CS 667

Reflection from Layered Surfaces due to Subsurface Scattering
Pat Hanrahan, Wolfgang Krueger, 1993

Presentation overview

- Introduction
- Formulation
- First-order approximation
- Monte-Carlo algorithm
- Results

Introduction - Goals

- Model for layered surfaces such as leaves and skin
- Comprehensive shading model suitable for a wide range of materials
- More accurate diffuse reflection than Lambert model
- Correspond to physical parameters
- Algorithm easily incorporated into rendering systems

Formulation

- Radiance as BRDF and BTDF
- Fresnel coefficients
- Description of materials
- Light transport equations

Radiance as BRDF and BTDF

- Reflected radiance:

$$L_r(\theta_r, \phi_r) = L_{r,s}(\theta_r, \phi_r) + L_{r,v}(\theta_r, \phi_r)$$

Lr,s - reflected radiance due to surface scattering
Lr,v - reflected radiance due to volume or subsurface scattering

- BRDF components:

$$f_r = f_{r,s} + f_{r,v}$$

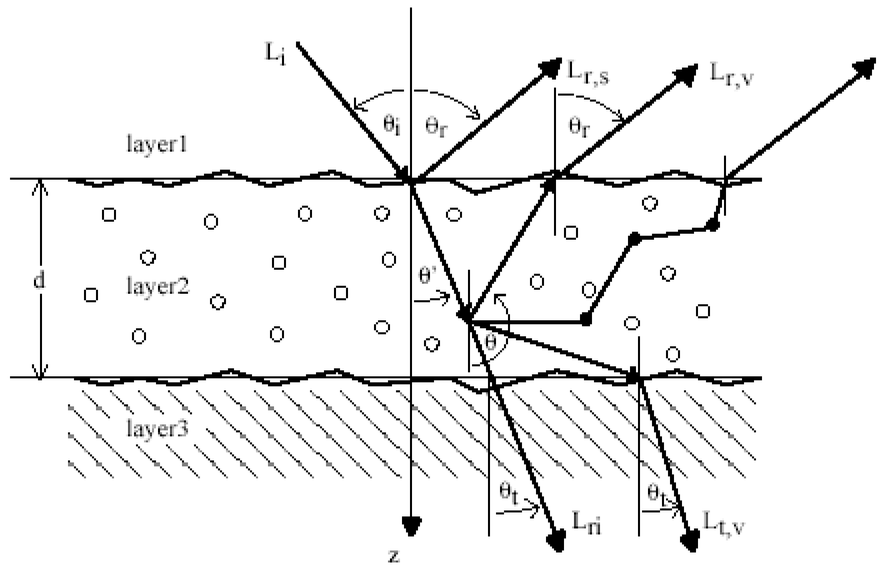
- Transmitted radiance:

$$L_t(\theta_t, \phi_t) = L_{ri}(\theta_t, \phi_t) + L_{t,v}(\theta_t, \phi_t)$$

Lri - reduced intensity
Lt,v - transmitted radiance due to volume or subsurface scattering

- BTDF components:

$$f_t = f_{ri} + f_{t,v}$$



BTDF

- Bidirectional transmission-distribution function
- "differential reflected radiance in the outgoing direction per differential incident irradiance in the incoming direction"
- Similar to BRDF except reflects transmitted distribution

$$f_t(\theta_i, \phi_i; \theta_t, \theta_t) \equiv \frac{L_t(\theta_t, \phi_t)}{L_i(\theta_i, \phi_i) \cos \theta_i d\omega_i}$$

- We can get the BRDF from traditional methods. What about BTDF?

