

# Practical Rendering of Multiple Scattering Effects in Participating Media

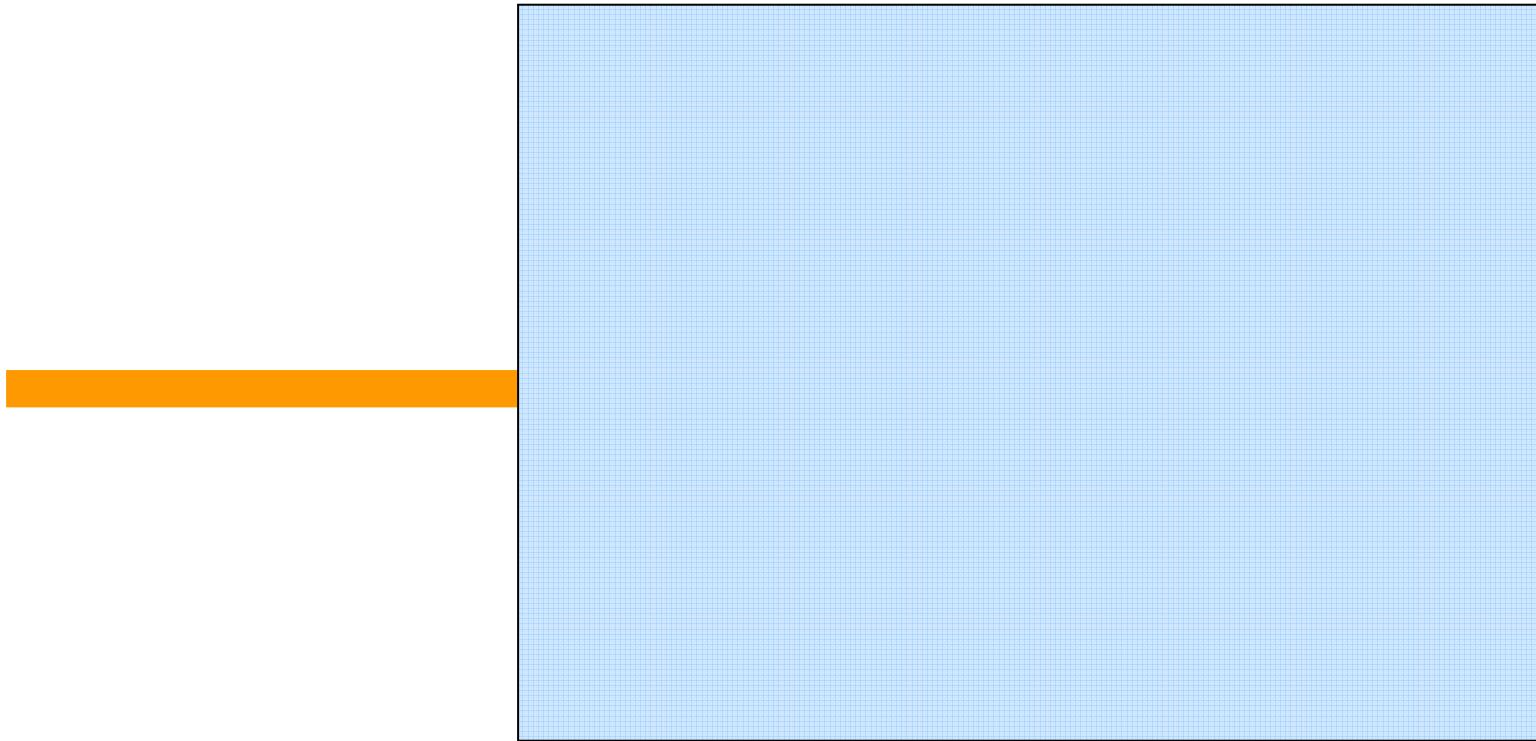
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Jerry Tessendorf, Ravi Ramamoorthi,  
and Shree Nayar

# Goal

- Want to accurately and efficiently compute effects of multiply-scattered light
- Assumption: direct and single-scattered light handled separately

