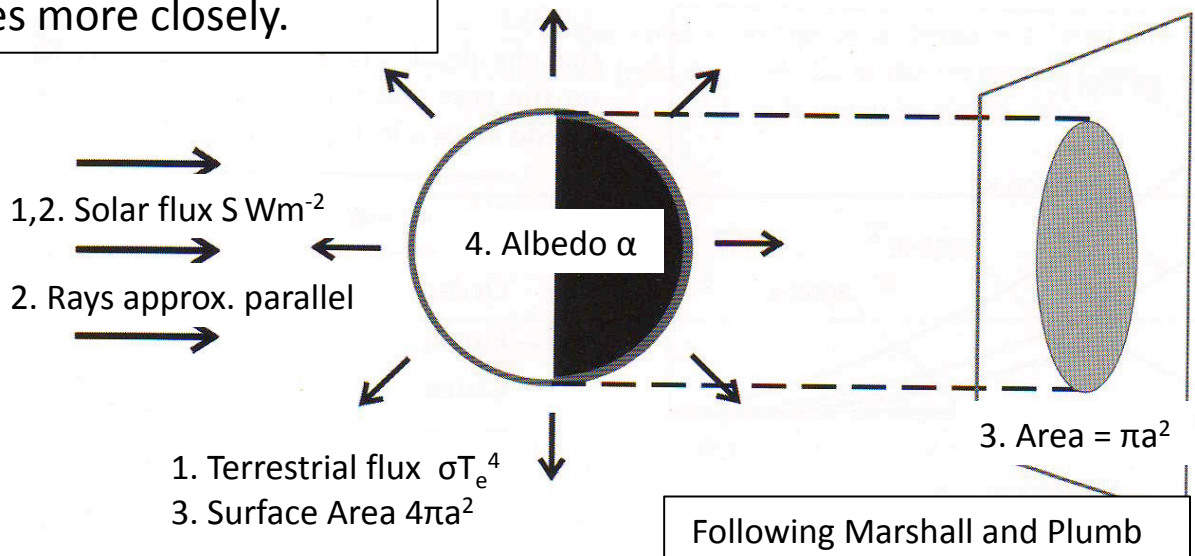
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# Aside: DE Viewpoint

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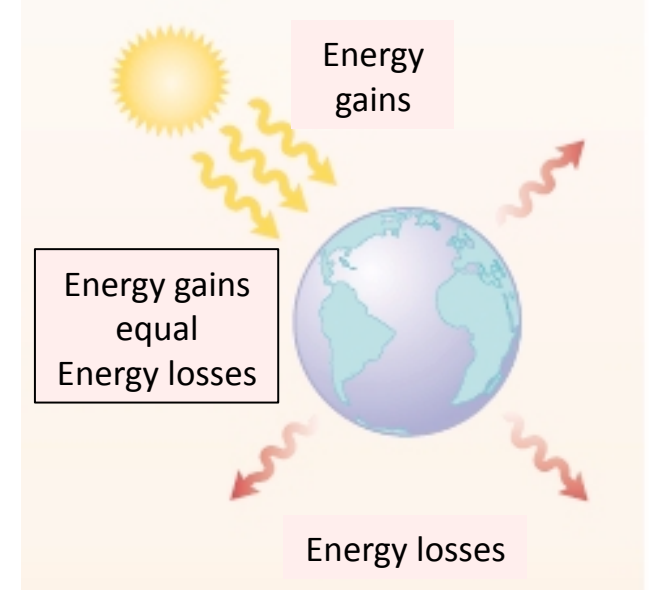
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Observed global average

surface temp  $T_s = 288\text{K} = 15^\circ\text{C}$



**Aside: as a differential equation for temp  $T_e$**

Sign( $dT_e/dt$ ) = energy gain – energy loss

$$= ((1-\alpha)S - \sigma T_e^4) \pi a^2$$

- when  $T_e = 255\text{K}$ ,  $dT_e/dt = 0$
- when  $T_e > 255\text{K}$ ,  $dT_e/dt < 0$
- when  $T_e < 255\text{K}$ ,  $dT_e/dt > 0$

**Energy balance corresponds to attracting equilibrium.**



# Energy Balance, No Atmosphere

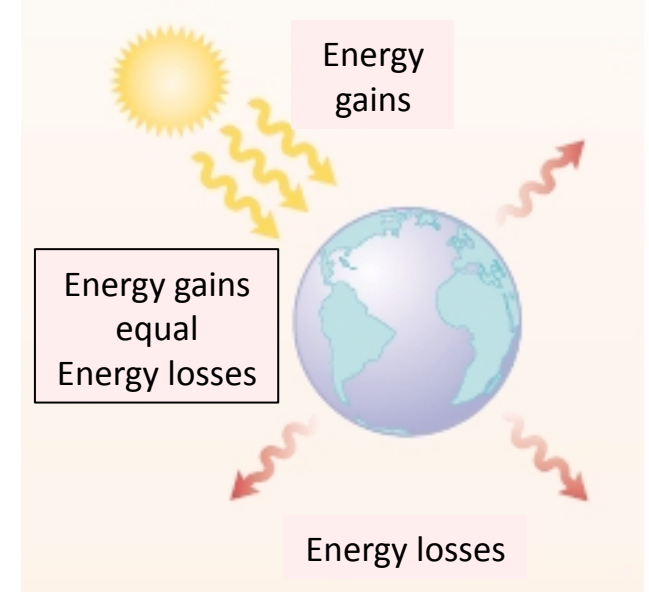
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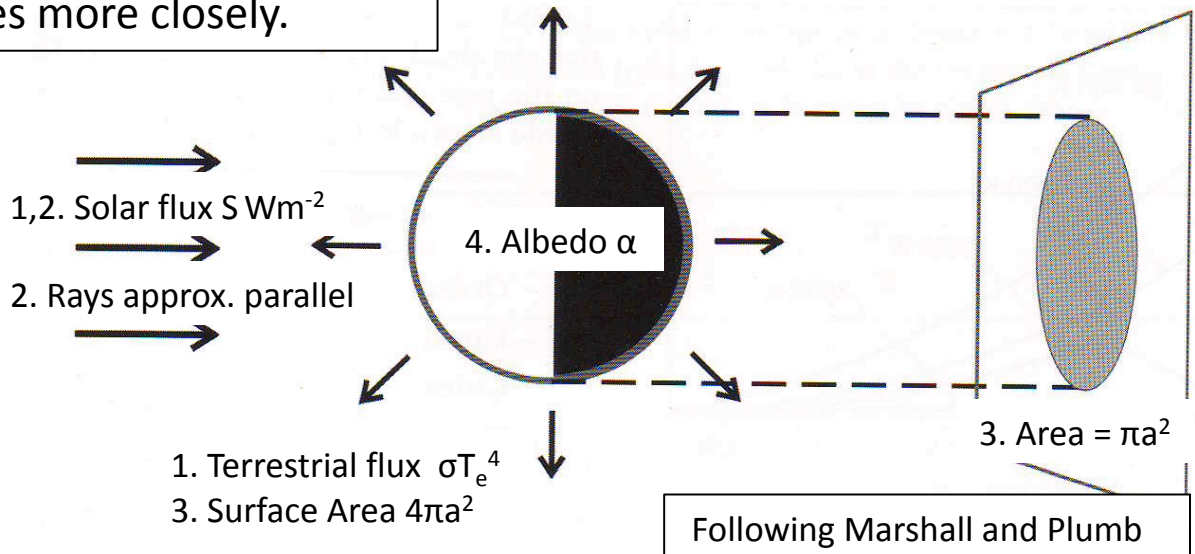
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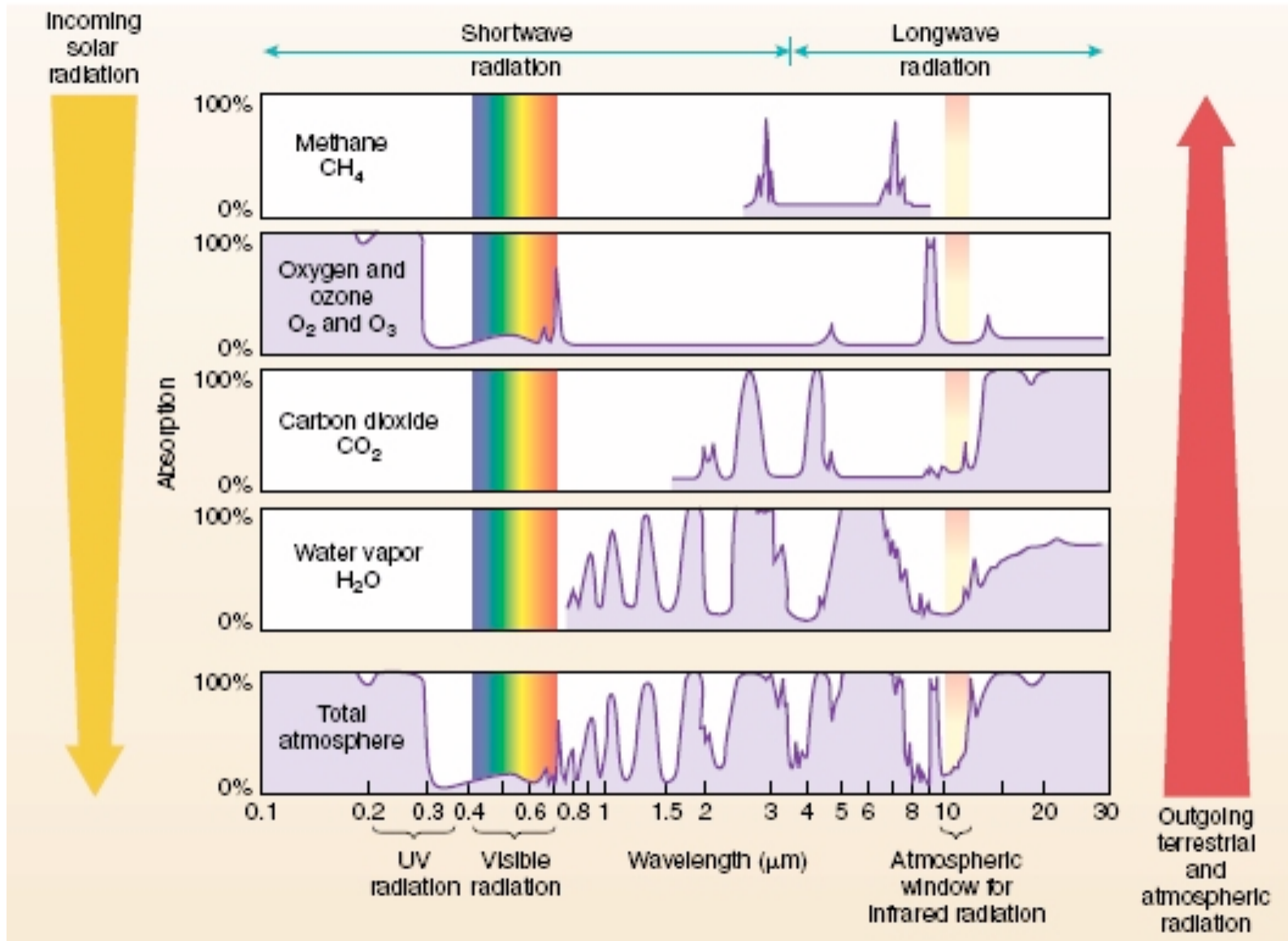
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# Atmospheric Absorption Spectrum

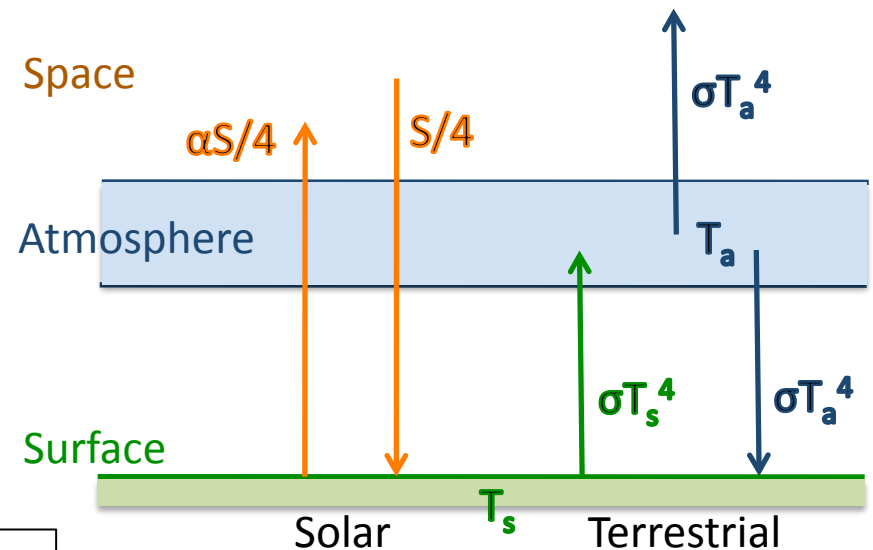
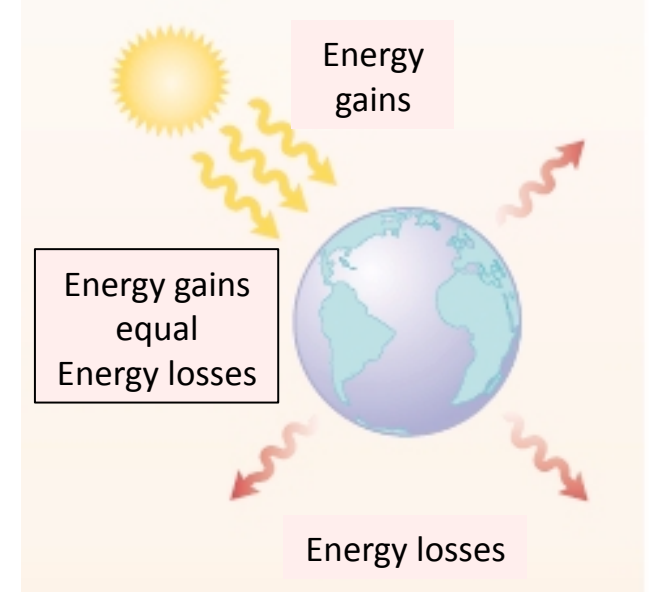