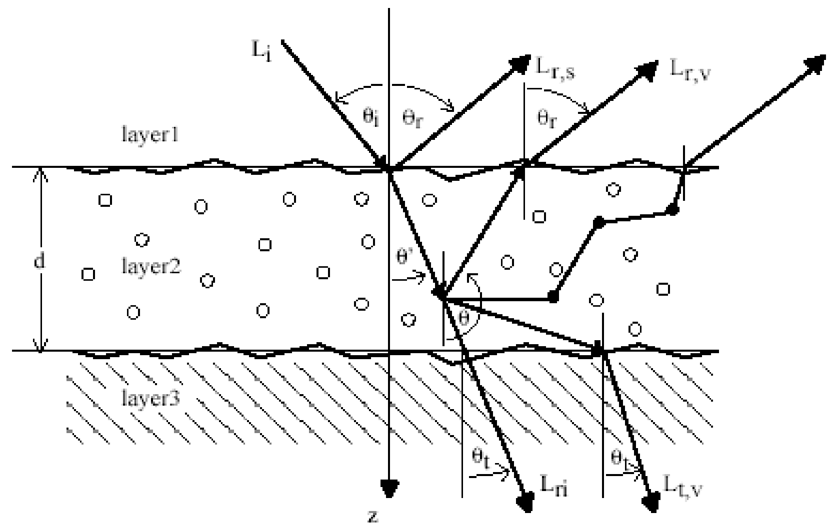
Fresnel coefficients

- For a planar surface:

$$L_r(\theta_r, \phi_r) = R^{12}(n_i, n_t; \theta_i, \phi_i \rightarrow \theta_r, \phi_r) L_i(\theta_i, \phi_i)$$
$$L_t(\theta_t, \phi_t) = T^{12}(n_i, n_t; \theta_i, \phi_i \rightarrow \theta_t, \phi_t) L_i(\theta_i, \phi_i)$$

where

$$R^{12}(n_i, n_t; \theta_i, \phi_i \rightarrow \theta_r, \phi_r) = R(n_i, n_t, \cos\theta_i, \cos\theta_t)$$
$$T^{12}(n_i, n_t; \theta_i, \phi_i \rightarrow \theta_t, \phi_t) = \frac{n_t^2}{n_i^2} T = \frac{n_t^2}{n_i^2}(1 - R)$$



- Fresnel reflection formulas which return the amount of reflection and transmission as well as their angles (Reflection Law and Snell's Law)
- Described in Ishimura's book *Wave Propagation and Scattering in Random Media*

Description of materials

- Index of refraction
- Absorption and scattering coefficients
- Henyey-Greenstein phase since can handle large particles

$$p_{HG}(\cos j) = \frac{1}{4\pi} \frac{1 - g^2}{(1 + g^2 - 2g \cos j)^{3/2}}$$

  - *j* = angle between incoming and outgoing direction
  - *g* = mean cosine of scattered light:

$$g = \int_{4\pi} p(\omega, \omega')\omega \cdot \omega' d\sigma(\omega')$$

$$g = 0 \Longrightarrow \text{isotropic scattering}$$
$$g > 0 \Longrightarrow \text{forward scattering}$$
$$g < 0 \Longrightarrow \text{backward scattering}$$

- Simplification: materials “homogeneous”

Description of materials

- Composite materials:

$$\sigma_a = \sum_{i=1}^{n} w_i \, \sigma_{a,i}$$

$$\sigma_s \, p(\cos j, g) = \sum_{i=1}^{n} w_i \, \sigma_{s,i} \, p(\cos j, g_i)$$

  - w_i = volume fraction of the volume occupied by matieral i

Light transport equations

- As previously presented:

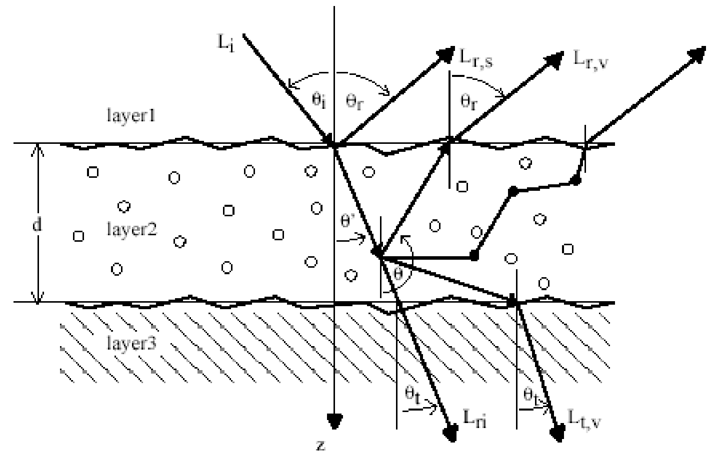
$$(\omega \cdot \nabla)L(x,\omega) = -\sigma_t(x)L(x,\omega) + \epsilon(x,\omega) + \sigma_s(x)\int_{4\pi} p(x,\omega,\omega')L(x,\omega')d\sigma(\omega')$$