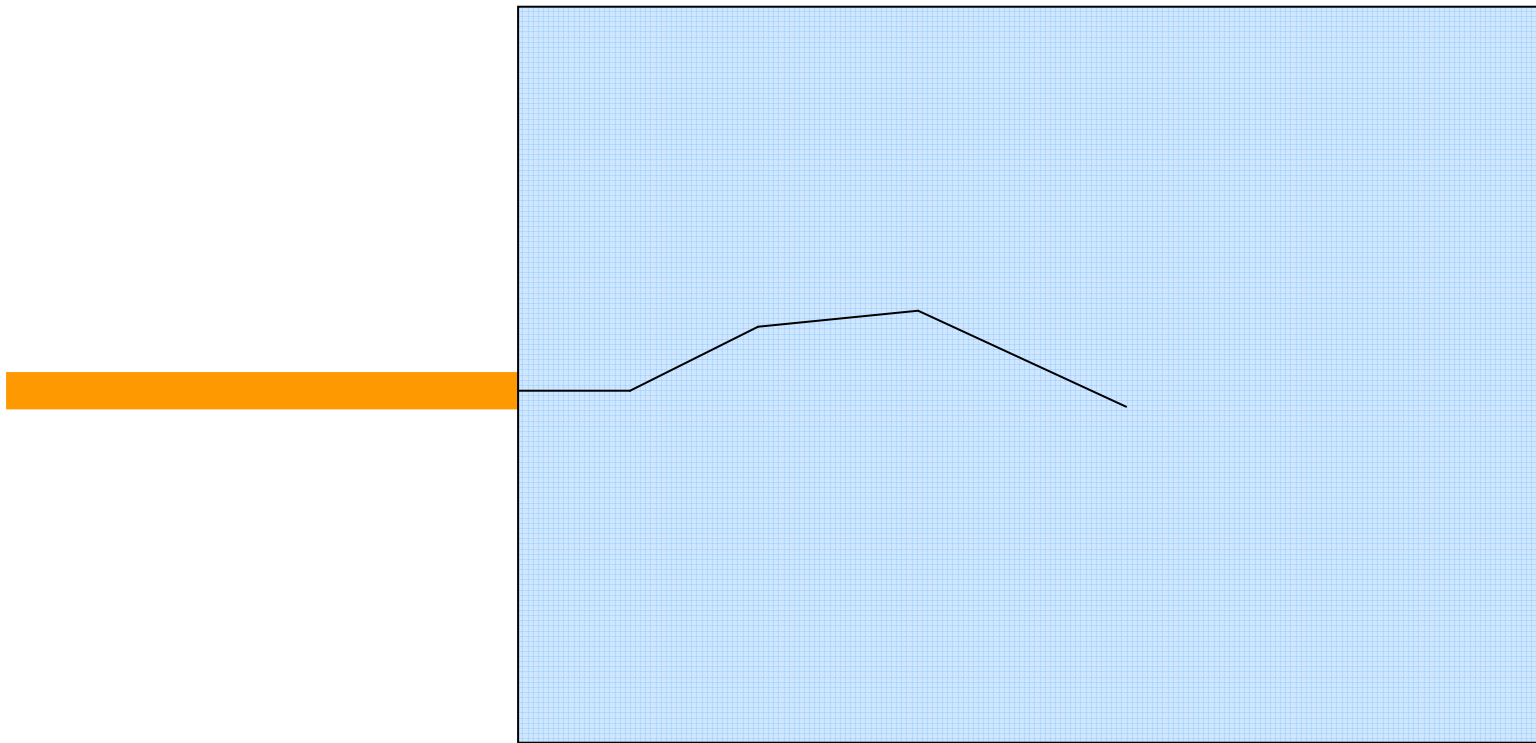
# Monte Carlo and Participating Media

- Beam of directed light



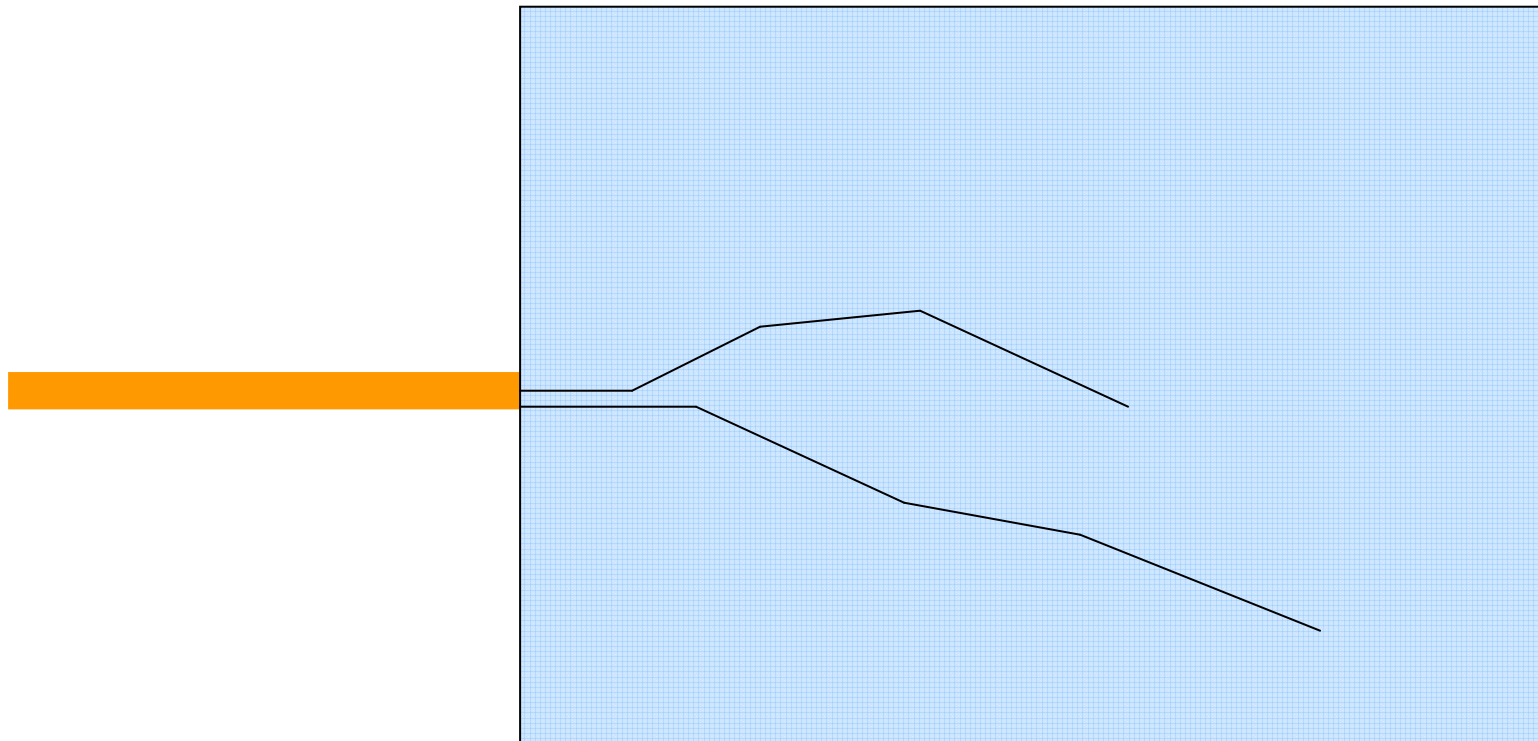