# Energy Balance, 1-Layer Atmosphere

**Hypotheses:** Earth's temperature is a consequence of

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Following Marshall and Plumb



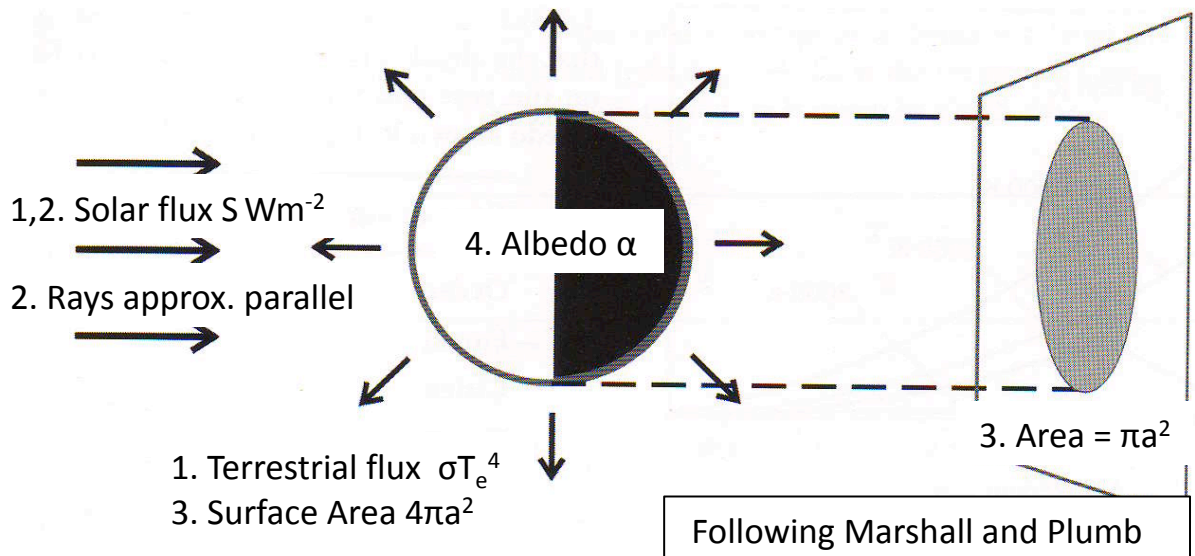
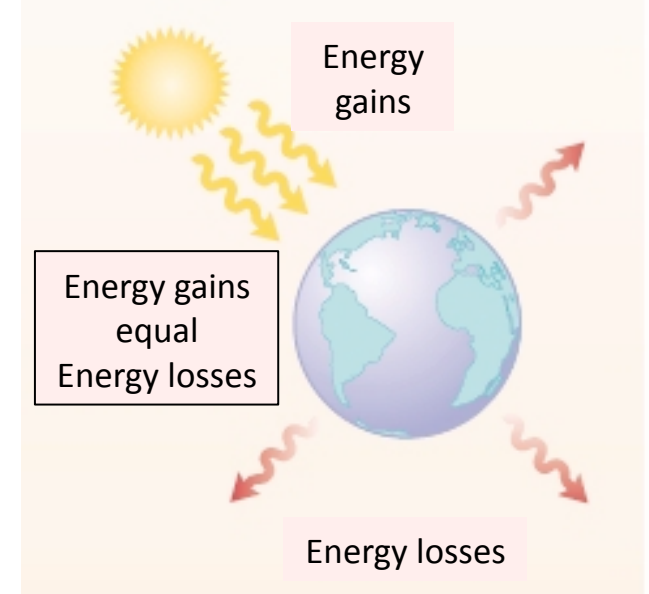
# Solar Flux

Recall total solar radiation (W)

$$\text{Solar flux} * \text{cross-sect area} = S\pi a^2$$

Solar flux ( $\text{W}/\text{m}^2$ ):

$$\text{Total solar in} / \text{surface area} = S\pi a^2 / 4\pi a^2 = S/4$$







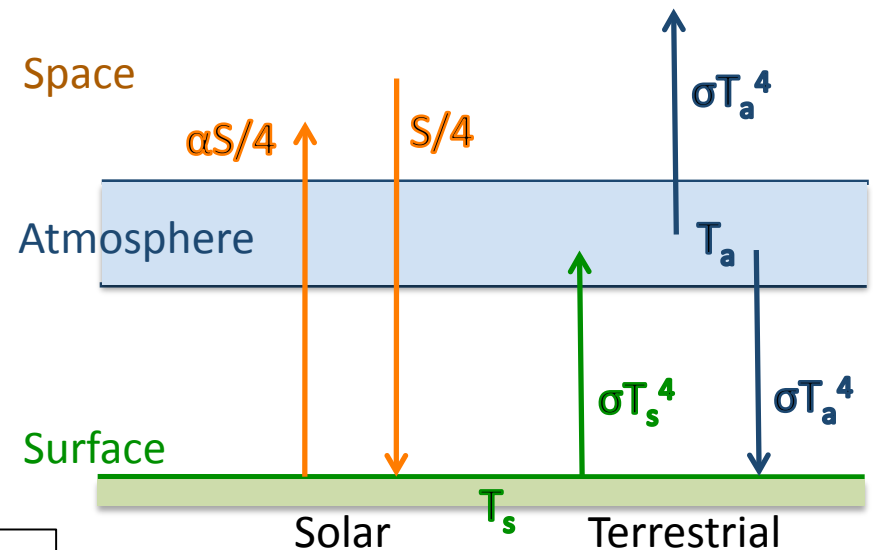
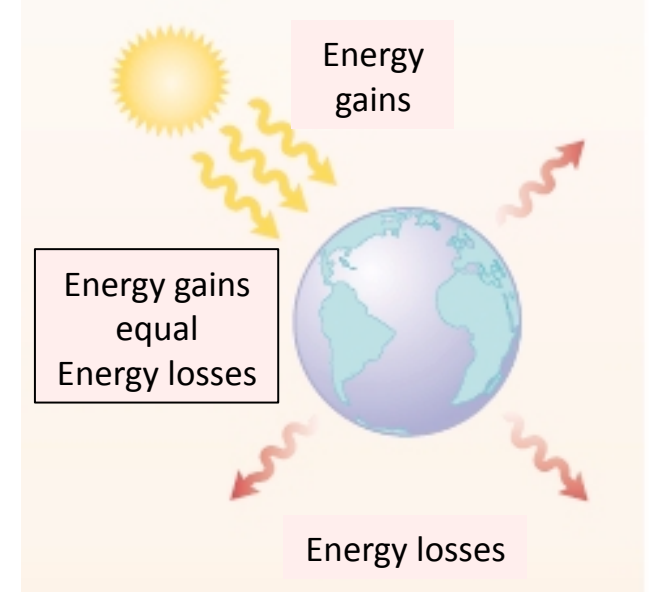
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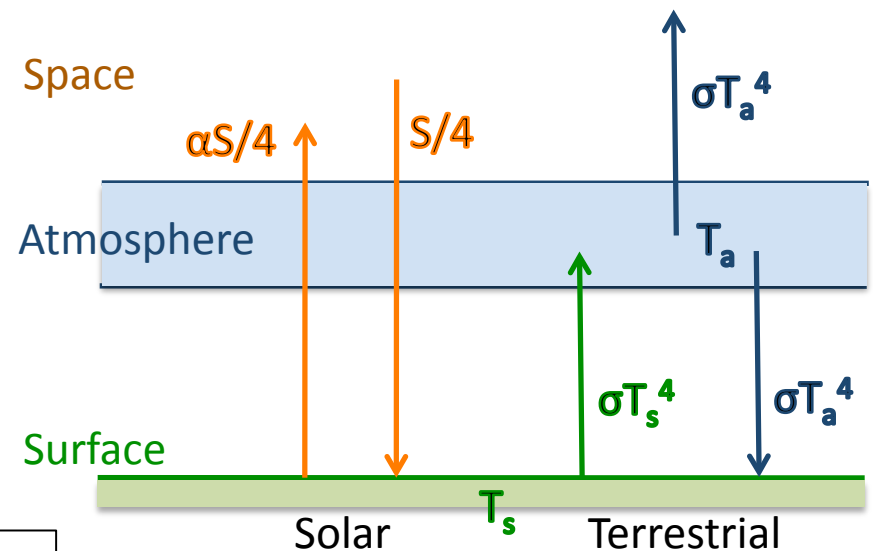
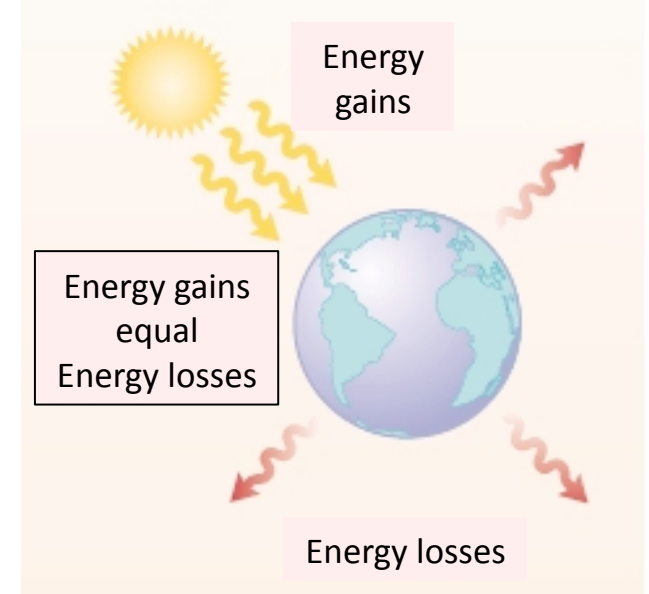
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$$(1-\alpha)S/4 = \sigma T_a^4$$



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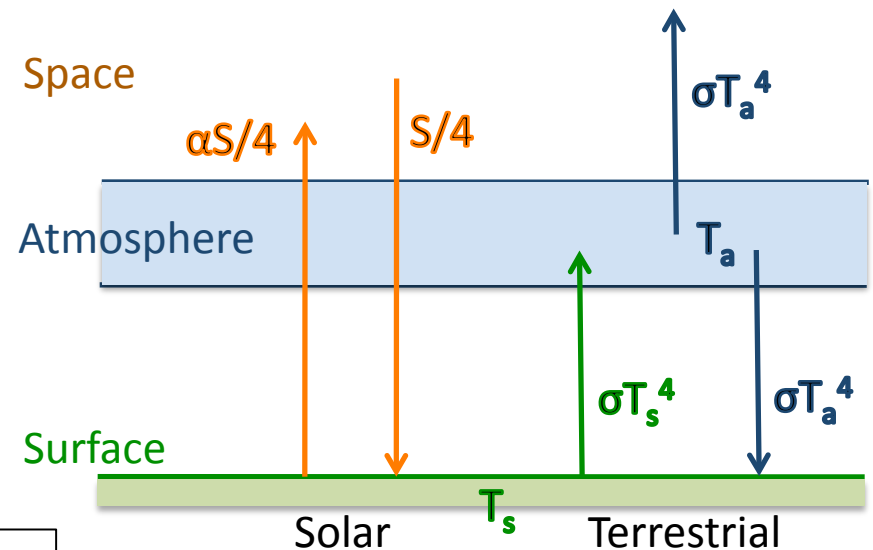
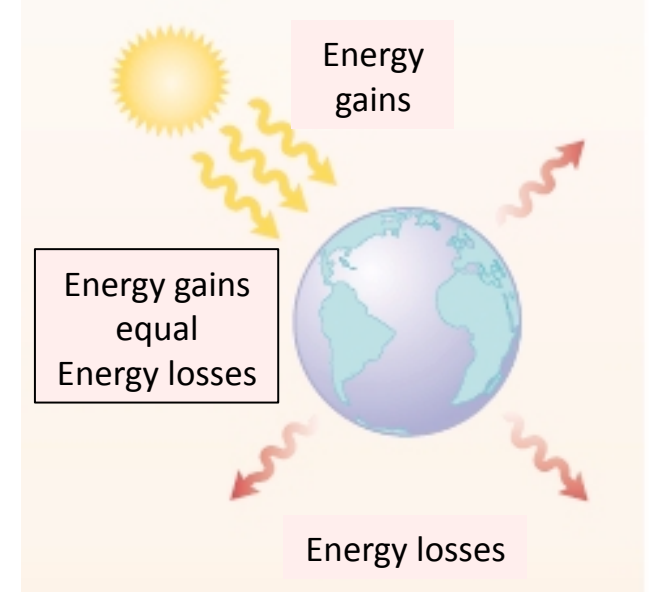
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So, as before,  $T_a = 255K$



