CS667 **Lecture 16: Fluid Simulation in Computer Graphics** 7 April 2005

Andrew Butts Lecturer: Steve Marschner

1 Introduction

In a computer graphics setting, the motion of fluids is difficult to achieve by hand or with simple procedures. Robustly and realistically simulating phenomena such as billowing smoke, water waves, and splashes requires solving the *Navier-Stokes equations*. Modern hardware is now cheap and fast enough to allow computer graphics folks to import fluid simulation methods from computational physics.

2 Some Fluid Terms

Here are some definitions of terms that pop up in the computer graphics fluid simulation literature.

"**Advect**" means to evolve some quantity forward in time given a velocity field, especially particle positions or masses defined on a grid. Just move the quantities according to the velocity field.

"**Lagrangian**" refers to methods which explicitly move fluid mass around, for example by advecting particles or triangle meshes.

"**Eulerian**" refers to methods in which all fluid quantities (mass, density, velocity) are defined on a grid. The quantities attached to the grid vertices or cells vary through time, but the grid remains fixed.

3 History of Methods in Graphics

The trend in fluid simulation for computer graphics has been toward models that are closer and closer to those used by the computational physics community. The classic "if it looks good enough, it is good enough" reasoning still applies, allowing CG folks to import computational physics methods while making simplifying/optimizing assumptions. But as demands for realism and desktop computational power increase, CG folks have relaxed these assumptions:

Random velocity fields: A barebones hack. Generate a 3D grid of random vectors and interpolate them to produce a velocity field. Good for Lagrangian advection of leaves or other particle-like objects that do not require much global fluid-like behavior.

Vortex methods: Like the random velocity field technique, but forces the velocity field to contain swirly fluid-like vortices by projecting an arbitrary velocity field into a divergence-free one.

Shallow water equations: Solves a simplified version of the Navier-Stokes equations, assuming the water configuration can always be expressed as a height field.

Full Navier-Stokes equations: Solves the Navier-Stokes equations fully in 3D, with or without surface boundary conditions. Captures a large range of unrestricted fluid motions (waves, swirly vortices, splashing). The equations themselves govern the flow of fluid within a fluid volume. All the really interesting motions arise as a result of the choice of boundary conditions, such as interactions with solid objects and other fluids.

4 Ultra-brief Intro to Fluid Mechanics

Fluid mechanics and dynamics is a two-semester sequence. We'll only gloss over the critical points, and only concern ourselves with one case: **viscous, incompressible fluids**.

We will not worry about:
Effects of molecular/statistical nature of fluids. *We only care about macroscopic effects.*
Compression, an increase in density due to pressure. *We're generally interested in low speeds.*
Discontinuities (shock waves). *Everything in sight is nice and differentiable.*

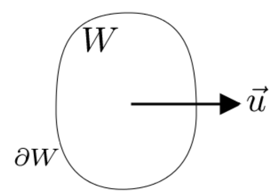
Physical basis for fluid flow

$$\begin{aligned} \text{velocity } \vec{u}(\vec{x}) \ (3 \text{ DOF}) \\ \text{density } \rho(x) \ (1 \text{ DOF}) \\ \text{pressure } p(x) \ (1 \text{ DOF}) \end{aligned}$$

Fluid is subject to the principles of:

Conservation of mass. *Advecting mass through the velocity field cannot change total mass.*
Balance of momentum. *Sum of internal and external forces = change in momentum.*
Conservation of energy. *The sum of kinetic and internal energy must remain constant.*

Conservation of Mass



To discuss conservation of mass of a velocity field $\vec{u}$, we can define a region of space W with boundary δW and boundary normal $\vec{n}$. If $\vec{u}$ and a density field ρ are defined for every point in W, then:

$$\begin{aligned} \text{Mass } &= \int_W \rho dW \\ \frac{\partial}{\partial t}\text{Mass } &= -\int_{\partial W} \rho(\vec{u}\cdot\vec{n})dA \end{aligned}$$