- Rewritten as:

$$L(x,\omega) = \underbrace{\int_y^x \alpha(x',x)\epsilon(x',\omega)dx' + \alpha(y,x)L_e(y,\omega)}_{source} + \underbrace{\int_y^x \alpha(x',x)\sigma_s(x')\left(\int_{4\pi} p(x',\omega',\omega)L(x',\omega')d\omega'\right)dx'}_{scattering}$$

- Simplification: assume all quantities only dependent on depth (*z*) and not on *x* or *y*
- Simplification: parallel light rays
- Change how we handle scattering

Reformulation

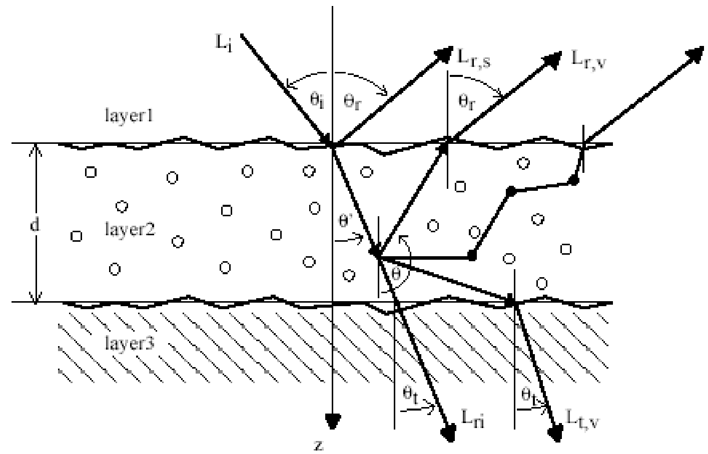


- Considering 3 layers
- Scattered radiance:

$$L(\theta, \phi) = L_{+}(\theta, \phi) + L_{-}(\pi - \theta, \phi)$$

- Boundary conditions for the equation
  - Top
  - Bottom

Top boundary



- Forward radiance:

$$L_+(z=0;\theta',\phi') = \int f_{t,s}(\theta_i,\phi_i;\theta',\phi')\,L_i(\theta_i,\phi_i)\,d\omega_i$$

  Planar surface:

$$L_+(z=0;\theta',\phi') = T^{12}(n_i,n_t;\theta_i,\phi_i \to \theta',\phi')L_i(\theta_i,\phi_i)$$

- Backward radiance:

$$L_{r,v}(\theta_r,\phi_r) = \int f_{t,s}(\theta,\phi;\theta_r,\phi_r)\,L_-(z=0;\theta,\phi)\,d\omega$$

  Planar surface:

$$L_{r,v}(\theta_r,\phi_r) = T^{21}(n_i,n_t;\theta,\phi \to \theta_r,\phi_r)L_-(z=0;\theta,\phi)$$