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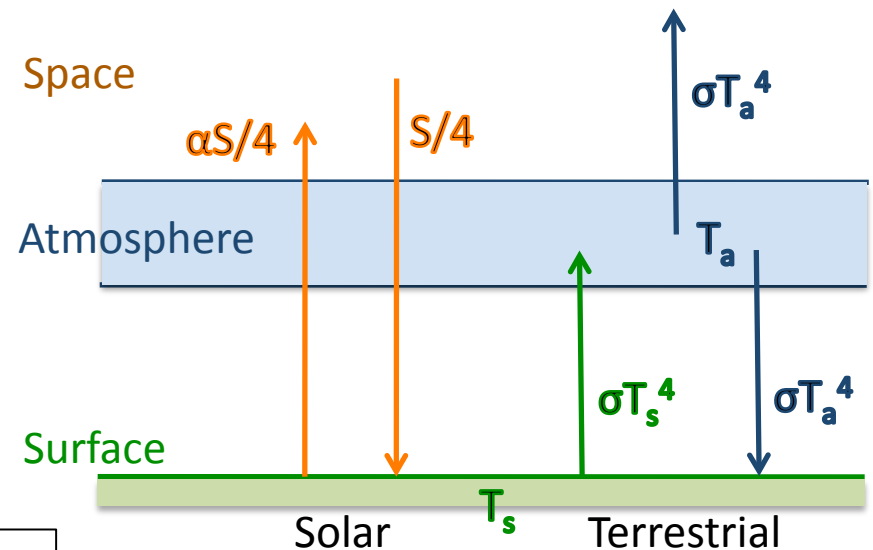
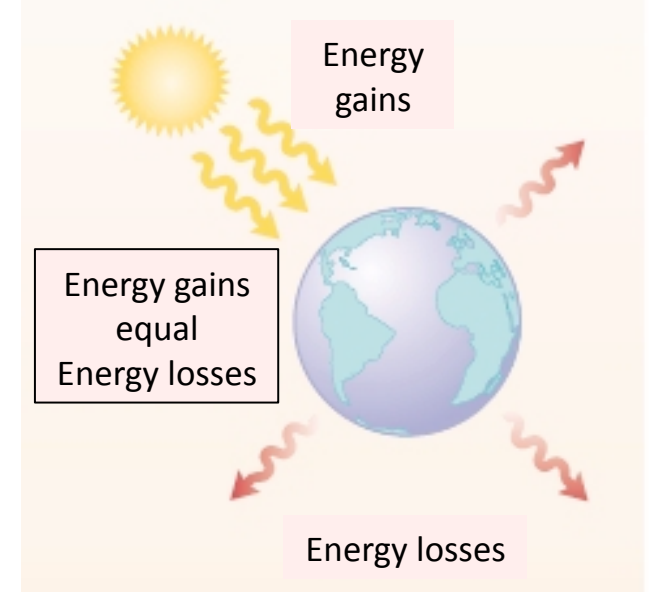
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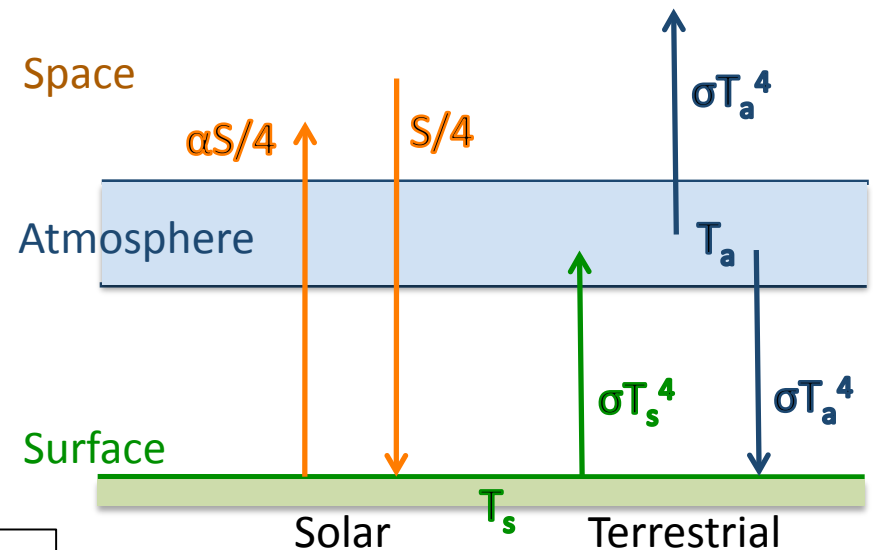
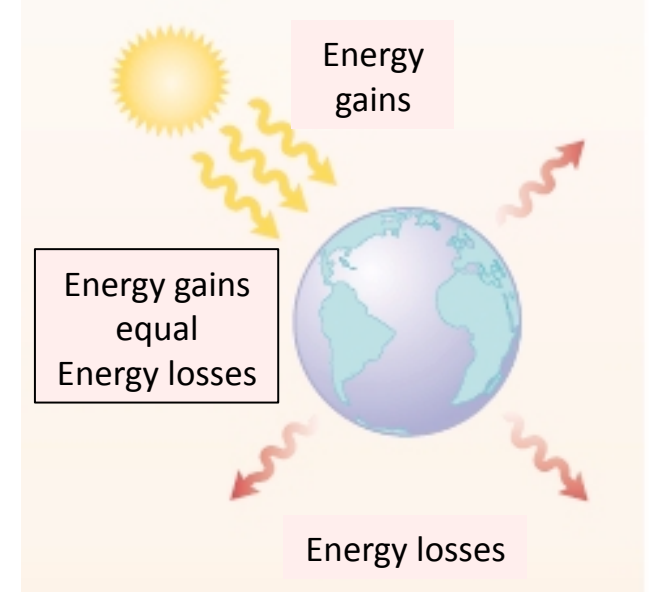
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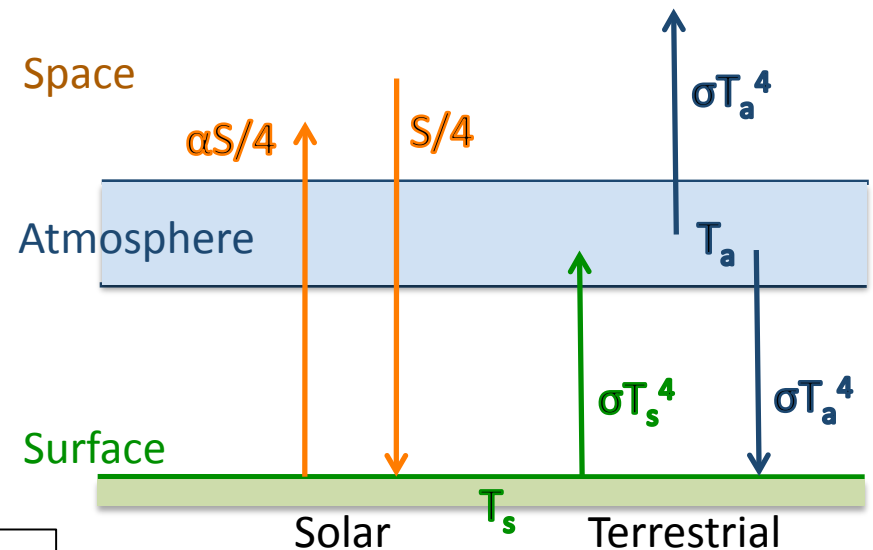
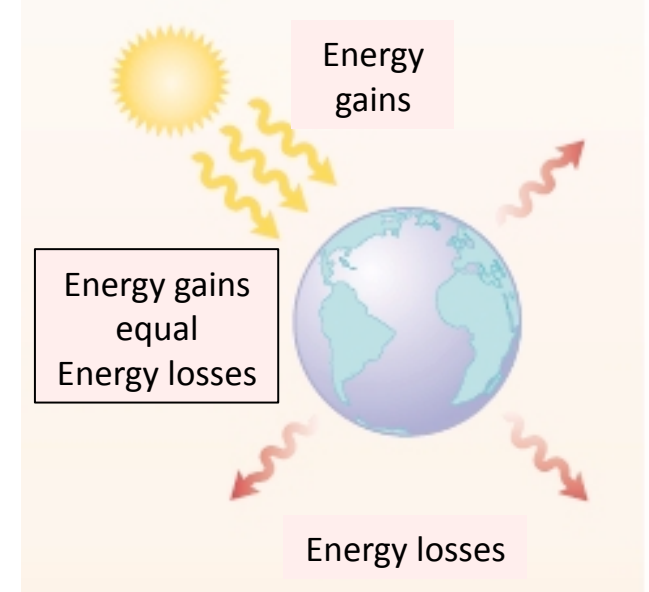
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So,  $T_s = 2^{1/4} T_a \approx 1.19 T_a = 303K = 30^\circ C$



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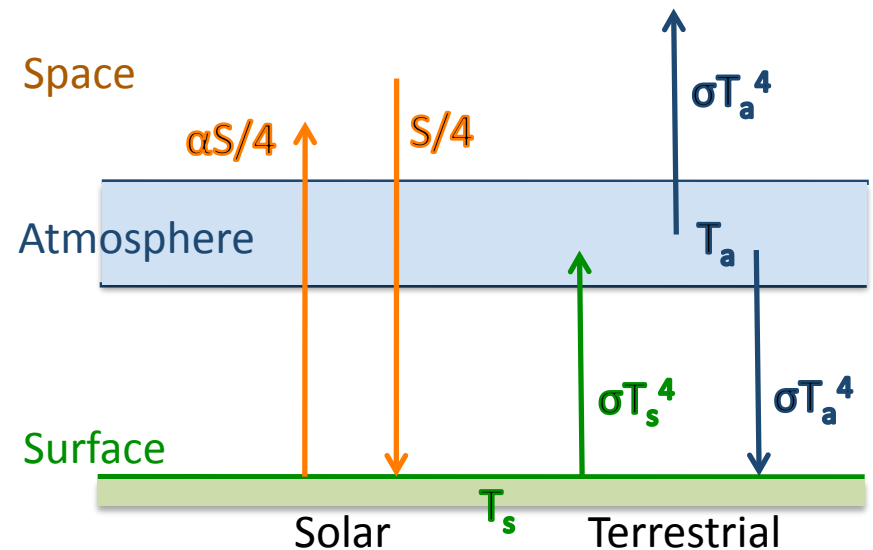
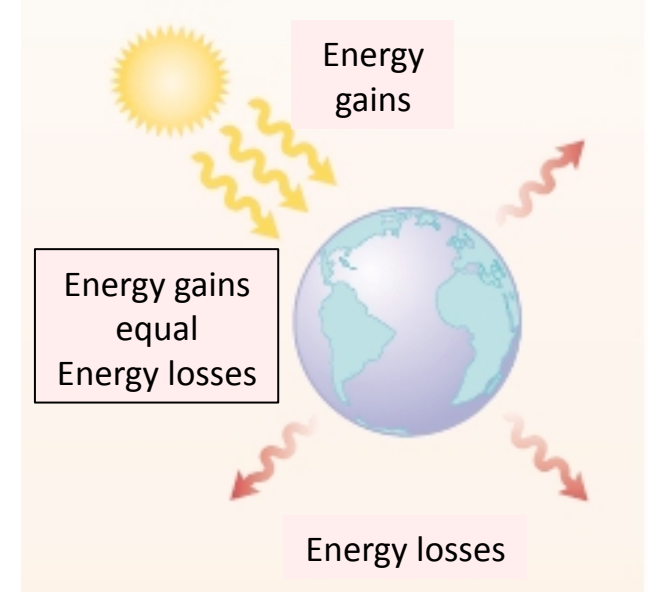
$$S/4 + \sigma T_a^4 = \alpha S/4 + \sigma T_s^4$$

$$\sigma T_s^4 = (1-\alpha)S/4 + \sigma T_a^4$$

$$= 2\sigma T_a^4$$

$$\text{So, } T_s = 2^{1/4} T_a \approx 1.19 T_a = 303K = 30^\circ C$$

**Surface is  $15^\circ C$  warmer than observed.**





# Energy Balance, 1-Layer Atmosphere

**Hypotheses:** Earth's temperature is a consequence of

1. Radiation, distance from sun, size, albedo
2. Single layer, uniform atmosphere
3. Atmosphere transparent to solar radiation
4. Atmosphere opaque to terrestrial radiation