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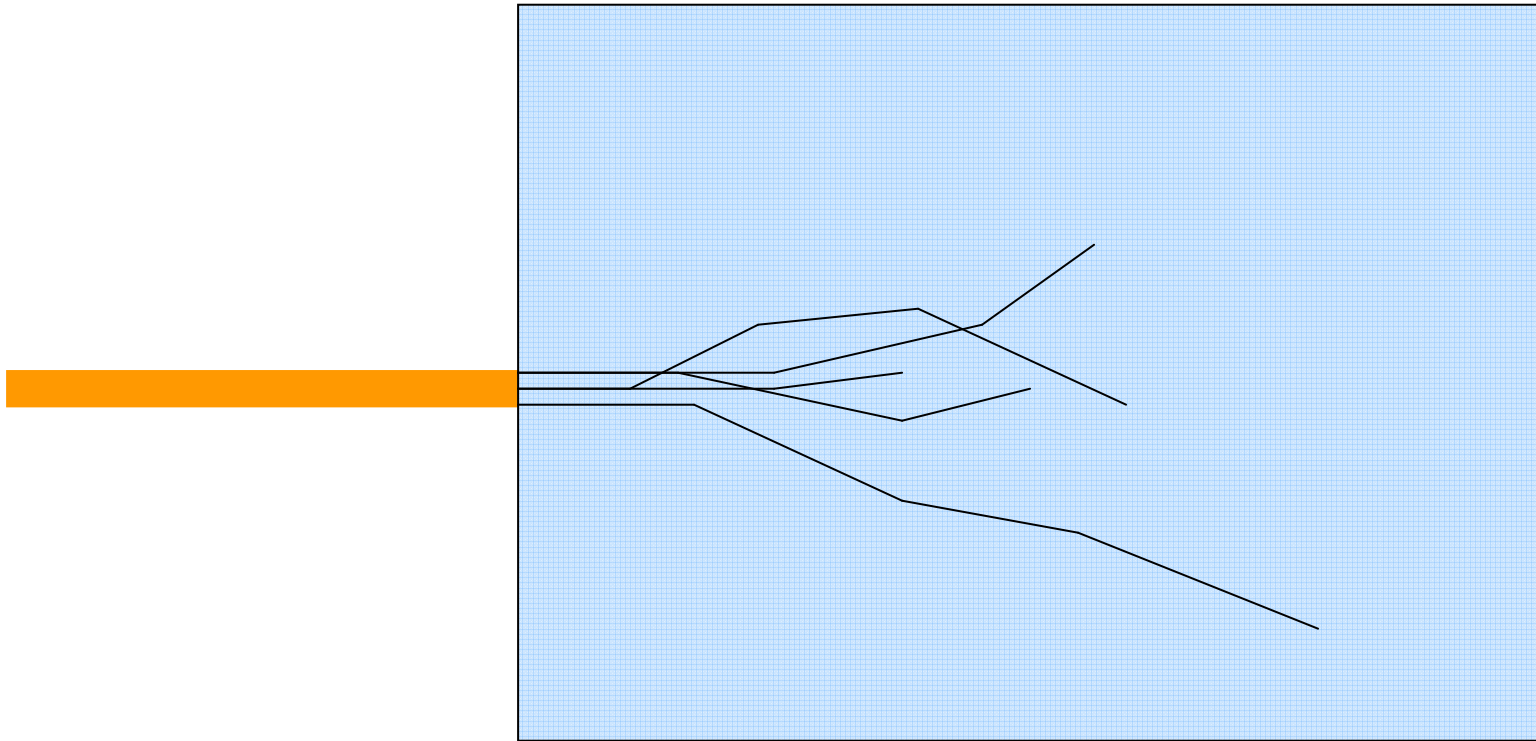
# Monte Carlo and Participating Media