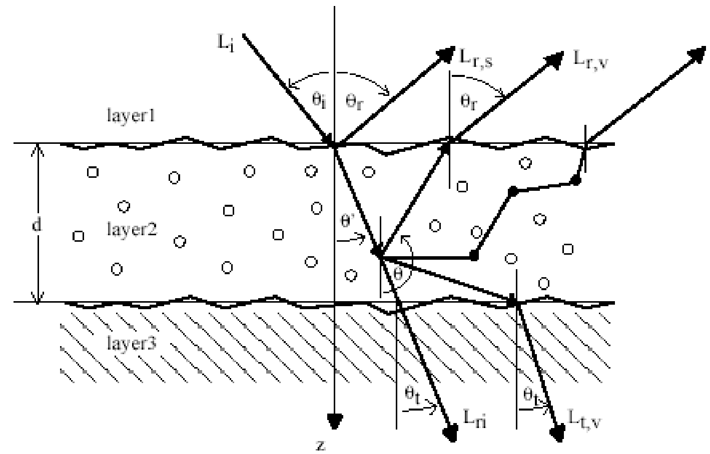
Bottom boundary

- Forward radiance:

$$L_{t,v}(\theta_t, \phi_t) = \int f_{t,s}(\theta, \phi; \theta_t, \phi_t)\, L_{+}(z = d; \theta, \phi)\, d\omega$$

Planar surface:

$$L_{t,v}(\theta_t, \phi_t) = T^{23}(n_2, n_3; \theta, \phi \rightarrow \theta_t, \phi_t) L_{+}(z = d; \theta, \phi)$$



First-order approximation

- Solving with Neumann sums
- 0th-order solution
- 1st-order solution
- Using 1st-order to approximate

Neumann sums

- Radiance as a sum of scattering events (Chandrasekhar):

$$L = \sum_{i=0}^{\infty} L^{(i)}$$

- Each term *i + 1* is an integral of the *i*th term:

$$L^{(i+1)}(z;\theta,\phi) = \int_0^z e^{-\int_0^{z'} \sigma_t \frac{dz''}{\cos\theta}} \int \sigma_s(z')\, p(z';\theta,\phi;\theta',\phi')\, L^{(i)}(z';\theta',\phi')\, d\omega' \frac{dz'}{\cos\theta}$$

0th-order solution

- No scattering events
- Only factors are Fresnel transmission and attenuation:

$$L_{t,v}^{(0)}(\theta_t, \phi_t) = T^{12} T^{23} e^{-\tau_d} L_i(\theta_i, \phi_i) \qquad \tau(z) = \int_0^z \sigma_t \, dz$$

Attenuation = $e^{-\tau_d}$

1th-order solution

- 1 scattering event
- Reflected radiance

$$L^{(1)}_{r,v}(\theta_r, \phi_r) = WT^{12}T^{21}p(\pi-\theta_r, \phi_r; \theta_i, \phi_i)\frac{\cos\theta_i}{\cos\theta_i+\cos\theta_r}(1-e^{-\tau_d(1/\cos\theta_i+1/\cos\theta_r)})L_i(\theta_i, \phi_i)$$

- For “semi-infinite” layers (optical depth = infinity):

$$L_{r,v}(\theta_r, \phi_r) = \frac{\cos\theta_i}{\cos\theta_i + \cos\theta_r}L_i(\theta_i, \phi_i)$$