**Solar flux ( $W/m^2$ ):**

$$\text{Total solar in / surface area} = S\pi a^2 / 4\pi a^2 = S/4$$

**Energy balance of total system:**

$$S/4 = \alpha S/4 + \sigma T_a^4$$

$$(1-\alpha)S/4 = \sigma T_a^4$$

$$\text{So, as before, } T_a = 255K$$

**Energy balance at surface:**

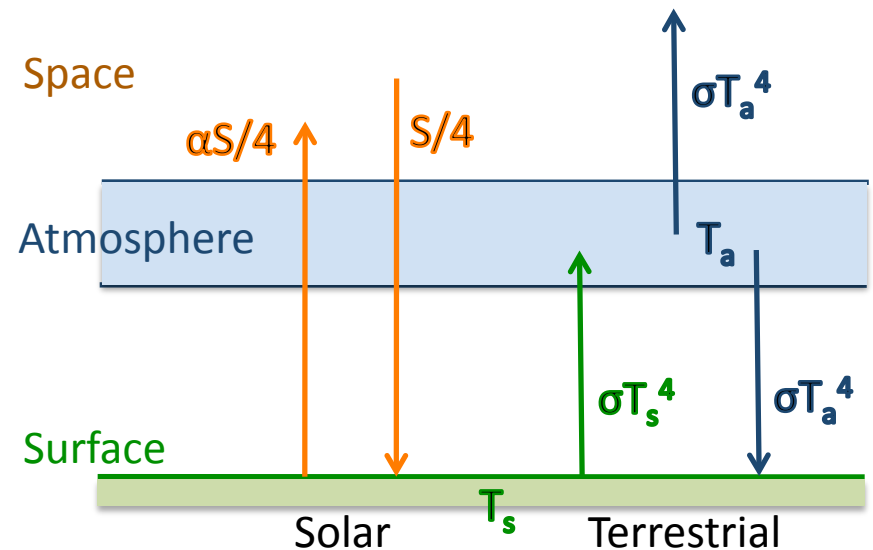
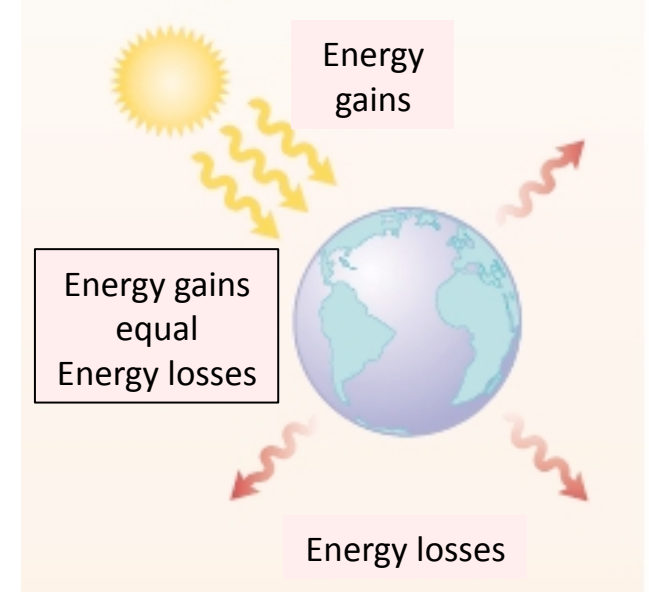
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$$= 2\sigma T_a^4$$

$$\text{So, } T_s = 2^{1/4} T_a \approx 1.19 T_a = 303K = 30^\circ C$$

**Note:** downwelling from atmosphere is at magnitude of solar flux.







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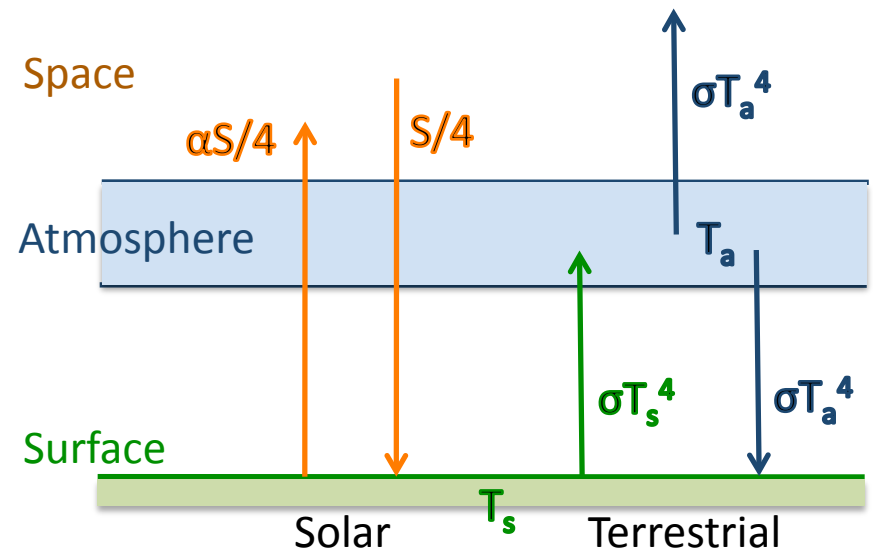
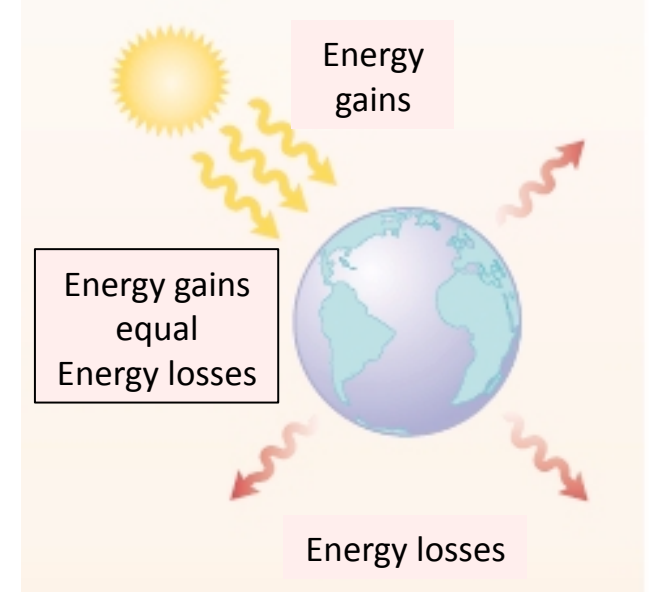
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**Note: Atmosphere is cooler than the surface.**





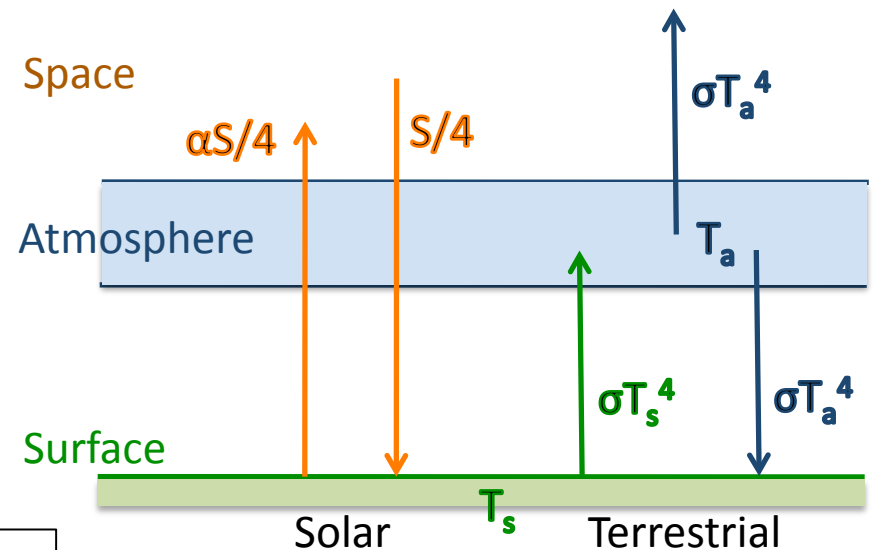
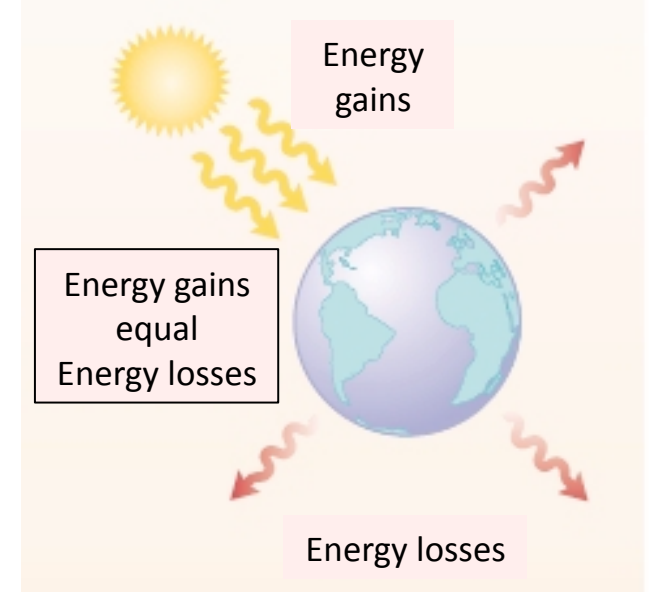
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**Conclusions:** With these hypotheses

- Top of atmosphere is 255K (observed 250K)
- Surface temp is 15°C too high
- Atmosphere is cooler than surface



Following Marshall and Plumb



# Energy Balance, With Atmosphere

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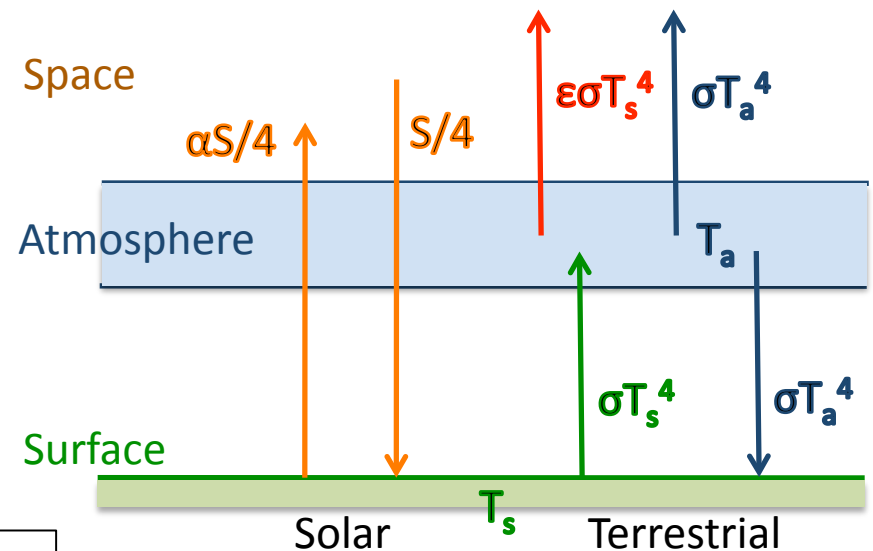
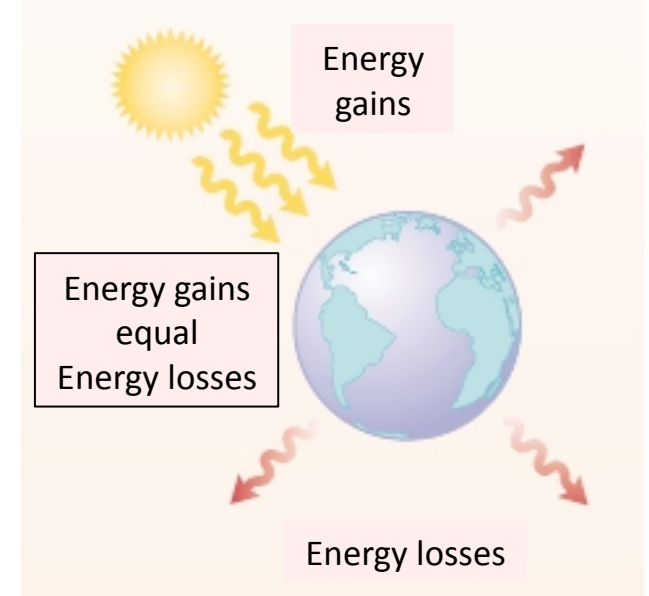
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**New Hypotheses:**

- Atmosphere partially opaque to terrestrial IR

**Conclusion:** With realistic  $\epsilon$ , surface temp is still too high



Following Marshall and Plumb



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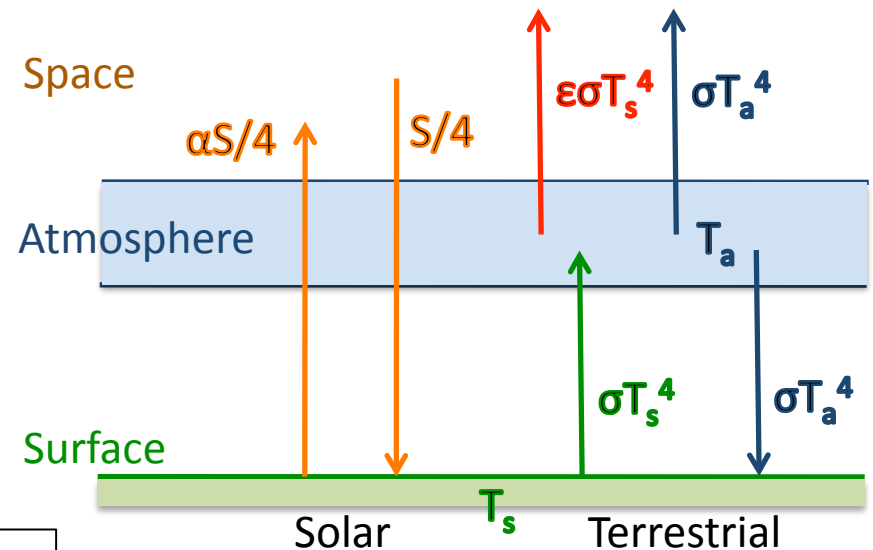
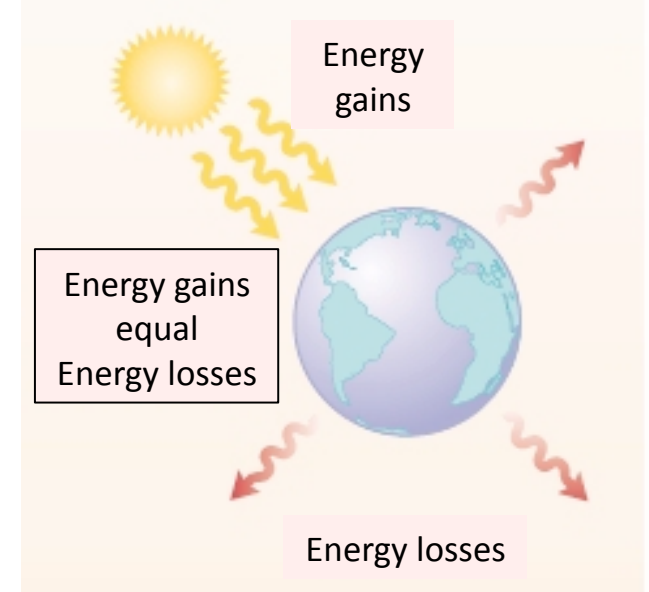
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**New Hypotheses:**

- Atmosphere partially opaque to terrestrial IR
- Many layers of atmosphere
- Accurate  $\epsilon$  for each layer
- Accurate  $\epsilon$  for each wavelength