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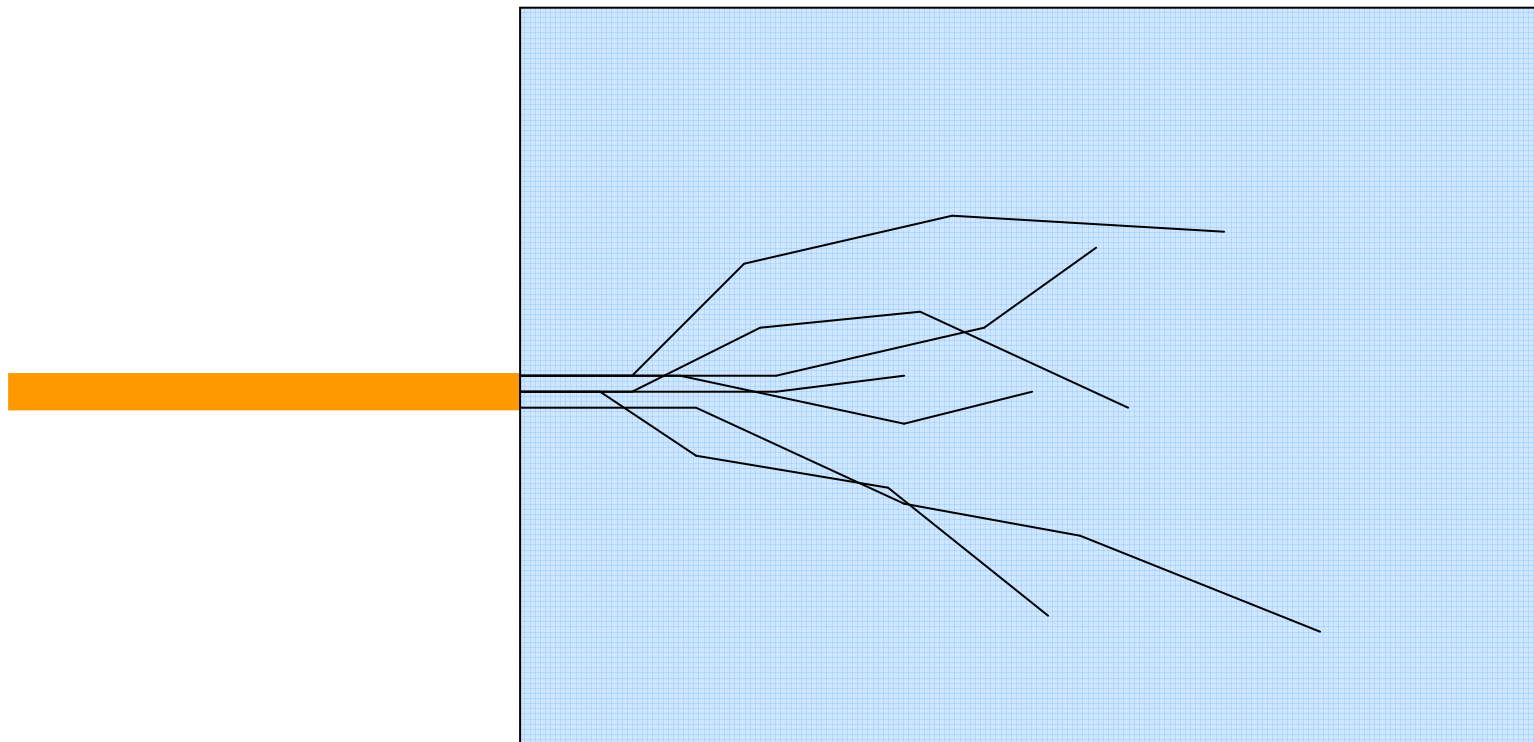
# Monte Carlo and Participating Media

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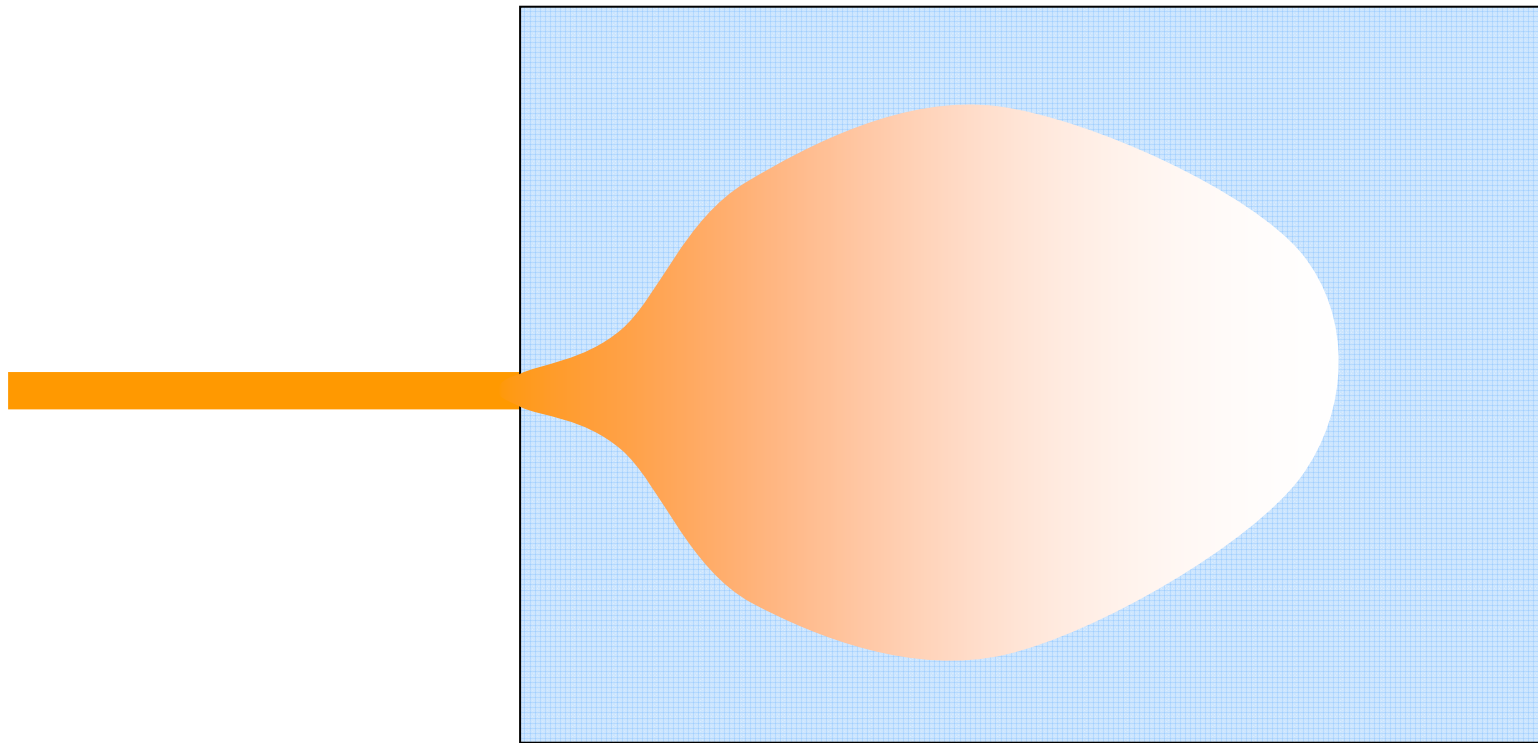
# Monte Carlo and Participating Media