1th-order solution

- 1 scattering event
- Transmitted radiance

$$L_{t,v}^{(1)}(\theta_t, \phi_t) =$$

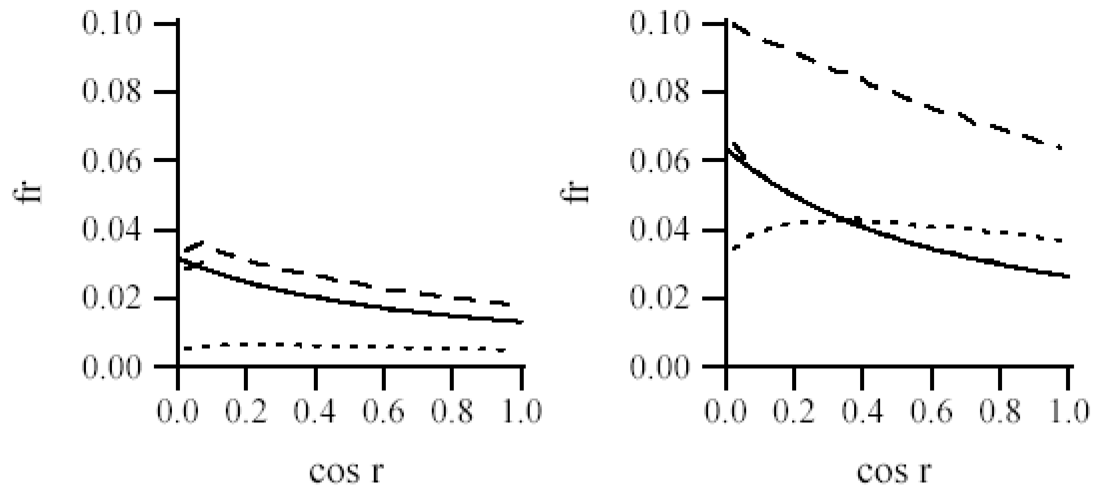
$$WT^{12}T^{23}p(\theta_t,\phi_t;\theta_i,\phi_i)\frac{\cos\theta_i}{\cos\theta_i-\cos\theta_t}(e^{-\tau_d/\cos\theta_i}-e^{-\tau_d/\cos\theta_t})L_i(\theta_i,\phi_i)$$

- For single layers (refraction of 1 = refraction of 3)

$$L_{t,v}^{(1)}(\theta_t, \phi_t) = WT^{12}T^{23}p(\theta_t, \phi_t; \theta_t, \phi_t)\frac{\tau_d}{\cos\theta_t}e^{-\tau_d/\cos\theta_t}L_i(\theta_t, \phi_t)$$

Using the 1st-order to approximate

$$L_{r,v}(\theta_r, \phi_r) = L^{(1)}(\theta_r, \phi_r) + L^m$$



Above: Graphs of the BRDF (*fr*) as a function of the angle of reflection for a semi-infinite slab with different albedos (on the left W = 0:4 and on the right W = 0:8) and an angle of incidence of 45. The solid line is the theoretical BRDF as given by Seeliger's Law (the superimposed dashed line is the computed 1st-order BRDF showing a good match). The top dashed curve is the total computed BRDF; The bottom dotted curve is the difference between the total BRDF due to multiple scattering events and the 1st-order BRDF.

The Monte-Carlo algorithm

- Compute the BRDF/BTDF
- Initialize
- Events
  - Step
  - Scatter
- Score

The Monte-Carlo algorithm

- Compute the BRDF/BTDF
- Initialize
- Events
  - Step
  - Scatter
- Score
- Adding it to an existing ray tracer

The Monte-Carlo algorithm

- Repeat calculation over various angles
- Can be thousands of samples

The Monte-Carlo algorithm

- Events - Repeat the following steps until the ray weight drops below some threshold or the ray exits the layer.

2A *Step*: First, estimate the distance to the next interaction:

$$d = -\frac{\log r}{\sigma_t}$$

Where r in this and the following formulas is a uniformly distributed random number between 0 and 1. Then, compute the new position:

$$\vec{p} = \vec{p} + d\,\vec{s}$$

And, finally set the particle weight to

$$w = w\frac{\sigma_s}{\sigma_s + \sigma_a}$$

Note: If d causes the particle to leave the layer, break from the repeat loop and adjust the weight using the distance to the boundary.

The Monte-Carlo algorithm

- Events - Repeat the following steps until the ray weight drops below some threshold or the ray exits the layer.

2B *Scatter*: First, estimate the cosine of the scattering angle for the Henyey-Greenstein phase function using the following formula.

$$\cos j = \frac{1}{|2g|}(1 + g^2 - (\frac{1 - g^2}{1 - g + 2gr})^2)$$

and $\cos\phi$ and $\sin\phi$ with $\phi = 2\pi r$. Then, compute the new direction:

$$\vec{t} = \begin{pmatrix} (\vec{s}.x \cos\phi \cos\theta - \vec{s}.y \sin\phi)/\sin\theta \\ (\vec{s}.y \cos\phi \cos\theta + \vec{s}.x \sin\phi)/\sin\theta \\ \sin\theta \end{pmatrix}$$

$$\vec{s} = \vec{s}\cos j + \vec{t}\sin j$$

Here, $\cos\theta = \vec{s}.z$ and $\sin\theta = \sqrt{1 - \vec{s}.z^2}$. Note: Care must be taken if $\sin\theta = 0$.