**Conclusion:** With realistic atmosphere, surface temp is still too high



Following Marshall and Plumb



# Energy Balance, With Atmosphere

**Hypotheses:** Earth's temperature is a consequence of

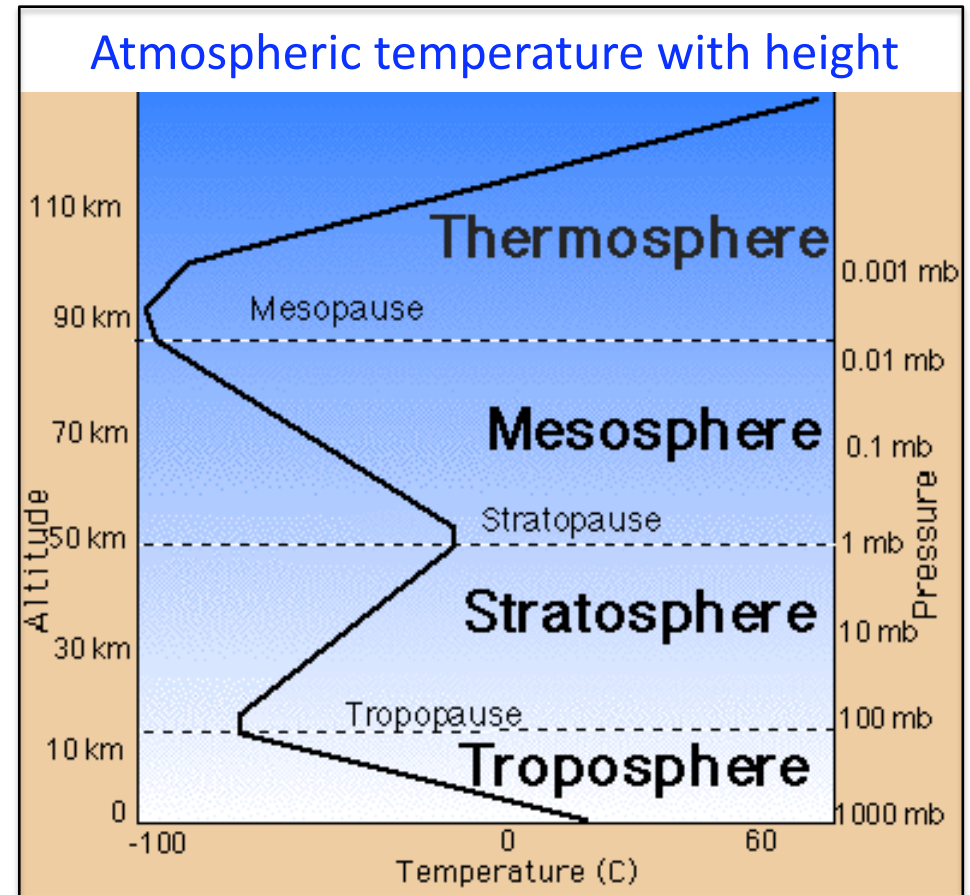
1. Radiation, distance from sun, size, albedo
2. Single layer, uniform atmosphere
3. Atmosphere transparent to solar radiation
4. Atmosphere opaque to terrestrial radiation

Do get qualitative reproduction of temperature distribution in atmosphere.

**New Hypotheses:**

- Atmosphere partially opaque to IR
- Many layers of atmosphere
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# Energy Balance, With Atmosphere

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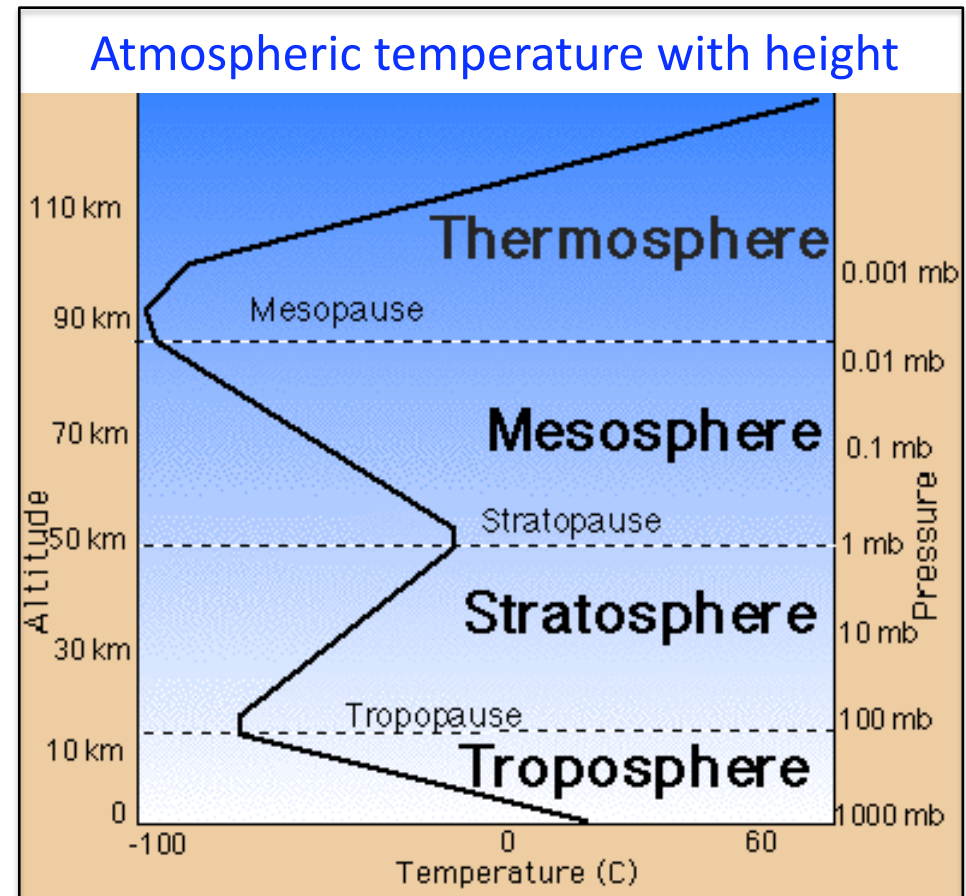
1. Radiation, distance from sun, size, albedo
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Radiative energy balance is NOT enough to explain surface temperature.

**New Hypotheses:**

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# Energy Balance, With Atmosphere

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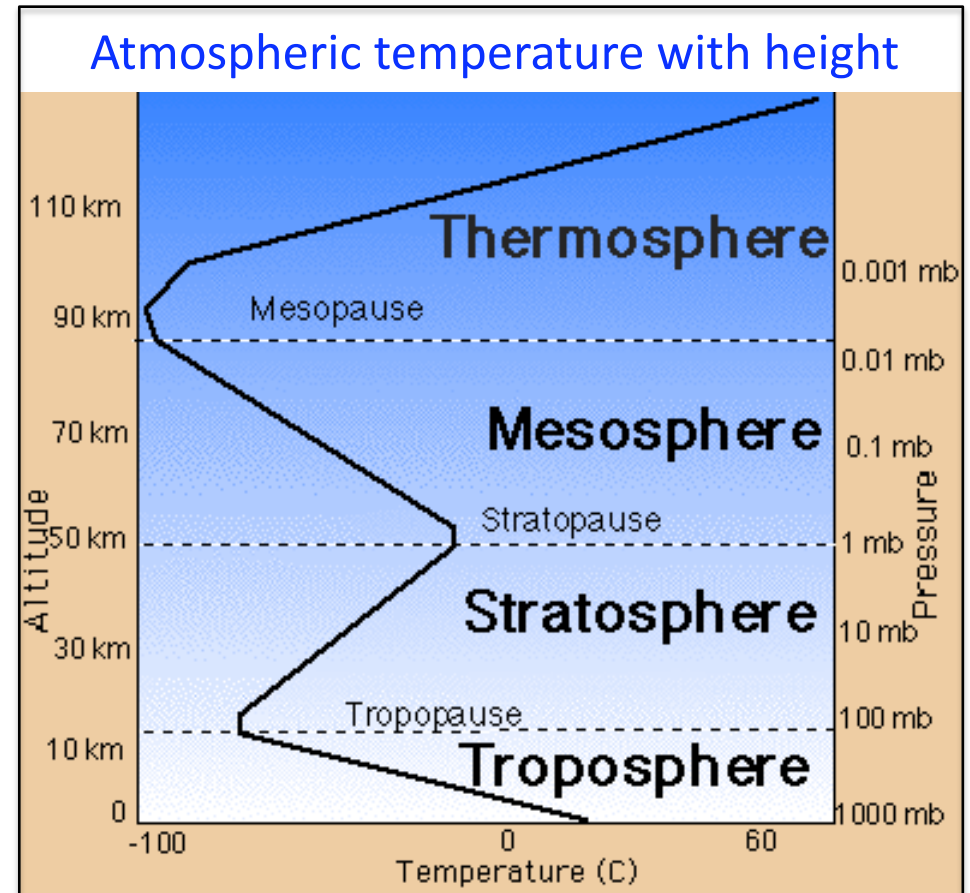
1. Radiation, distance from sun, size, albedo
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**Why? Cold fluid above hot fluid is an unstable equilibrium, so convection sets in... fluid dynamics and weather in troposphere**

**New Hypotheses:**

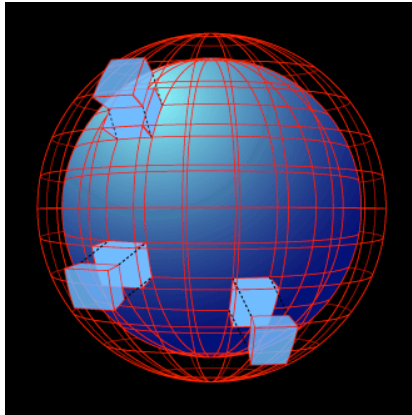
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**Conclusion:** With realistic atmosphere, surface temp is still too high





# Numerical Climate Model



$$\rho \frac{D\vec{u}}{Dt} = -\rho(2\Omega \times \vec{u}) - \nabla p + \rho g \vec{k} + \Gamma$$

+

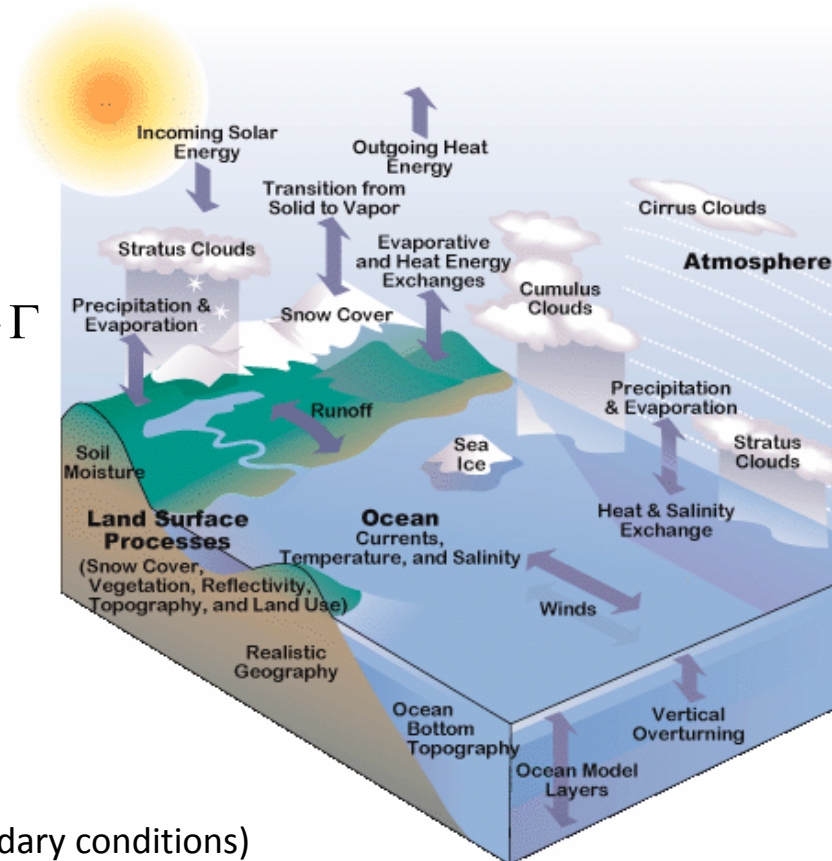
- conservation of mass
- water vapor (atmosphere)
- salinity (ocean)
- **conservation of energy**  
*brings in all other processes*

Discretize (put on grid)

→ connect pieces of model (boundary conditions)