- Beam of directed light



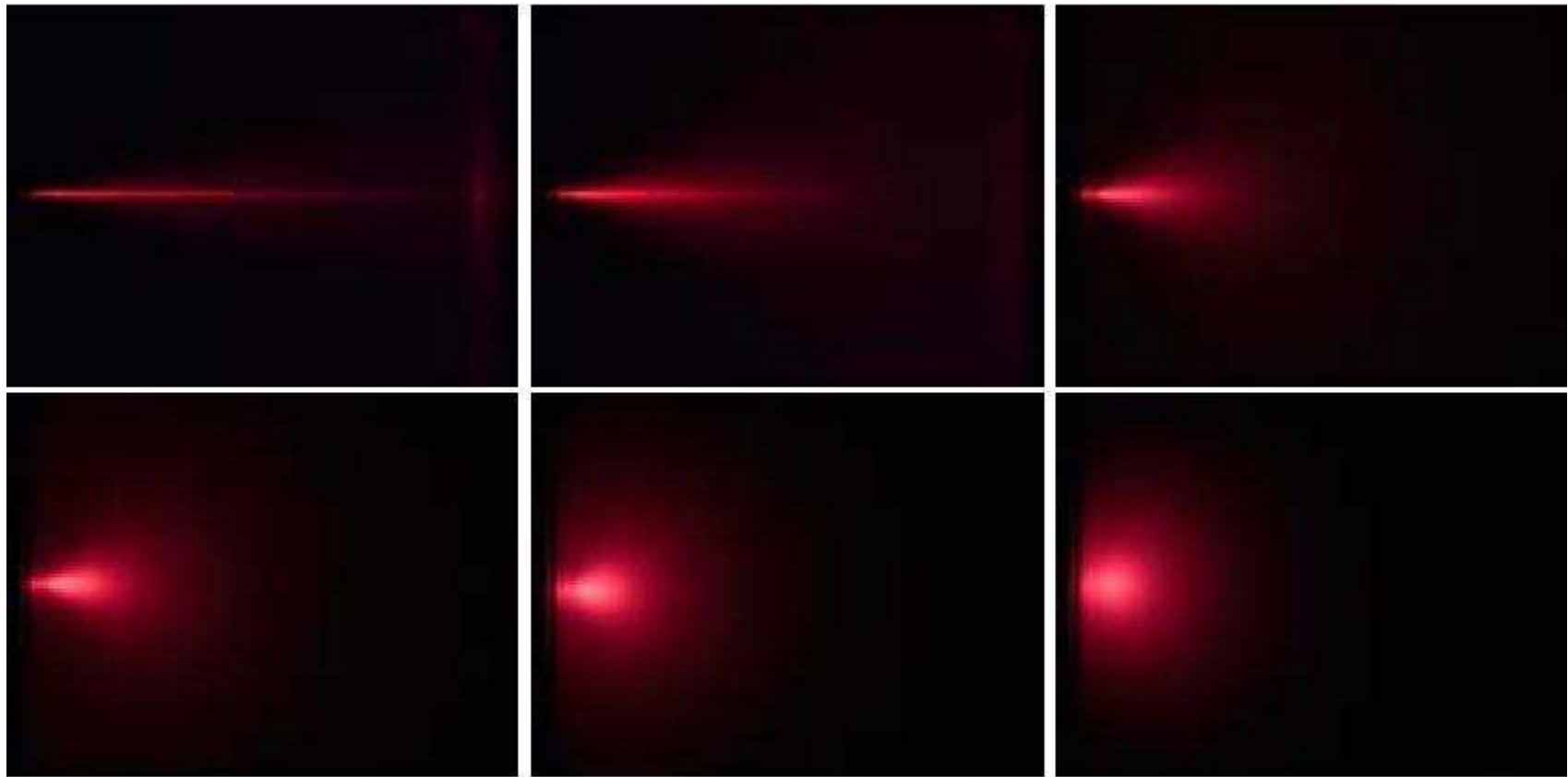
# Monte Carlo and Participating Media

- Beam of directed light





# Experimental Validation



from Premoze et al, Practical Rendering of Multiple Scattering Effects in Participating Media