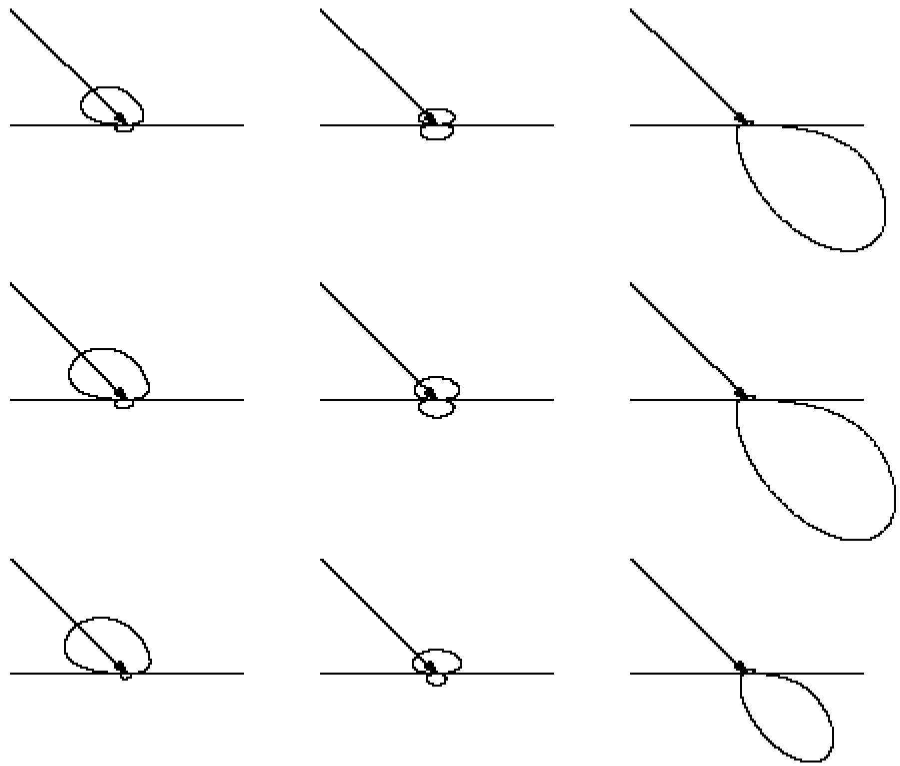
The Monte-Carlo algorithm

- Score - Divide the sphere into regions of equal solid angle and add the weight of the particle to the weight associated with the bin in which it is contained.

The Monte-Carlo algorithm

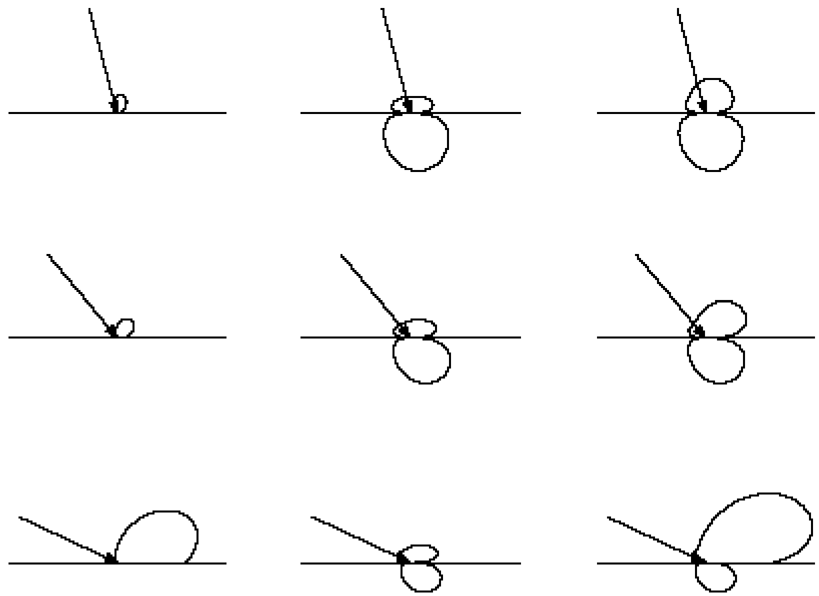
- How to incorporate into an existing ray tracer
  - Call subsurface procedure when necessary
  - Send a ray to the light at each scattering event
  - Weight by phase function and attenuation
  - Benefit is no precomputation necessary
  - Introduces bias