→ initialize

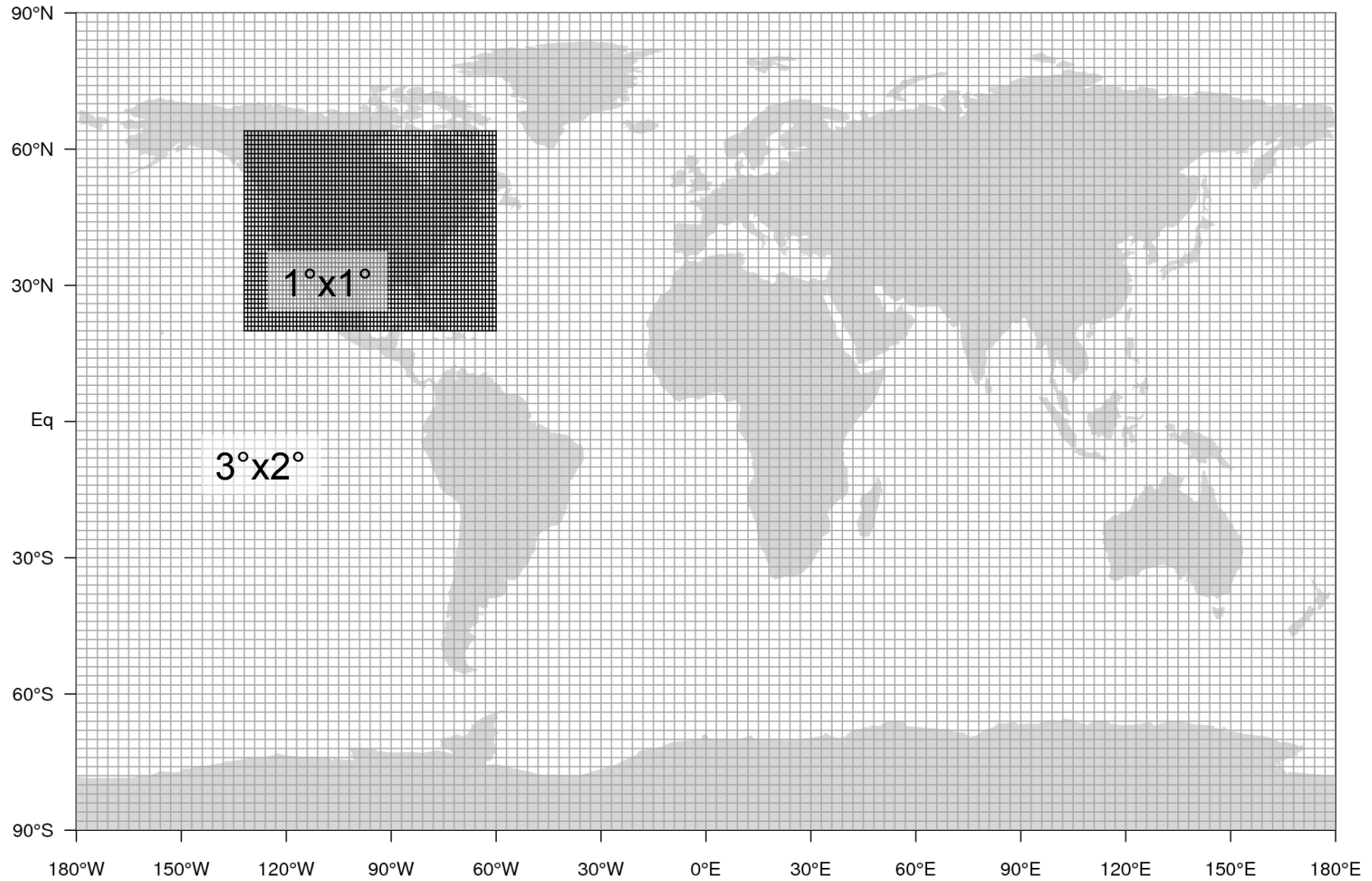
→ solve computationally







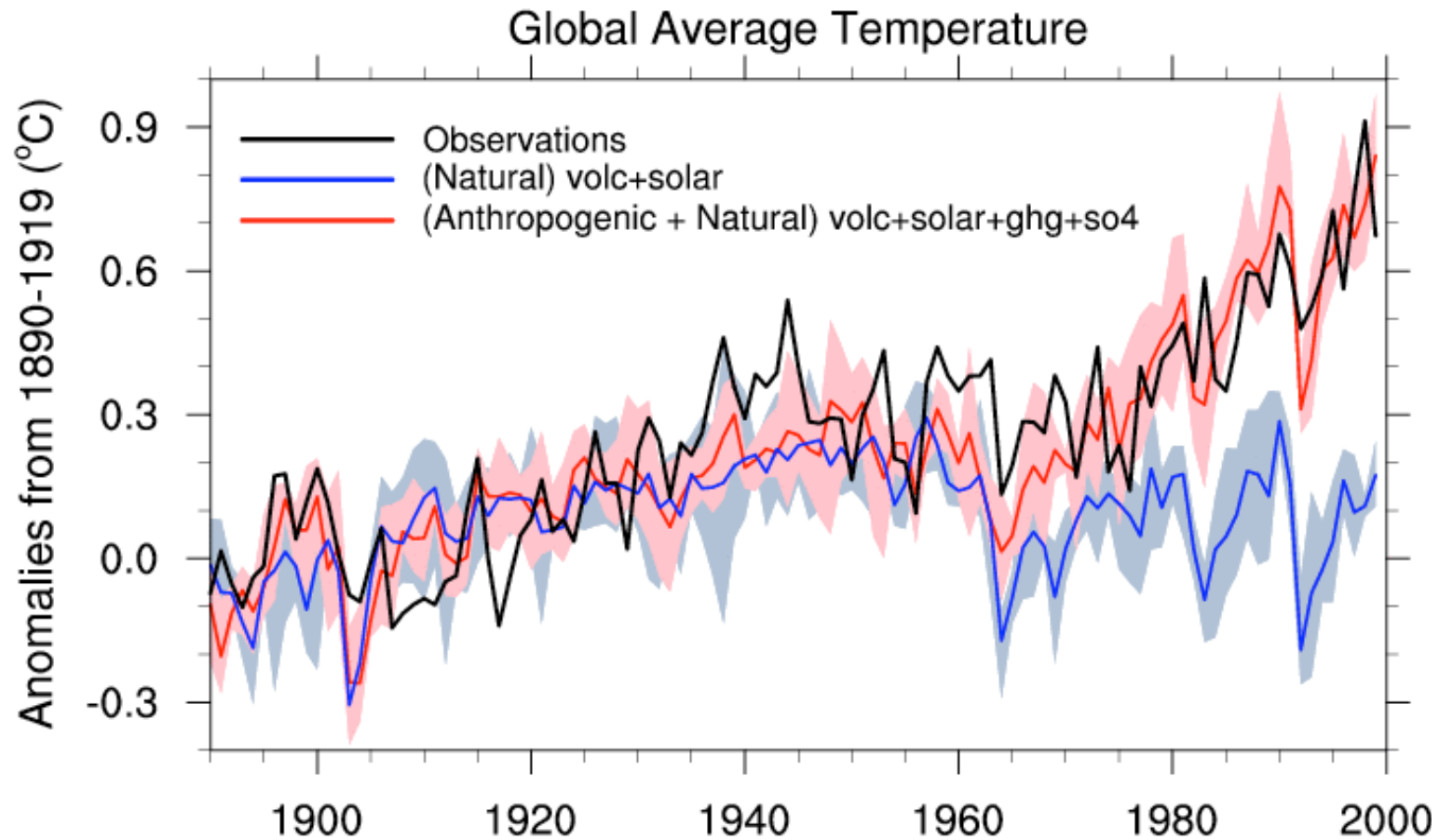
# Climate Model Grid Size





# Hypothesis Testing

## Model Solutions with Human Forcing



NCAR

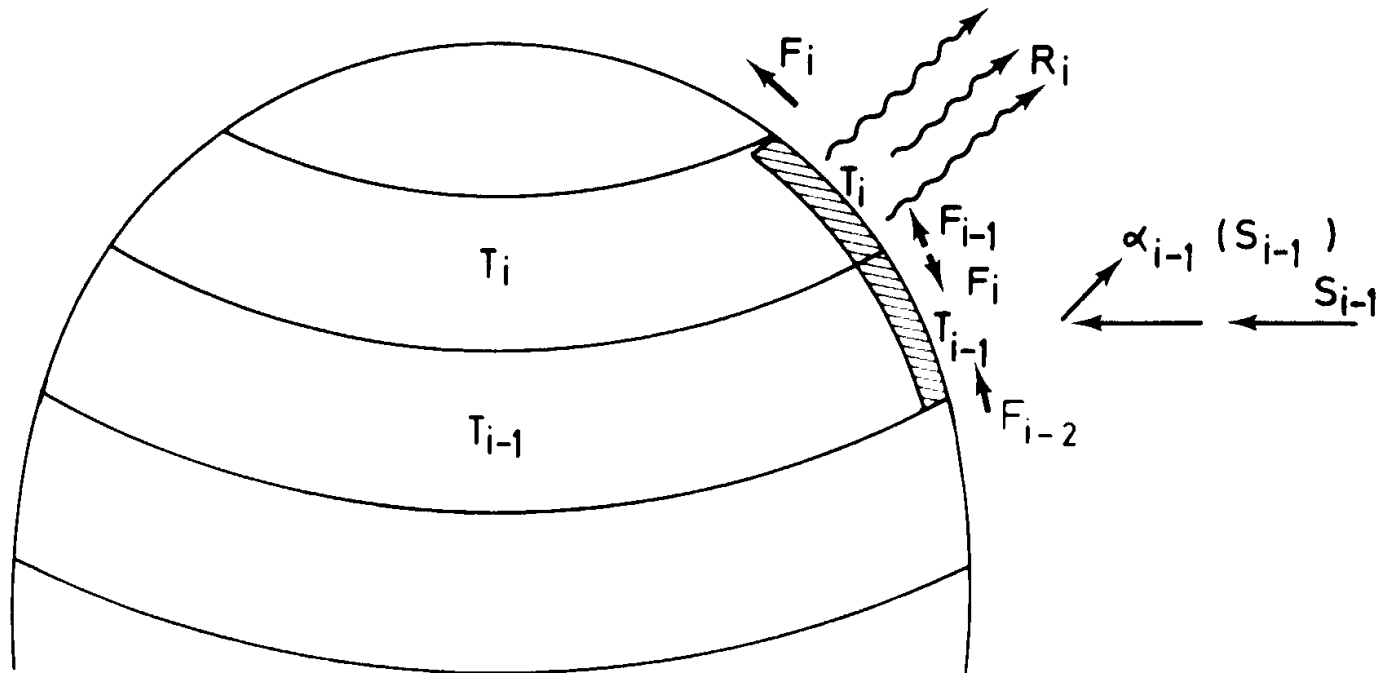
J. J. Hack/A. Gettelman: June 2005



# 1-Dimensional Energy Balance

## Budyko-Sellers-North. Hypotheses:

1. Rotational symmetry of earth
2. Energy balance calculated for each latitude  $x_i$ , with temp  $T_i$
3. Atmospheric dynamics provide heat transport between layers
4. Albedo depends on  $T_i$  (ice or not) – captures ice-albedo feedback



Solve equations for  $T_i$  profile, and for location of ice line



# 1-D Energy Balance Equations

Latitude  $x$ , temperature  $T$

Solar Radiation

Atmospheric and oceanic heat transport.

$$\frac{Q}{4} S(x)(1 - \alpha(x)) = A + BT(x) + \nabla \cdot \vec{F}.$$

Albedo: reflected light increases if  $T(x)$  drops below freezing!

Linearization of Outgoing Terrestrial Radiation

Assume that atmospheric and oceanic movements smooth the temperature profile

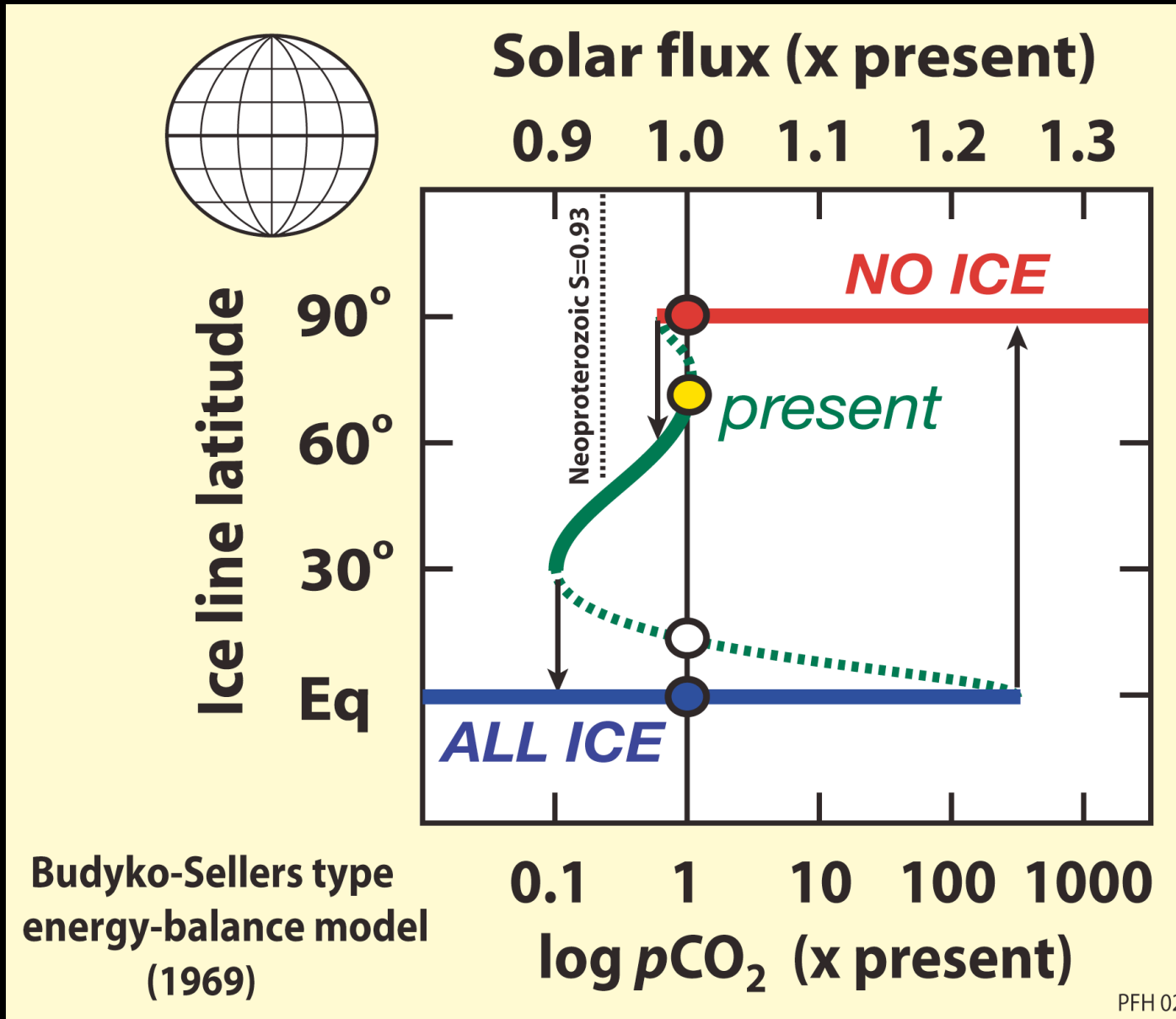
$$\nabla \cdot \vec{F} = C(T - \bar{T}),$$