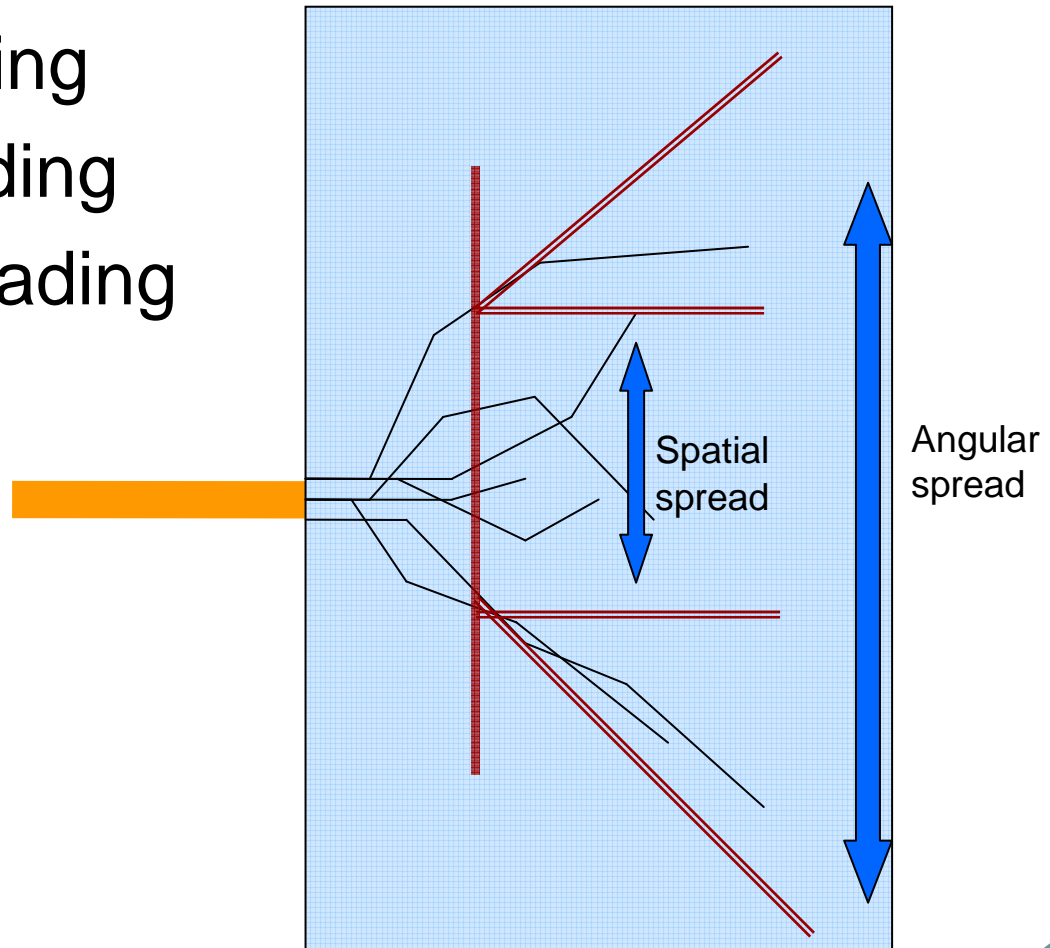
Laser beam shining into water with varying amounts of milk added

# Problems

- Very Slow
  - From a few hours to a few days
- Difficult to arrive at the obvious result

# Important features of the solution

- Spatial spreading
- Angular spreading
- Temporal spreading
  - Not as important



# Radiative Transfer Equation

- Uses alternative (but equivalent) expression for radiative transfer

$$L(s, \mathbf{x}, \vec{\omega}) = \int_{\mathbb{H}^2} \int_{\mathbb{R}^3} G(s, \mathbf{x}, \mathbf{x}', \vec{\omega}, \vec{\omega}') L_0(\mathbf{x}', \vec{\omega}') \delta \mathbf{x}' \delta \vec{\omega}'$$

- **G**: Represents light at  $(\mathbf{x}, \omega)$  due to light emitted from  $(\mathbf{x}', \omega')$  following a path of length  $s$