Results



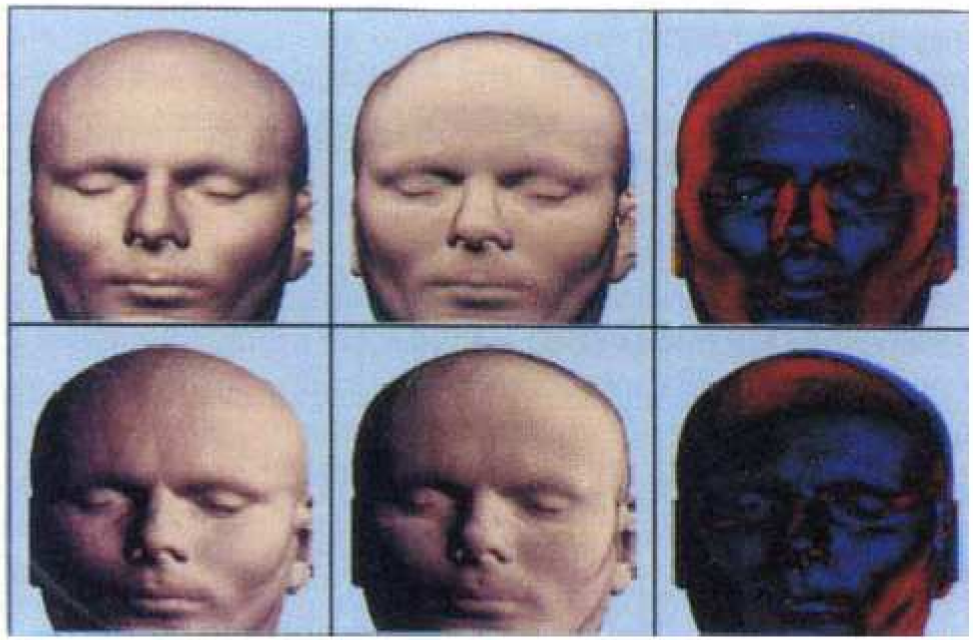
Above: First-order solutions for subsurface reflectance and transmission for different values of g and d. From left to right the phase function shifts from predominately backward scattering (g = 0:3) to isotropic scattering (g = 0:0) to forward scattering (g = 0:6). From top to bottom the optical depth of the layer increases from 0.5 to 1.0 to 2.0.