$$\nabla \cdot \vec{F} = -D \frac{d}{dx} (1 - x^2) \frac{dT}{dx}.$$

Abbot



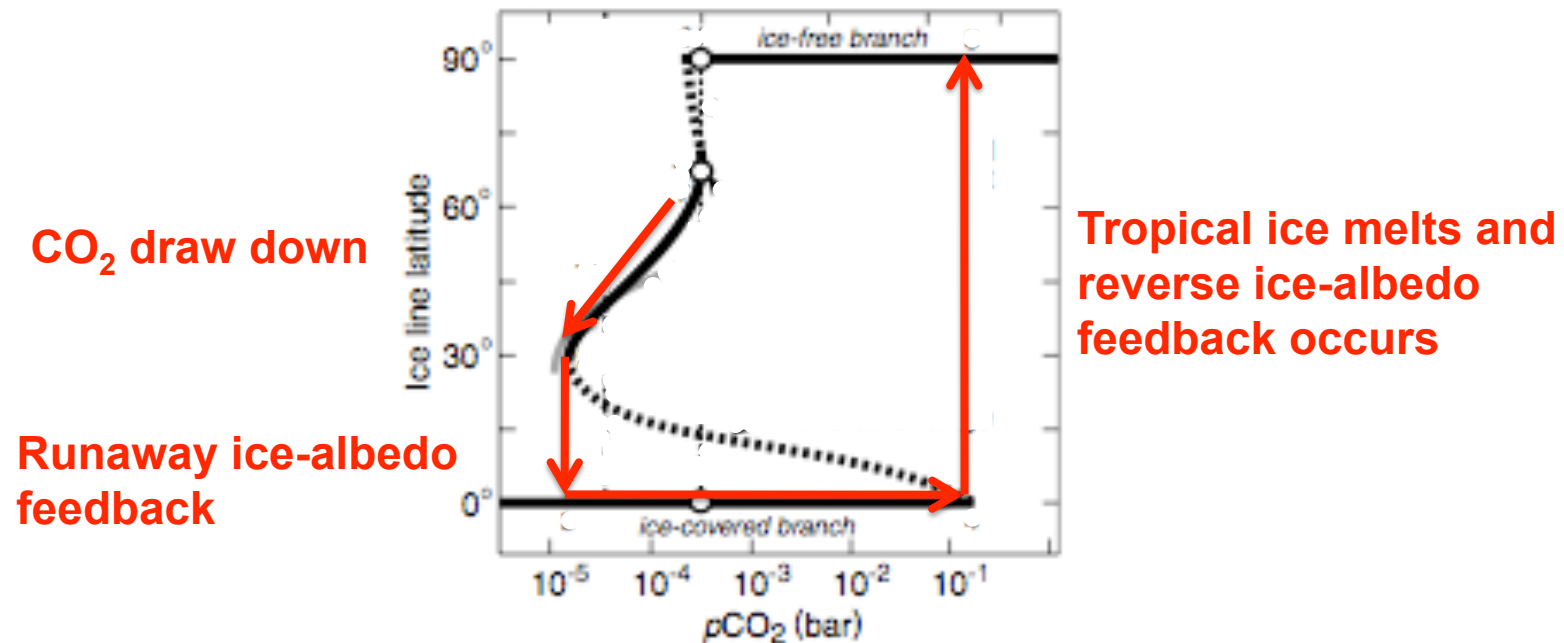
# Budyko-Sellers Model Predictions





# Snowball Earth: 700M yrs ago

Energy Balance Models produces a bifurcation diagram that might explain the geological observations.

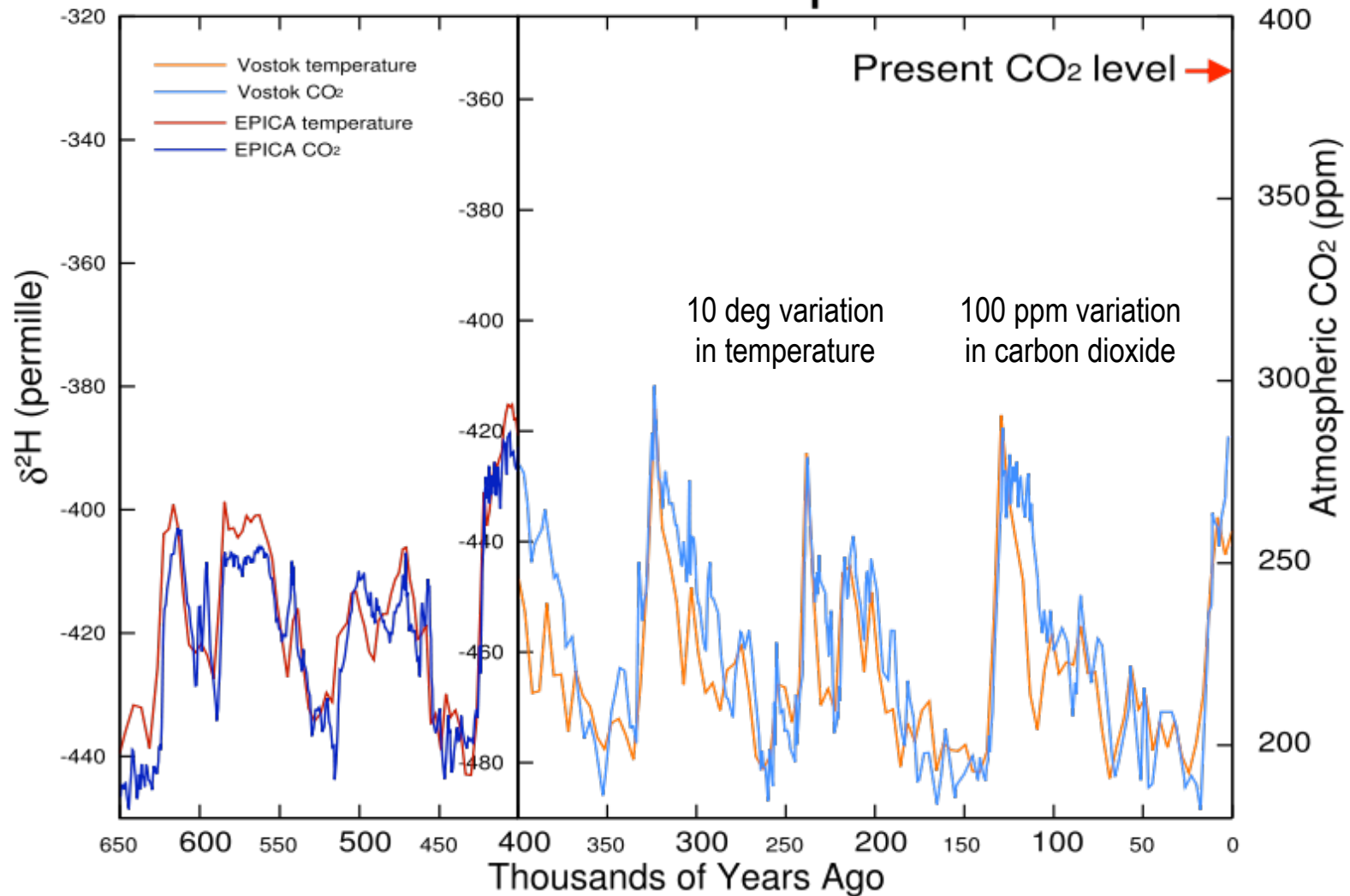


Very low weathering allows CO<sub>2</sub> to build up to ~10% of atmosphere over 1-10 million years

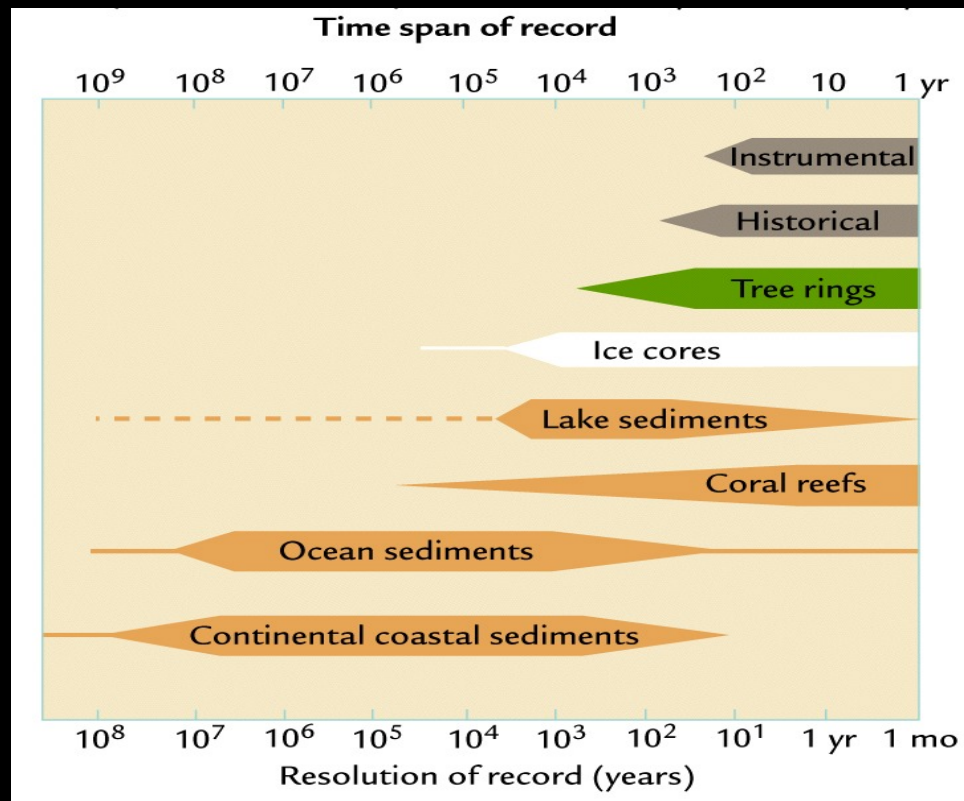


# More recent: last 600K yrs

## Carbon Dioxide and Temperature Records



# Climate archives: what information do we have?



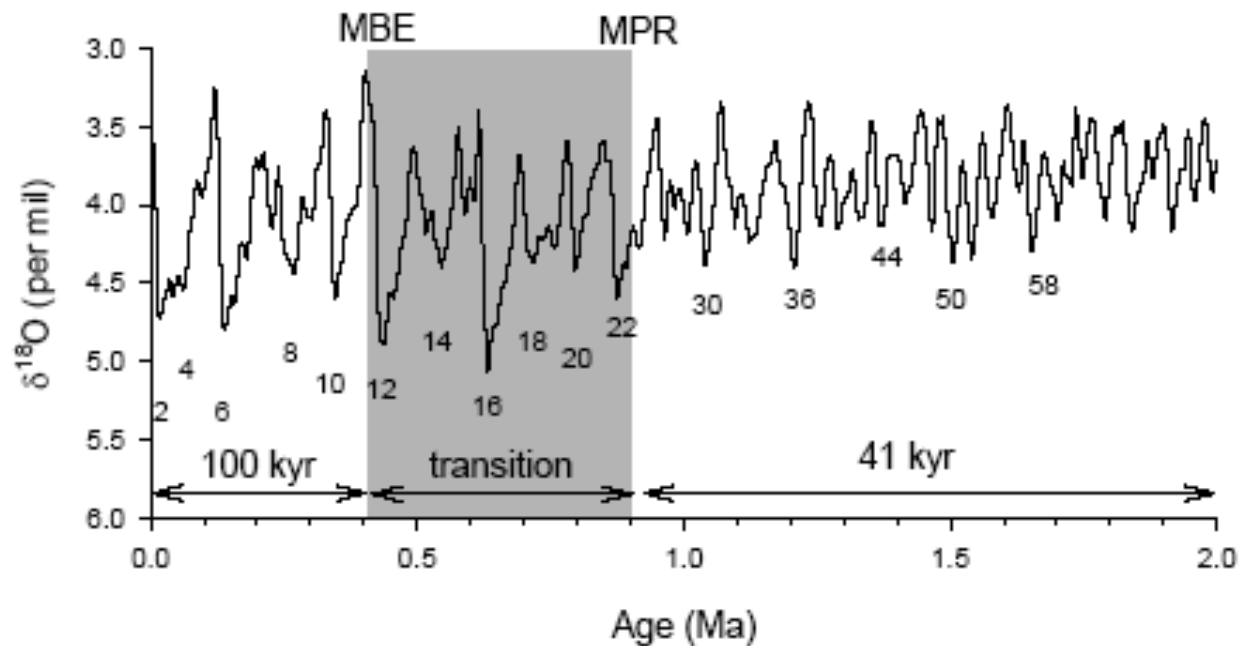
Ruddiman, W. F., 2008. Earth's Climate: past and future