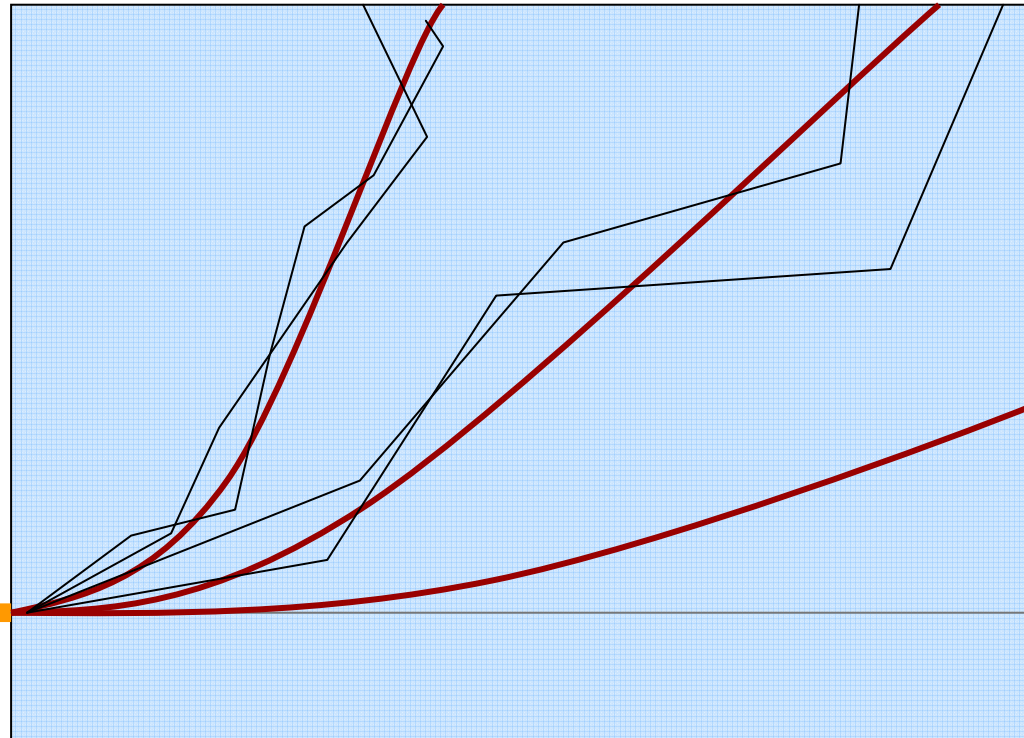
# More on the Green Propagator

- Idea: Break it into multiple pieces
  - Direct / Single scattering events ( $G_d$ )
  - Multiple scattering events ( $G_s$ )
- This paper: efficient approximation of  $G_s$

# Path Integral

- Express  $G_s$  as an integral over all paths
- Integration measure: infinite-dimensional path space (very messy math)
- Only a small number of paths are actually important
  - Call these the Most Probable Paths (MPP)
  - Sample over paths “near” to MPPs

# Most Probable Paths



# Most Probable Paths

- Account for spatial spreading:

$$w(s) = \sqrt{\frac{\langle \theta^2 \rangle l s^2}{24 \left( 1 + \frac{\sigma_a l^2 \langle \theta^2 \rangle}{12 \sigma_s} \right)}}$$

- Angular spreading?



# Inhomogeneous mediums

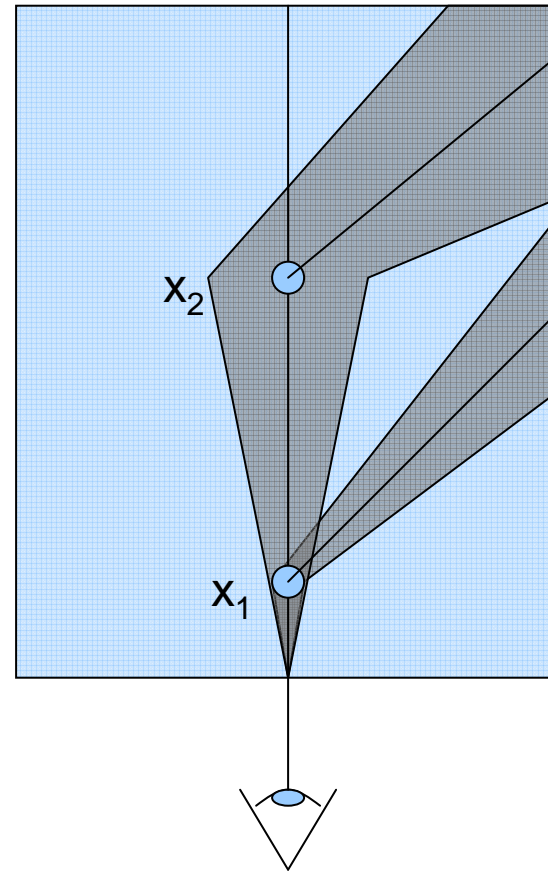
- Easy to adapt to inhomogeneous mediums (why?)
- MPPs are affected by heterogeneity

# Algorithm

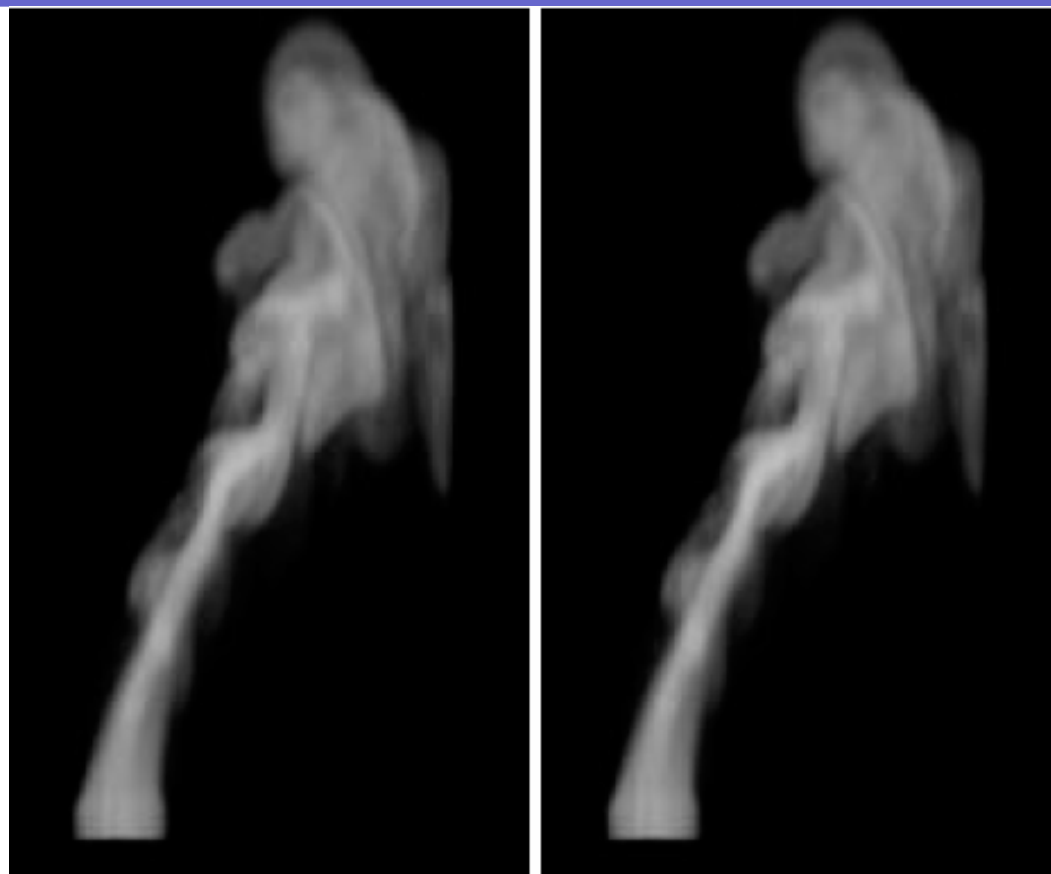
- Precomputation:
  - Light attenuation in voxel space
  - Blurred contribution per light
    - “light pyramid”
  - (Inhomogeneous mediums) Scattering coefficient  $l$  per voxel per light
  - Quick ( $\sim 10$  secs for  $128^3$  voxels)