Results



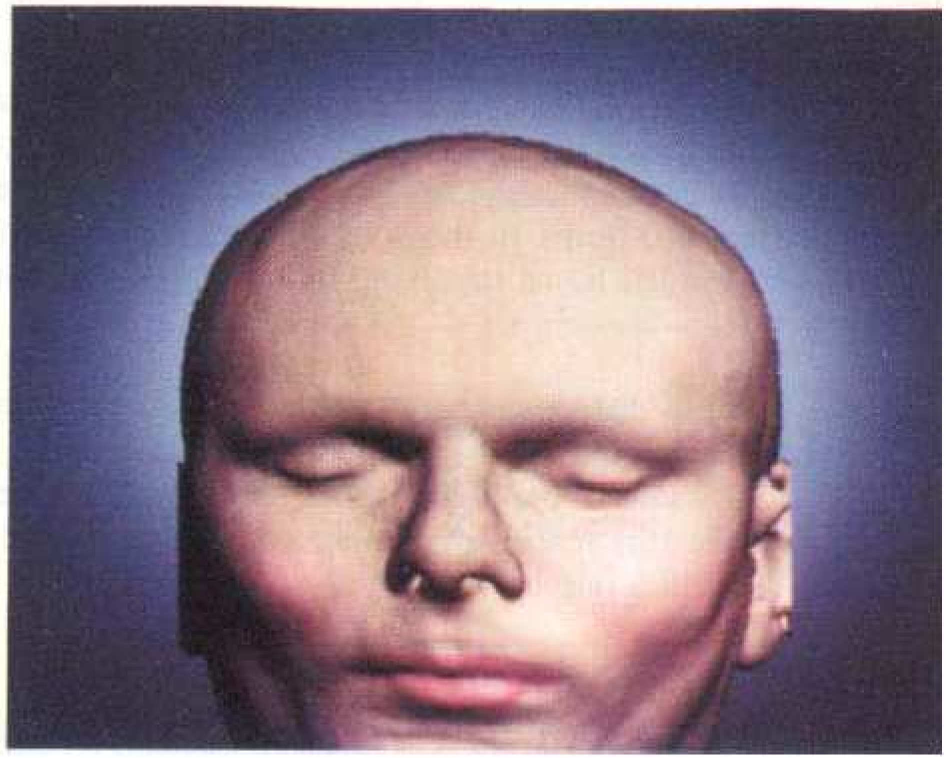
Above: Solutions for BRDF and BTDF. In the left column is the surface specular reflection and in the middle is the subsurface reflection and transmission. On the right is the sum of surface and subsurface modulated by the Fresnel coefficients. From top to bottom the angle of incidence increases from 10 to 40 to 65 degrees.

Results



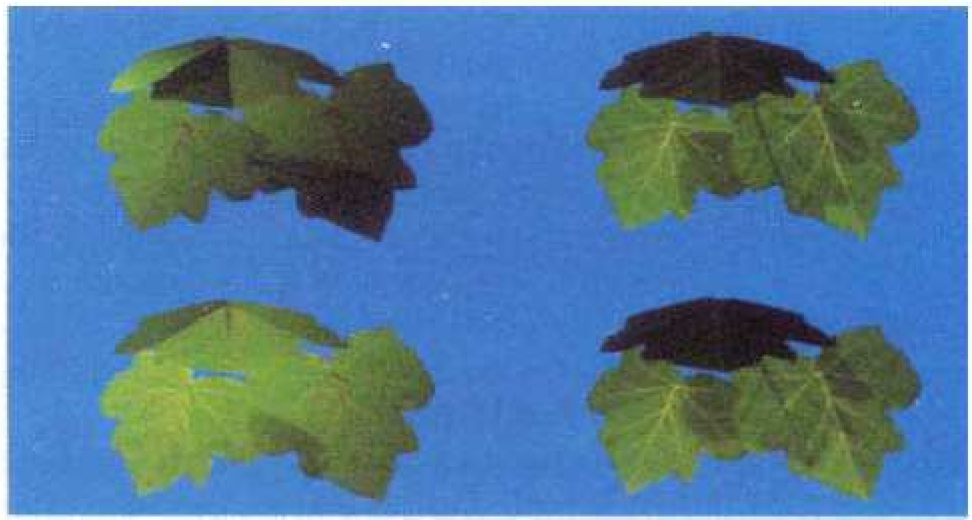
The left column shows the results for the Lambert scattering for angles 0 and 45 degrees, and the middle column is rendered for the new model. The right column gives the relative difference of both models, red indicates more reflection from the new model, and blue vice versa.

Results



Human face with variation in subsurface blood concentration, an oily outer layer and Gaussian variation in parameters to create the "freckles."

Results



A cluster of leaves. A series of leaf images under different simulated lighting conditions. On the left are two backlit images, on the right, front lit.

Final Comments

- Model relevant to materials with low indices of refraction
- Parameters of model can be determined from experiments
  - But does the model correctly reflect these parameters?
- Layering can potentially model very complex surfaces