# Temperature Record - 2M yrs

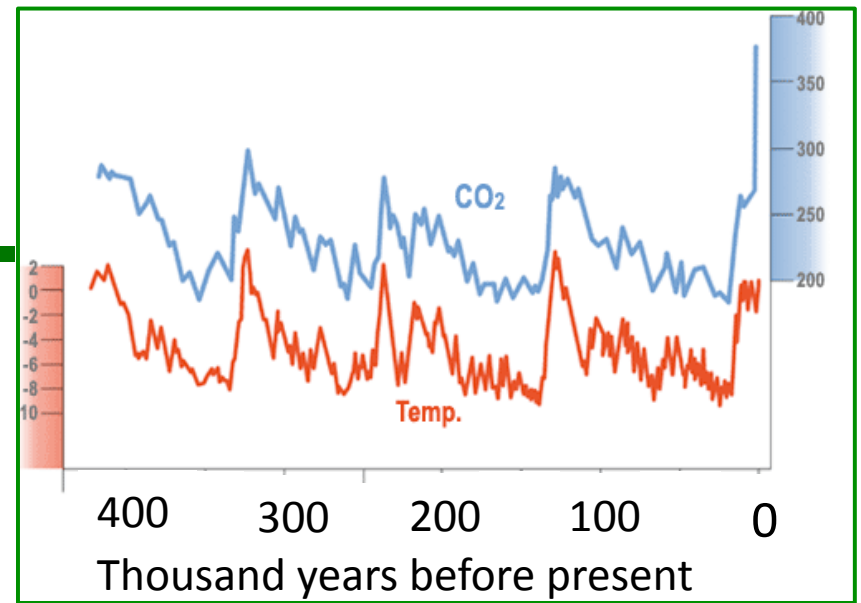
## Glacial-interglacial cycles



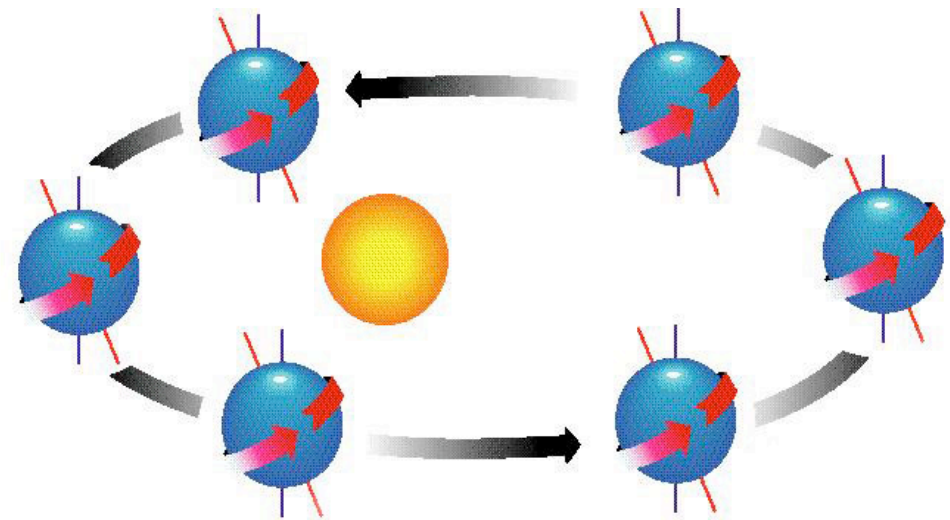


# Sun

Milankovich cycles:  
Variation in orbit & axis,  
20K, 40K and 100K years



Combine Budyko model  
with Milankovich cycles



As orbit changes slightly, amount of solar radiation received changes.

- Can explain *timing* of end of ice ages
- Cannot explain speed or amplitude of rise



# Sun



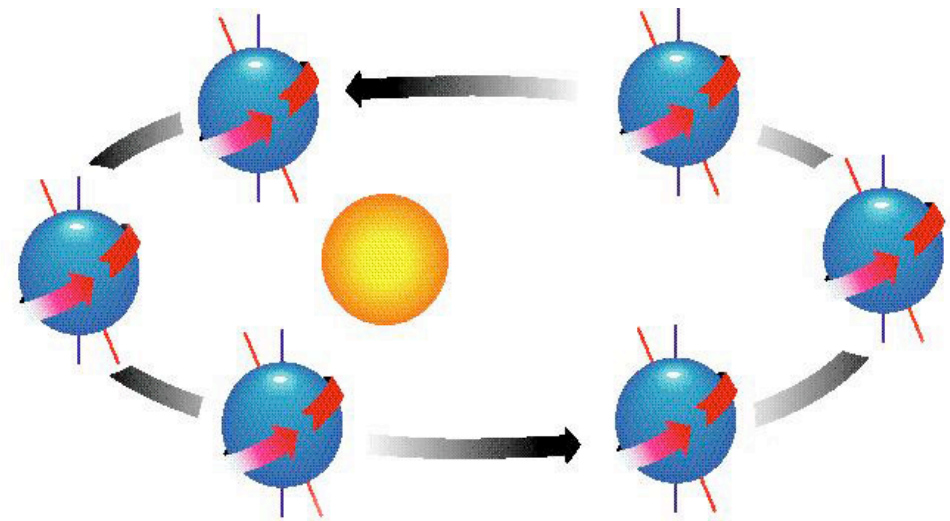
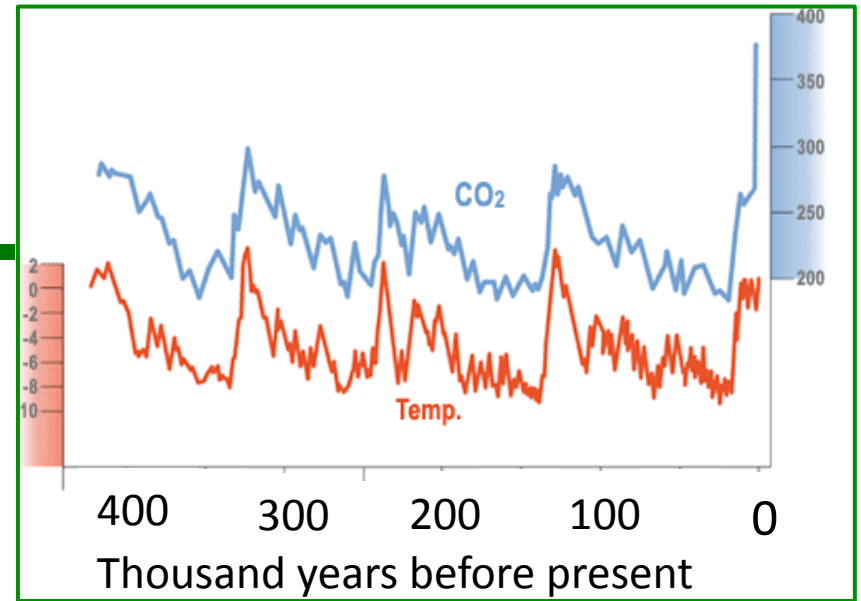
Milankovich cycles:  
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Temp

Green-  
house  
Gases

Geol.  
Carbon  
Cycle

CO<sub>2</sub>

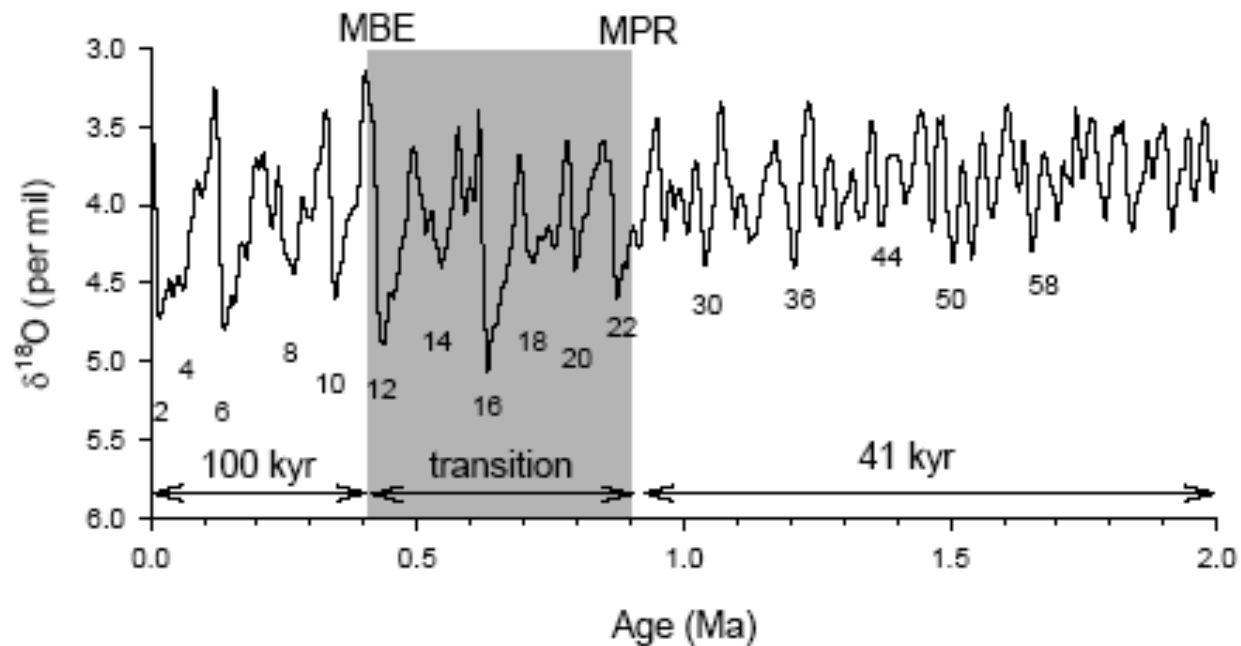


Include Temp-CO<sub>2</sub> feedback from geological carbon cycle to help understand speed and amplitude of rise.



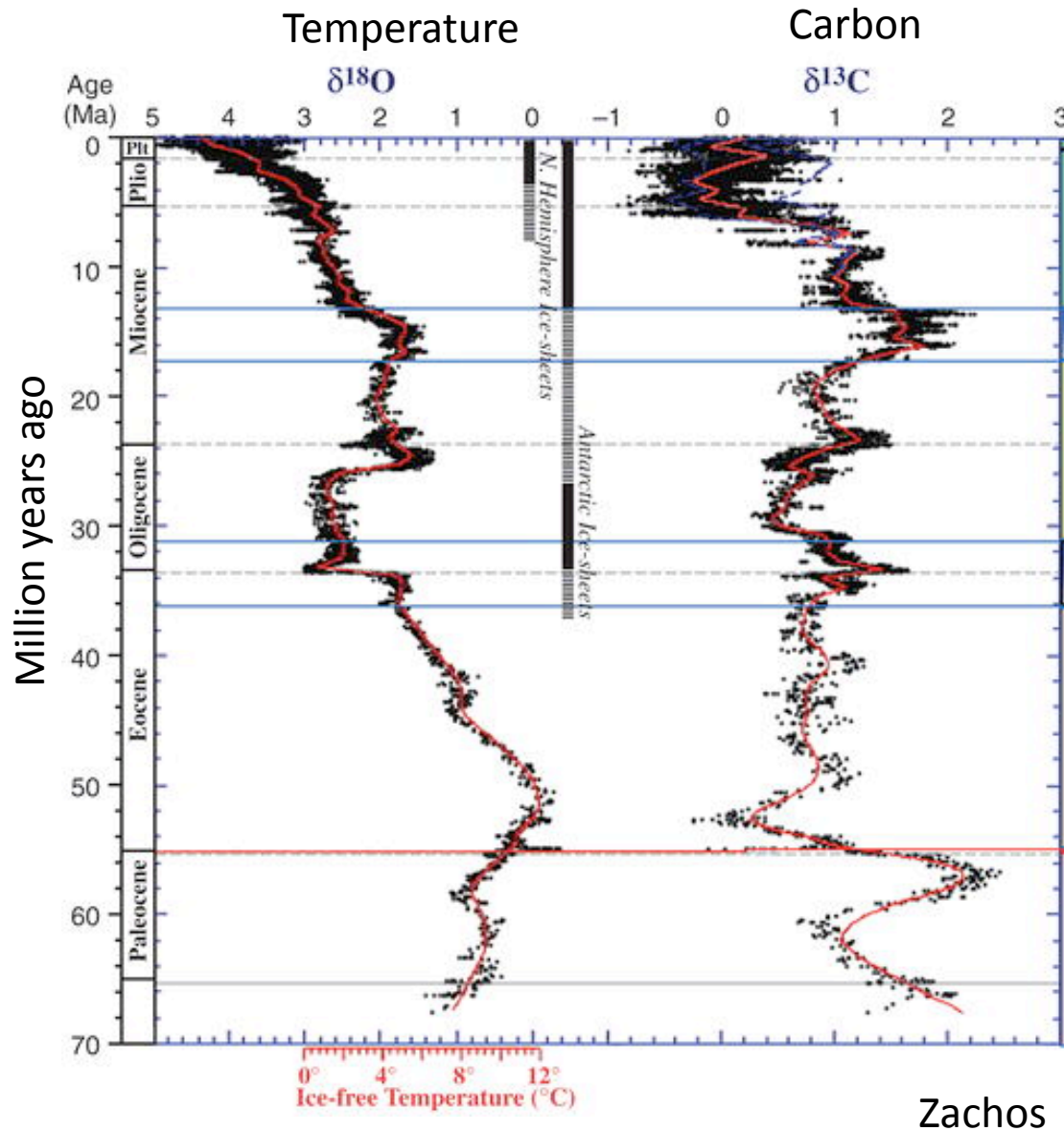
# Temperature Record - 2M yrs

## Glacial-interglacial cycles





# Paleoclimate Record – 70M years



What models capture this behavior?

Especially abrupt changes.

Instructive to develop a hierarchy of models of increasing complexity.

When do robust behaviors of simple models persist as we increase complexity?

Use different levels of model hierarchy to give insight into feedback interactions between climate processes.