The integral over ∂W of $\vec{u}$'s flow through it is positive when mass is *leaving* W, hence the negative sign to express change in mass contained in W. The limit of $\frac{\partial}{\partial t}$Mass as W shrinks to a point yields a PDE:

$$\rho_t = -\rho(\nabla\cdot\vec{u})$$

since the integral of $\vec{u}$ over ∂W for small W is precisely the divergence $\nabla\cdot\vec{u}$. For incompressible fluids, the density ρ is constant, yielding instead

$$\nabla\cdot\vec{u} = 0$$

Balance of Momentum

A fluid volume element is subject to external forces (i.e. gravity) and internal forces (strain). In an ideal fluid (zero viscosity), there is no *shear strain*, leaving pressure as the only internal force, given by the following equation (note its similarity to $F = ma$):

$$F_{\text{internal}} = \rho \frac{D\vec{u}}{Dt}$$
$$-\nabla p + \rho \vec{g} = \rho \frac{D\vec{u}}{Dt}$$

where $\vec{g}$ is the acceleration due to gravity. The negative sign in front of the pressure gradient is explained by the tendency of fluid mass to flow from high-pressure regions to low-pressure regions. $\frac{D}{Dt}$ is the *convective* or *Lagrangian* derivative. It differs from the ordinary derivative $\frac{d}{dt}$ in that it expresses the change over time of some characteristic of a fluid element, which is generally moving, advected by $\vec{u}$. In contrast, $\frac{d}{dt}$ expresses the change of some characteristic at a *fixed position*. The operator is defined as $\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{u} \cdot \nabla$.

In a viscous (non-ideal) fluid, a new dissipative term appears, generally acting to blur $\vec{u}$ over time:

$$-\frac{\nabla p}{\rho} + \nu \nabla^2 \vec{u} + \vec{g} = \frac{D\vec{u}}{Dt}$$

Conservation of Energy

Energy in a fluid can be:
Kinetic: Mass is moving
Internal: Potential energy due to compression, or heat due to kinetic energy dissipated by viscosity

Ideal fluids exhibit no compression or viscosity. The assumption of only kinetic energy is equivalent to assuming an ideal fluid.

Counting DOFs

Fluid has 5 degrees of freedom (in 3D):
velocity on 3 axes
pressure (a scalar field)
density (a scalar field)

Constraints on the system are provided by equations:
mass conservation: 1 equation
momentum balance (F=ma): 3 equations (1 vector equation)
energy conservation / incompressible assumption: 1 equation

5 Fluid Boundary Conditions

Solid walls
Normal component of fluid velocity is zero, so no mass can move through the wall.
For ideal fluids, there is no viscosity (drag) at the wall. Fluid freely slips past.
For viscous fluids, the tangential component of fluid velocity is zero as well (no-slip boundary).

Free surface

The typical fluid interface boundary condition in computer graphics is fluid-nothing instead of fluid-fluid (the fluid nature of air is not considered). That is, the region containing the fluid is explicitly tracked and the fluid motion is computed only within this region.

Volume-of-fluid tracking: in a 3D grid, each cell contains a scalar describing how much fluid volume is present in the cell

Mesh tracking: explicitly track the evolving interface with a polygonal mesh.

Particle fluid: advect dense sets of particles along with the fluid, representing fluid mass.

Level sets: advect a signed distance function to the fluid interface. Tends to lose volume due to small features getting blurred out by the grid the function is discretized on. The volume loss problem has been addressed by also advecting fluid particles and periodically recomputing the signed distance function.

Surface tension, in reality a result of an imbalance of inter-particle forces at the fluid interface, may be approximated with a force defined on the interface that is proportional to the mean curvature of the interface. Such a force would have a tendency to smooth out the surface.

6 Miscellaneous phenomena

It's worth noting that the typically fluid-like swirly-vortex phenomenon only happens in viscous fluids, arising because of the blurring effect viscosity has on fluid velocities.

Turbulence is a chaotic motion that can occur in a fluid with a high *Reynolds number* R:

$$R = \frac{LV}{\nu}$$

where ν is the viscosity constant (larger is thicker) and L and V are characteristic lengths and velocities, roughly indicating the scale at which the fluid is observed and the speed of typical motions.