# Algorithm

- Ray marching down viewing ray
- Approximate curved MPPs from light sources with line segment
- Calculate spatial spreading along path
- Look up multiple scatter in light volume based on spatial spread



# Results



Left: New algorithm (10 minutes). Right: Monte Carlo (3 hrs)

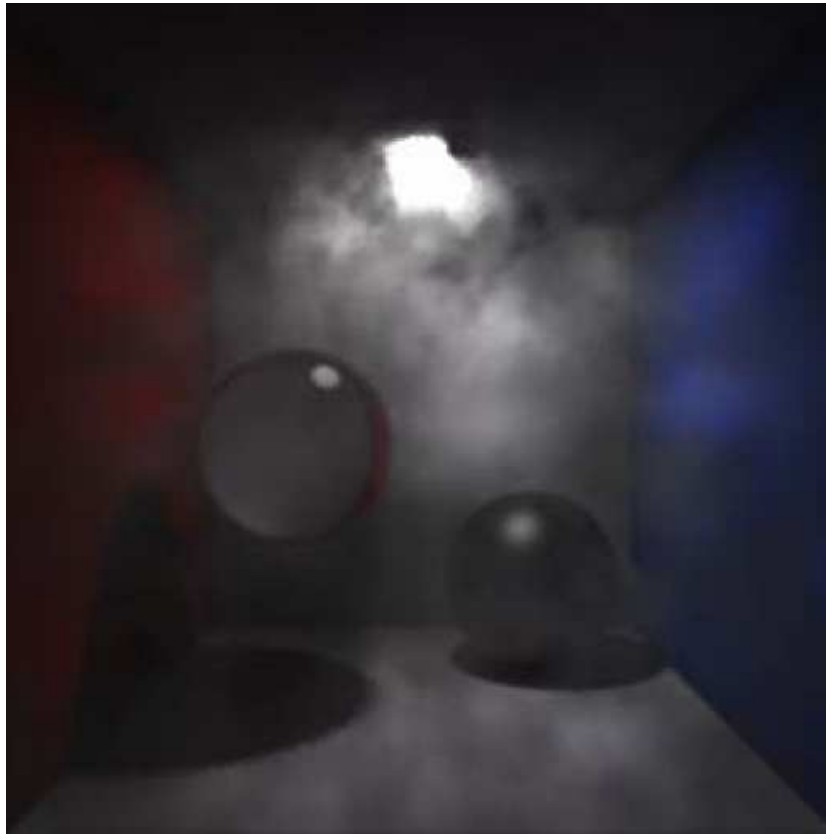
(source: Premoze et al)

# Results



Homogeneous medium (source: Premoze et al)

# Results



The requisite Cornell Box (30 min) (source: Premoze et al)

# Potential drawbacks

- Produces some noticeable differences w.r.t Monte Carlo
- Attenuation is a very rough approximation
- Precomputation per-light per-voxel
  - Prohibitively expensive for large numbers of lights?

# Conclusions

- Very fast algorithm
  - Direct / single-scatter dominates render time
- Produces very nice (but not exact) results
- Works well for both homogeneous and inhomogeneous materials
- Still need to handle direct / single-scatter light separately



Questions?

## More Math for the Strong of Heart

- Average square of the scattering angle:

$$\langle \theta^2 \rangle = 2\pi \int_0^\pi \theta^2 P(\vec{\omega}, \vec{\omega}') \sin \theta \delta\theta$$

# More Math for the Strong of Heart

- Full radiative transfer equation:

$$L(\mathbf{x}, \vec{\omega}) = \int_l dl \exp\left(-\frac{c}{b}l(MPP_l)\right) \exp(-A_c(MPP_l)) P_{MS}(l, \vec{\omega} - \vec{\omega}') \\ \times \int_{plane \perp MPP} gauss(\mathbf{x}', w(l)) L_0(\mathbf{x}', \vec{\omega}') \delta \mathbf{x}'$$